Math 2263
Fall 2014
Midterm 1
October 2, 2014
Time Limit: 50 minutes

Name (Print): _____
Student ID: _____
Section Number: 001 _____
Teaching Assistant: _____
Signature: _____

This exam contains 7 problems. Answer all of them. Point values are in parentheses. You must show your work to get credit for your solutions - correct answers without work will not be awarded points.

Do not give numerical approximations to quantities such as $\sin 5$, π, $\ln(3)$ or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{2} = 0$, $e^0 = 1$, and so on.

1	15 pts	
2	12 pts	
3	15 pts	
4	10 pts	
5	15 pts	
6	15 pts	
7	18 pts	
TOTAL	100 pts	

1. (a) (6 points) Find the point at which the given lines intersect:

$$L_1 : x = 1 + t, \quad y = 1 - t, \quad z = 2t \qquad \text{and} \qquad L_2 : x = 4 + 2s, \quad y = 1 + s, \quad z = 1 - s.$$

(b) (9 points) Find an equation for the plane which contains both lines.

2. (12 points) Find an equation for the surface in (x, y, z)-space obtained by rotating the ellipse $x^2 + 4y^2 = 1$ of the (x, y)-plane **about the x-axis**.

3. (a) (5 points) Find the domain of the function $f(x, y) = \sqrt{1 - x^2} - \sqrt{y}$.

(b) (10 points) Evaluate the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + xy - y^2}{x^2 - y^2}$$

or state that it does not exist, giving reasons.

4. (10 points) Suppose $z = f(x, y)$ is a function with partial derivatives $f_x(0, 3) = -1$ and $f_y(0, 3) = 2$. If x and y are both functions of t:

$$x = 1 - t \quad \text{and} \quad y = 2t + t^2,$$

find $\frac{dz}{dt}$ at $t = 1$.

5. (15 points) For the function $f(x, y) = e^{-y} \sin 2x$, find the second partial derivatives

$$f_{xx} = \frac{\partial^2 f}{\partial x^2}, \quad f_{xy} = \frac{\partial^2 f}{\partial y \partial x} \quad \text{and} \quad f_{yy} = \frac{\partial^2 f}{\partial y^2}.$$

6. (15 points) The point $(x, y, z) = (2, -1, 0)$ lies on the surface S:

$$x^2 - 3y^2 + xz - 4z^2 = 1.$$

Find the equation of the tangent plane to the surface S at $(2, -1, 0)$, in the form $ax+by+cz = d$.

7. The temperature at any point (x, y) is given by $T(x, y) = 10 - x^2 - 2y^2$.

   (a) (8 points) Find the rate of change of temperature at point $P = (1, 0)$ in the direction toward the point $(0, 2)$.

   (b) (10 points) In which direction does the temperature increase fastest at P? Find the maximum rate of increase of the temperature at P.