Math 2263: Practice problems for Midterm 3

Problem 1. (20 points) Evaluate the integral $\iint_R \sin(x^2 + 2y^2)dA$, where R is the region in the first quadrant bounded by the ellipse $x^2 + 2y^2 = 1$.

Problem 2. (20 points) Evaluate the integral $\iint_R \frac{(x-y)^2}{(x+y+2)^2} dA$, where R is the square with vertices at $(\pm 1, 0)$ and $(0, \pm 1)$.

Problem 3. (15 points) Show that the line integral

$$\int_C 2x\sin(y)dx + (x^2\cos y - 3y^2)dy$$

is independent of path and evaluate the given integral for any path C from $(1, 0)$ to $(-1, \pi/2)$.

Problem 4. (10 + 10 points) Compute the integral $\int_C \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$

(a) Over any closed curve C not enclosing the origin.

(b) Over the circle of radius a centered at $(0, 0)$.

Problem 5. (20 points) Find the flux of a vector field $\vec{F} = 2x\vec{i} - 2y\vec{j} + \vec{k}$ outward (away from the z-axis) through the surface cut from the bottom of the paraboloid $z = x^2 + y^2$ by the plane $z = 1$.

Problem 6. (15 points) Find the potential function f for

$$\vec{F} = (e^x \cos y - \ln z)\vec{i} + (-e^x \sin y + 2yz)\vec{j} + (-\tfrac{x}{z} + y^2)\vec{k}$$

Problem 7. (5+10 points) Show that $\vec{F} = \langle e^y + y^2e^x, xe^y + 2ye^x \rangle$ is conservative without finding the potential function. Find a potential function of $\vec{F}$.

Problem 8. (13+12 points) Let D be the region in the first quadrant bounded by $x = 0$, $y = 0$ and $x + y = 1$. Let C be the boundary of D, oriented counterclockwise. Evaluate the following integral in two ways: (a) directly and (b) using Green's theorem.

$$\oint_C x^2 dx + xy dy.$$

Problem 9. (15+10 points) Let $\vec{F} = \langle e^{-x} \sin y, xe^{-z} \sin y, xe^{-z} \cos y \rangle$.

(a) Find curl$\vec{F}$ and div$\vec{F}$.

(b) Determine, with justifications, whether $\vec{F}$ is conservative, and whether $\vec{F}$ is the curl of another vector field.

Problem 10. (20 points) Evaluate, using Green's theorem, $\oint_C y^3 dx - x^3 dy$ where C is the boundary of annulus $1 \leq x^2 + y^2 \leq 2$ with positive orientation. Also sketch the curve C showing its orientation.

Problem 11. (15 points) Find the surface area of the portion of the half-cone $z = \sqrt{x^2 + y^2}$ that lies below the plane $z = 2$.