

1. Consider a continuous time Markov Chain (CTMC) on state space $\{1, 2, 3\}$ with the following rate matrix

$$Q = \begin{bmatrix} -3 & 2 & 1 \\ 3 & -7 & 4 \\ 2 & 5 & -7 \end{bmatrix}.$$

- (a) Starting from state 1 at time $t = 0$, what is the probability that the CTMC does not make any jump up to time $t = 10$?
- (b) Starting from state 2 at time $t = 0$, what is the probability that the CTMC makes its first jump to state 1?
- (c) Starting from state 2 at time $t = 0$, what is the probability that the CTMC makes its first jump to state 1 AND the first jump happens before time $t = 10$?
2. Consider a discrete time MC $(Y_n)_{n \geq 0}$ on the state space $\{1, 2\}$ with the transition matrix

$$P = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$$

and an independent Poisson process $(N(t))_{t \geq 0}$ with rate 10. Find the rate matrix Q of the CTMC $(X_t)_{t \geq 0}$ defined by

$$X_t = Y_{N(t)}.$$