Math and the Cosmos:

The New Mathematical Gravitational Astronomy

Arvid T. Lonseth Lecture, Corvallis, 10 May 2005

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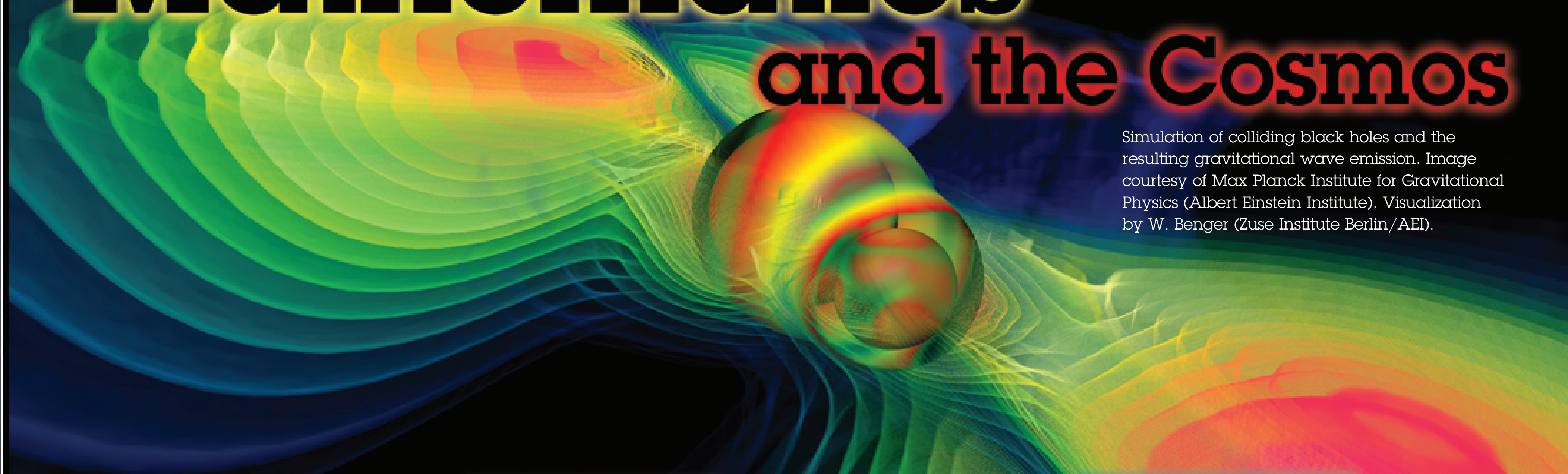
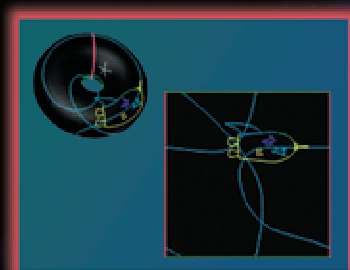
Mathematics and the Cosmos



Simulation of colliding black holes and the resulting gravitational wave emission. Image courtesy of Max Planck Institute for Gravitational Physics (Albert Einstein Institute). Visualization by W. Benger (Zuse Institute Berlin/AEI).

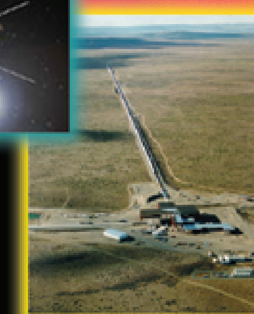
Mathematics is at the core of our attempts to understand the cosmos at every level: Riemannian geometry and topology furnish models of the universe, numerical simulations help us to understand large-scale dynamics, celestial mechanics provides a key to comprehending the solar system, and a wide variety of mathematical tools are needed for actual exploration of the space around us.



A model of a two-dimensional finite universe without edges. Image courtesy of Key Curriculum Press. www.keypress.com

Artist's conception of the Interplanetary Superhighway. Courtesy of Dr. Martin Lo, NASA/JPL, Caltech. The artist is Cic Koenig.

Artist's rendition of the Cassini spacecraft approaching Saturn. Courtesy of NASA/JPL, Caltech.



LIGO gravitational wave detector. Photo courtesy of LIGO Laboratory.

Mathematics Awareness Month – April 2005

www.mathaware.org

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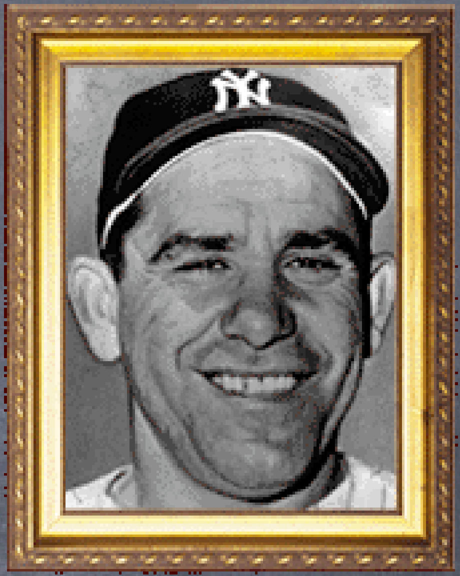
You can see a lot just by looking.

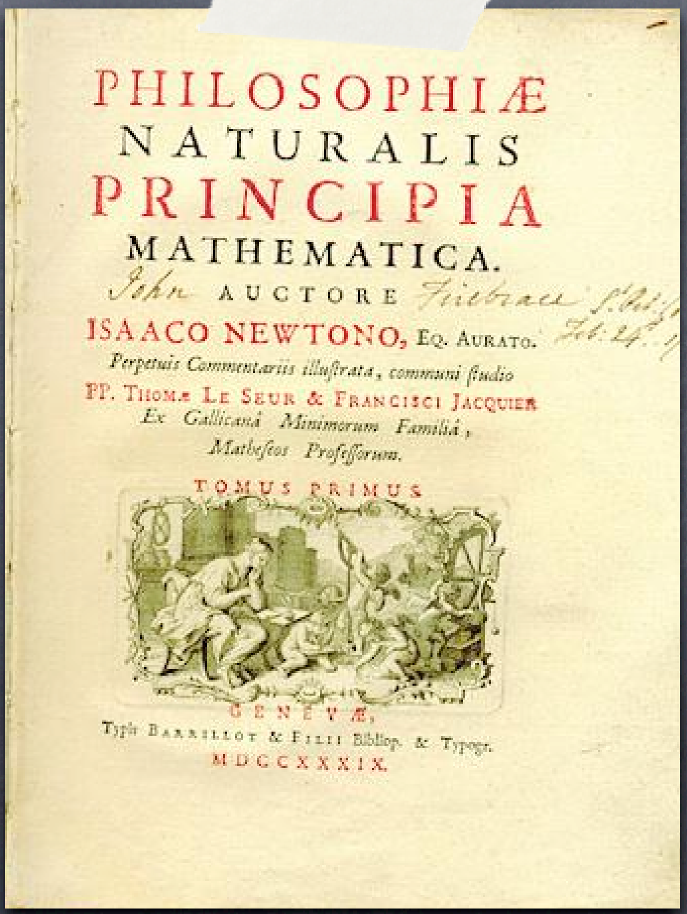
- Yogi Berra



But you can see a lot more by thinking!

Celestial Mechanics

- 1665: Isaac Newton's Annus Mirabilis (1687: Principia)
- 1744: Leonhard Euler Theoria Motuum Planetarum et Cometarum
- 1809: Karl Friedrich Gauss Theoria Motus Corporum Coelestium
- 1799-1825: Pierre Laplace: Traité de Méchanique Céleste
- 1892-1899: Henri Poincaré: Les Méthodes Nouvelles de la Méchanique Céleste


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.
AUCTORE
ISAACO NEWTONO, Eq. Aurato.
Perpetuis Commentariis illustrata, communi studio
PP. Thomæ Le Seur & Francisci Jacquier
Ex Gallicanâ Minimorum Familiâ,
Matheseos Professorum.
TOMUS PRIMUS.
GENEVÆ,
Typis Barrillot & Filii Bibliop. & Typogr.
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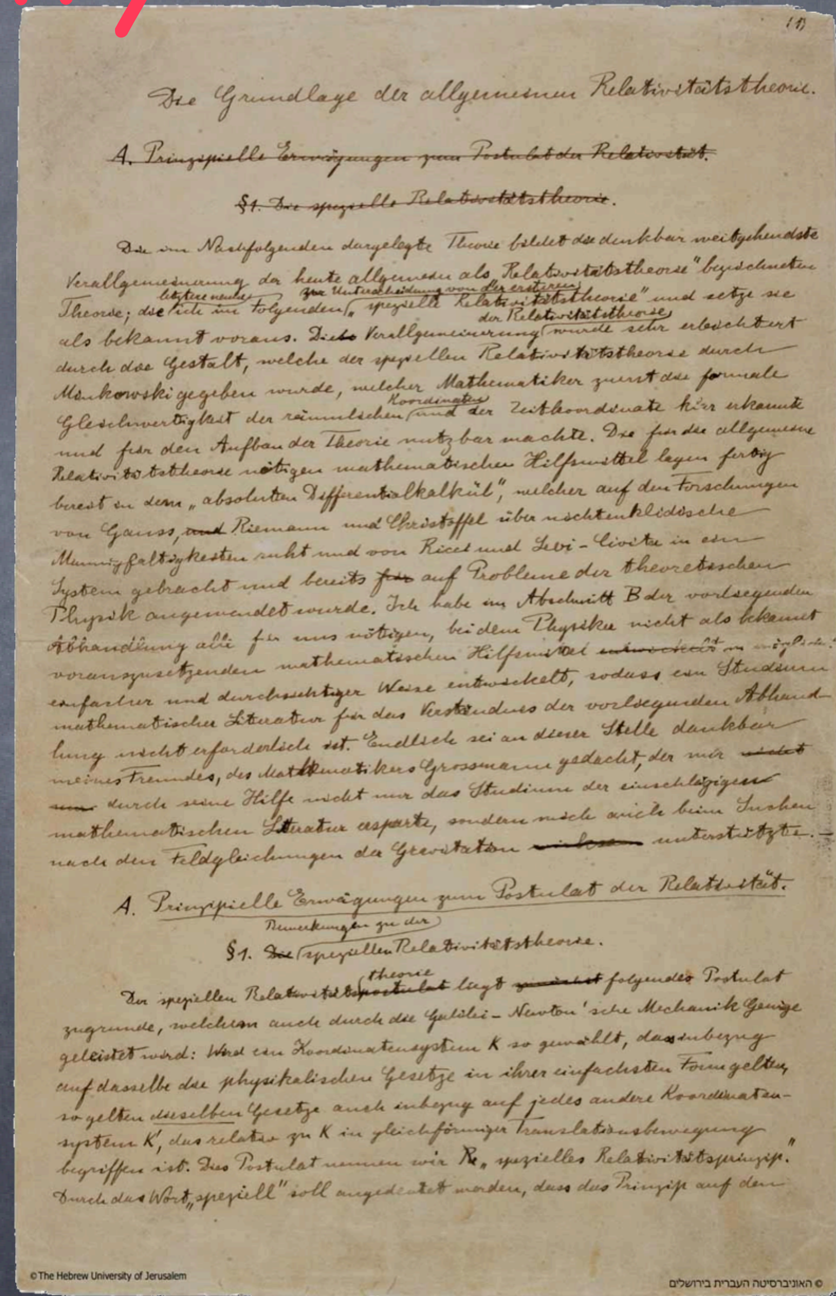
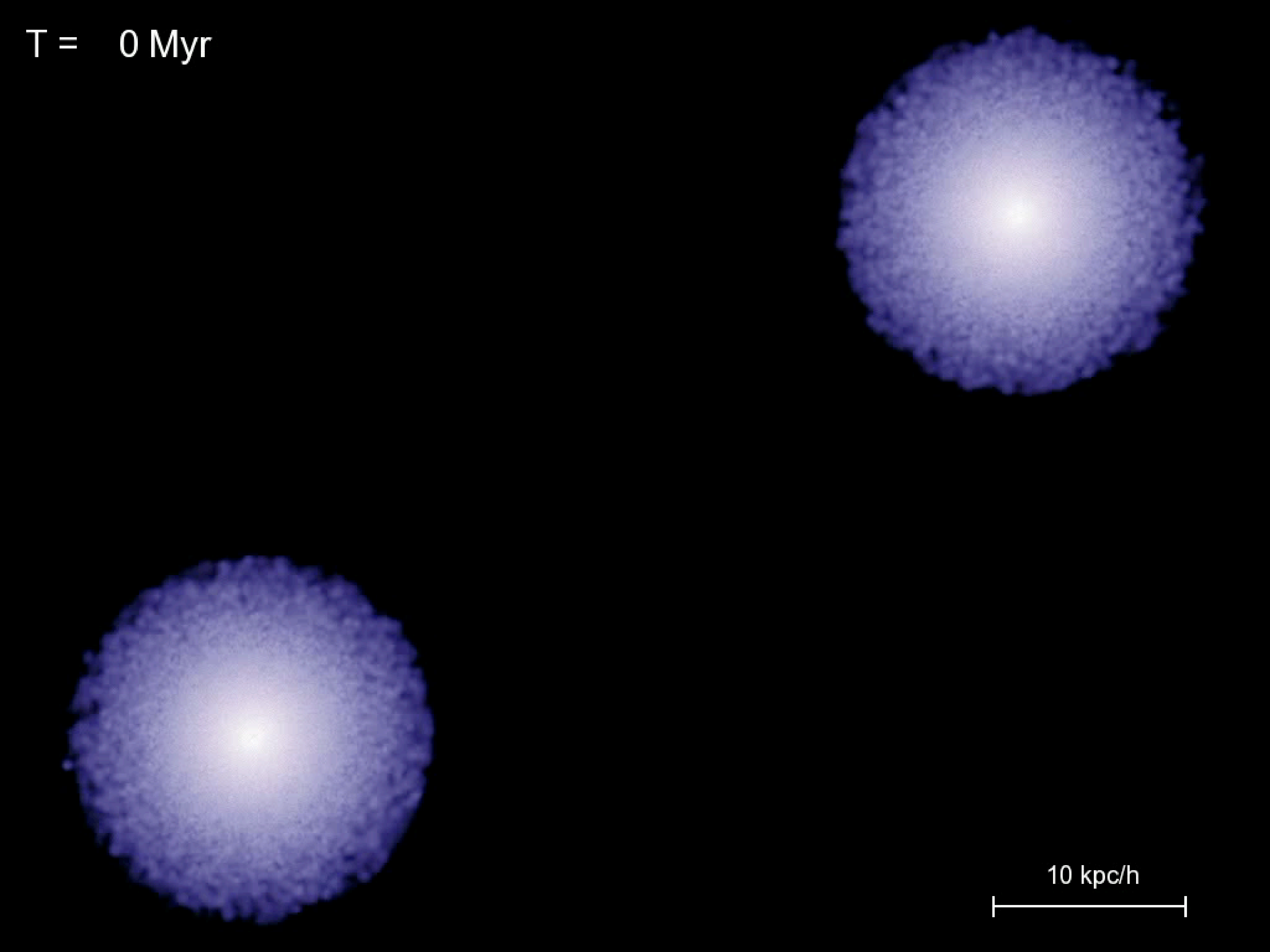
Relativity

- 1898: Poincaré: La mesure du temps
- 1905: Einstein's Annus Mirabilis
  - molecular size
  - Brownian motion
  - special relativity
  - $E=mc^2$
  - quantum theory of light
- 1907: Hermann Minkowski: Raum und Zeit
- 1916: Einstein: Die Grundlage der allgemeinen Relativitätstheorie




T = 0 Myr
10 kpc/h

Special Relativity

- No preferred observer, cannot detect constant uniform motion by physical experiment

Galileo

- Speed of light the same to all observers, independent of motion of the observer or the source

Michelson, Morley, de Sitter

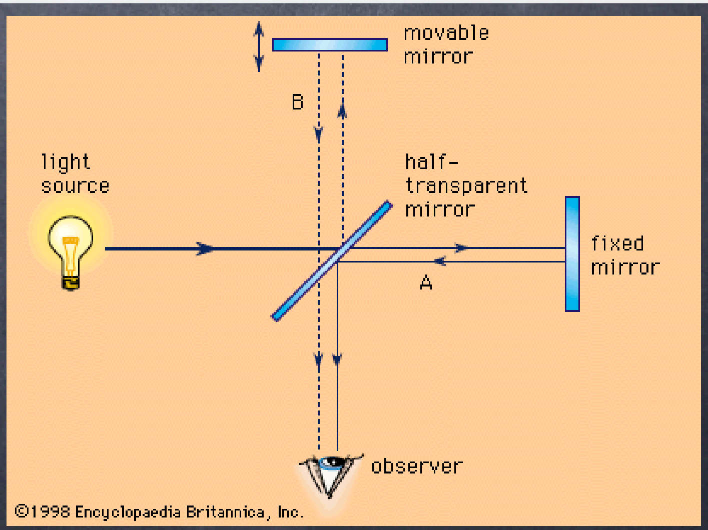
Speed of light

In 1600 Galileo tried measuring light speed with shuttered lanterns, but failed.

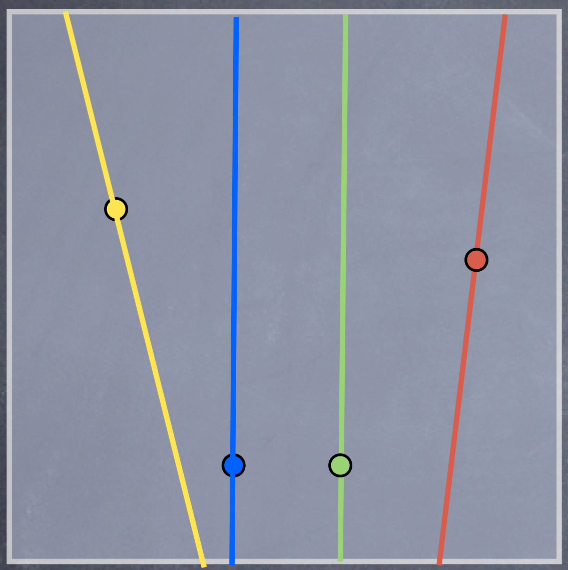
1676	Roemer	Jovian lunar eclipses	133,000
1728	Bradley	stellar aberration	187,000
1849	Fitzeau	toothed wheel	195,000
1973	Evanson et al	lasers	186,282.397

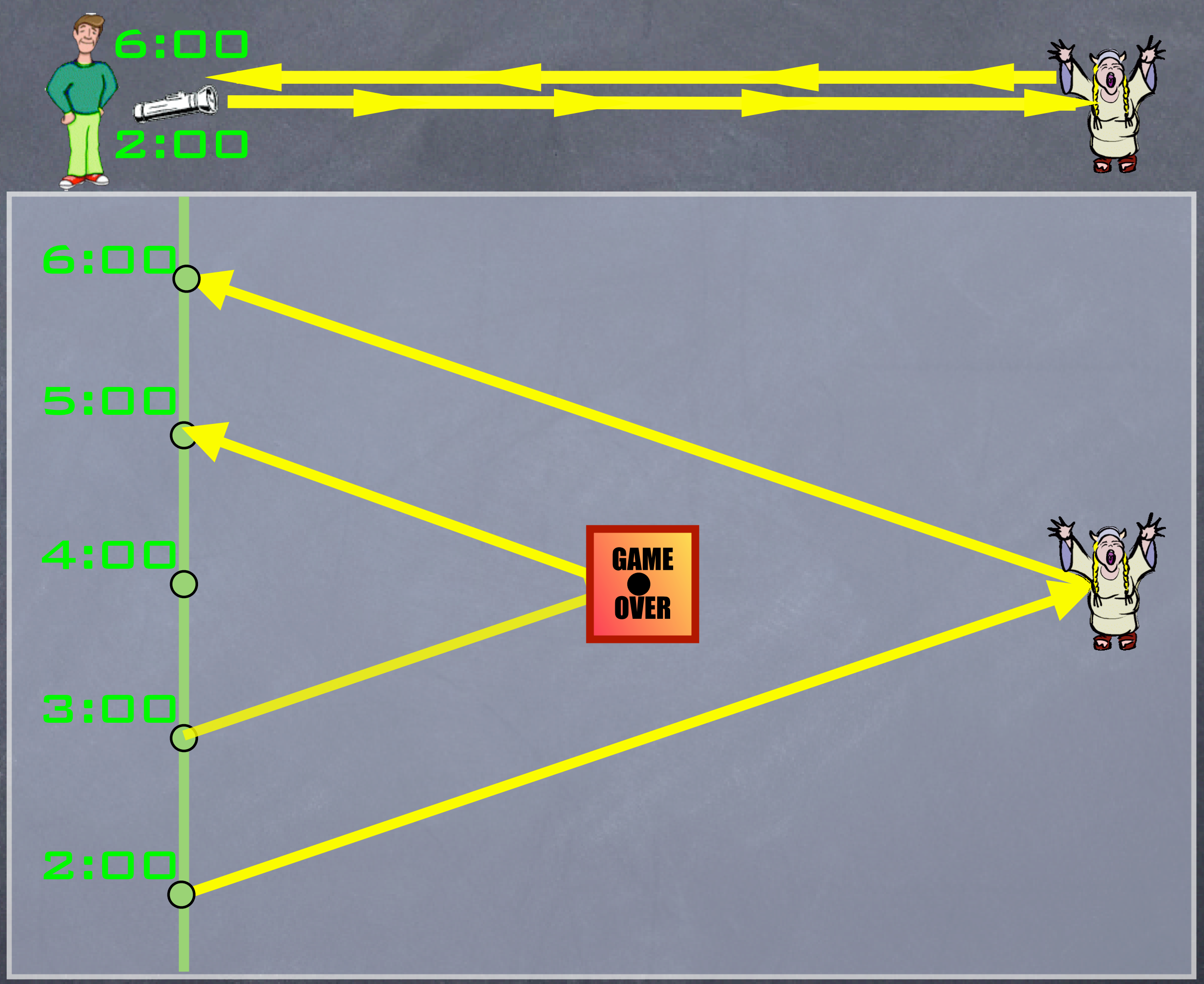
1881 Michelson: Speed doesn't change if you move

1913 de Sitter: or if the source moves towards you

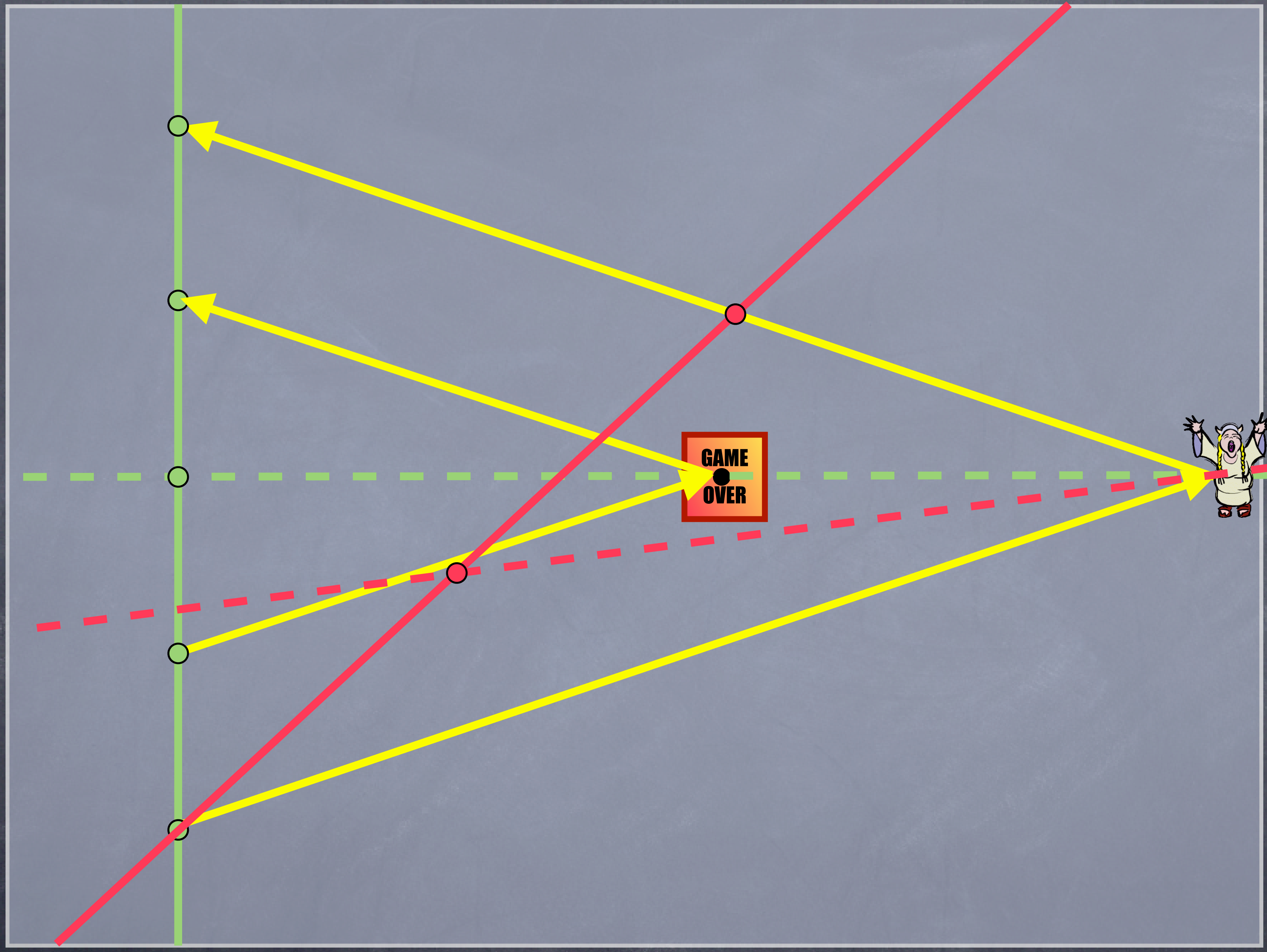


Spacetime



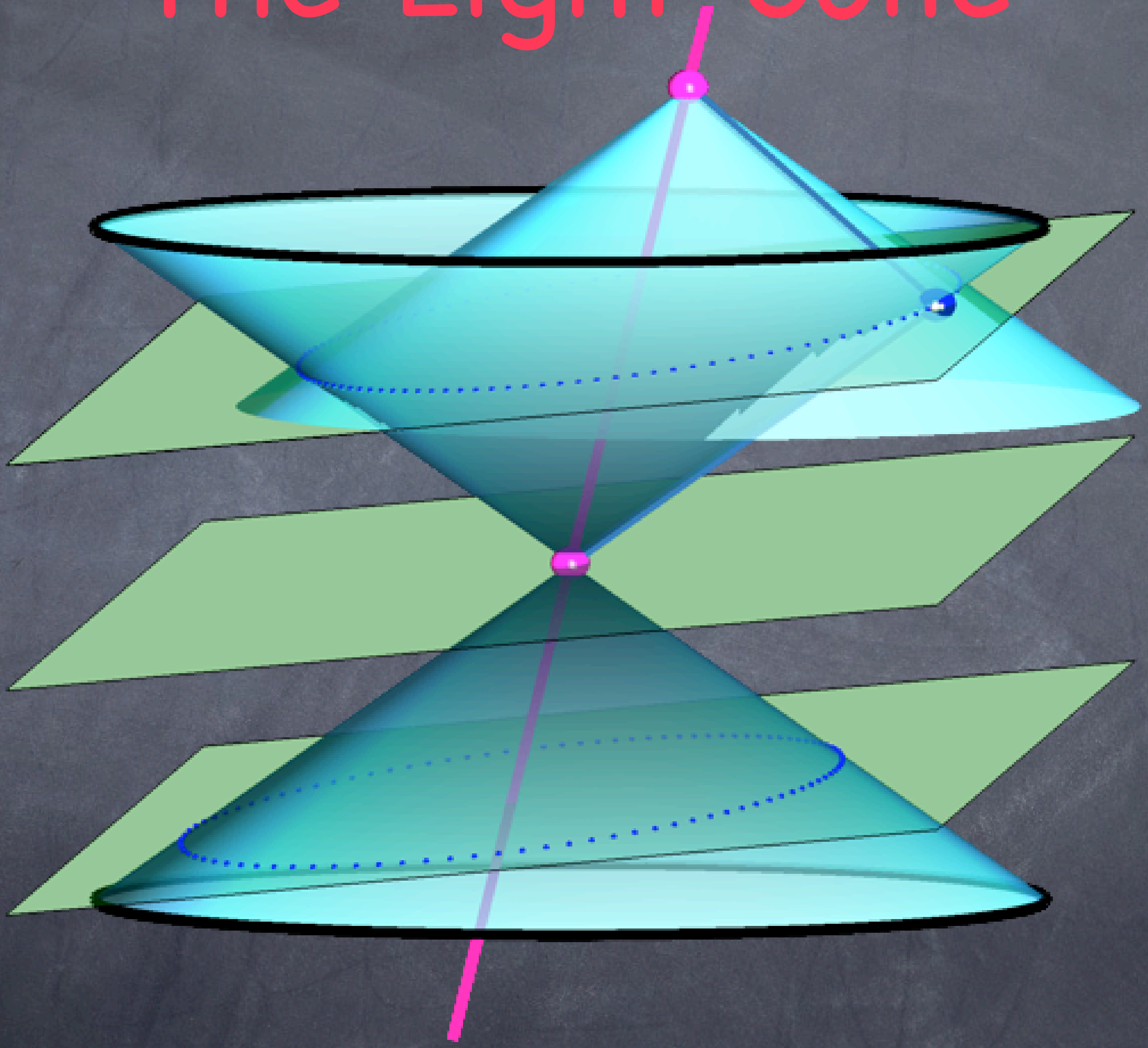


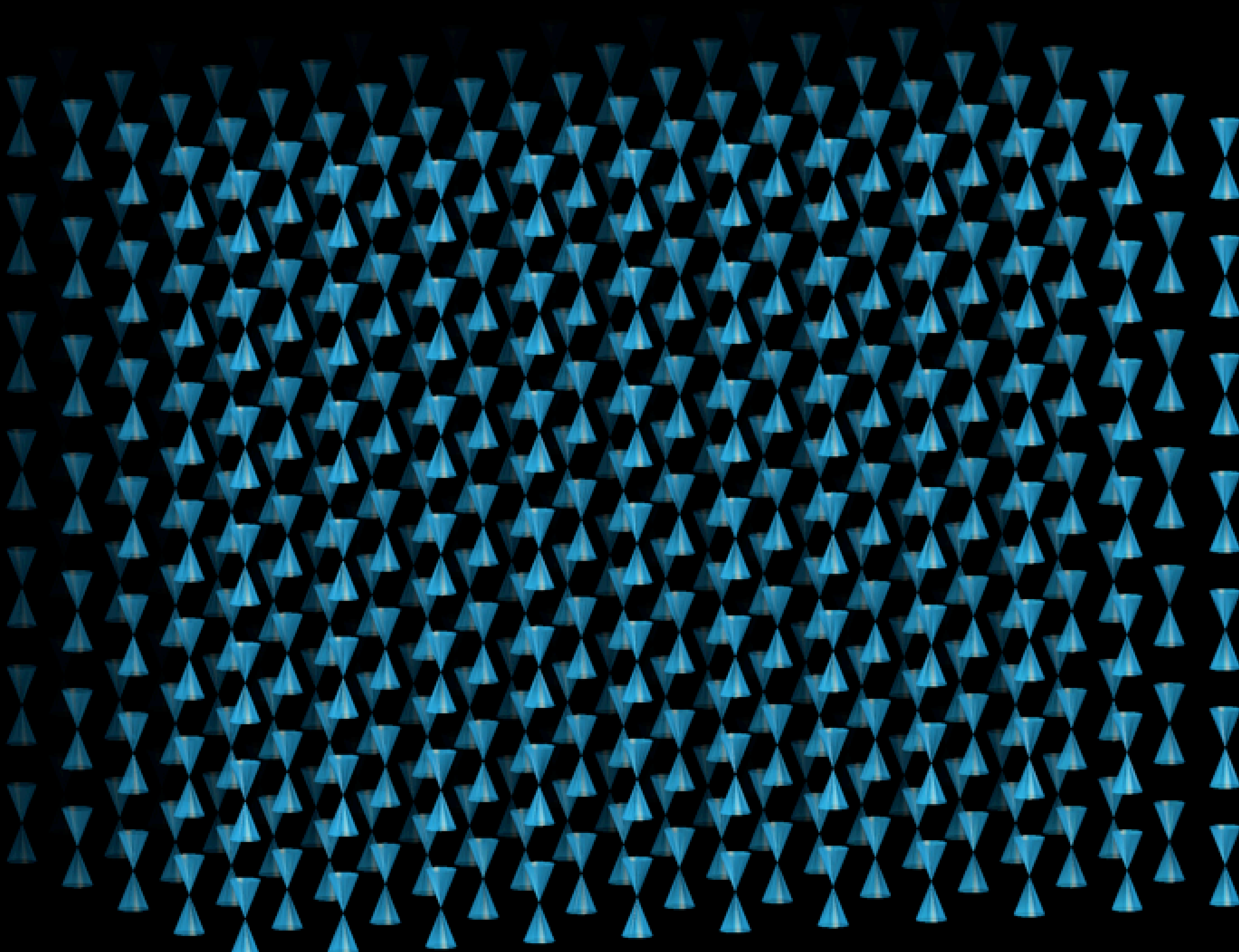
Green says: It's over when the fat lady sings.



Red says: It ain't over when the fat lady sings.

The Light Cone



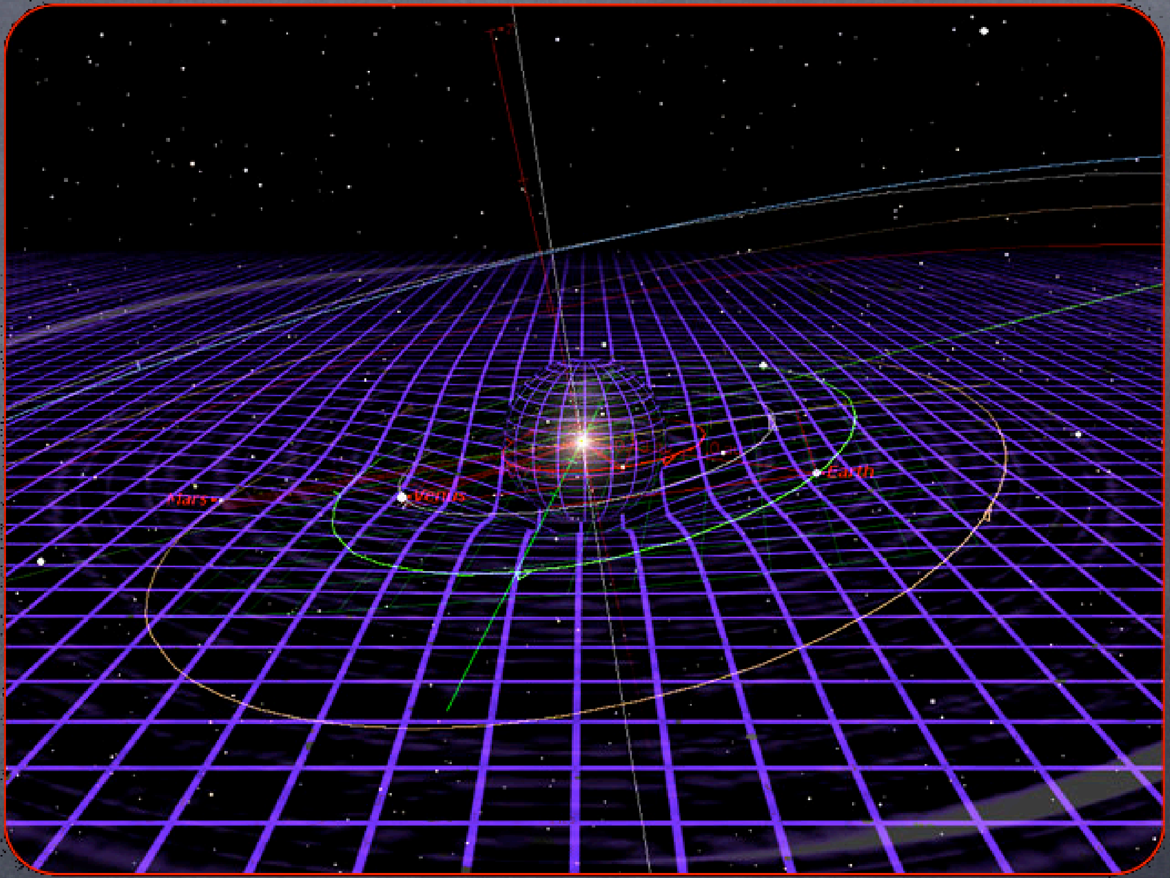


"Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve independence."

– Hermann Minkowski, 1908

General Relativity

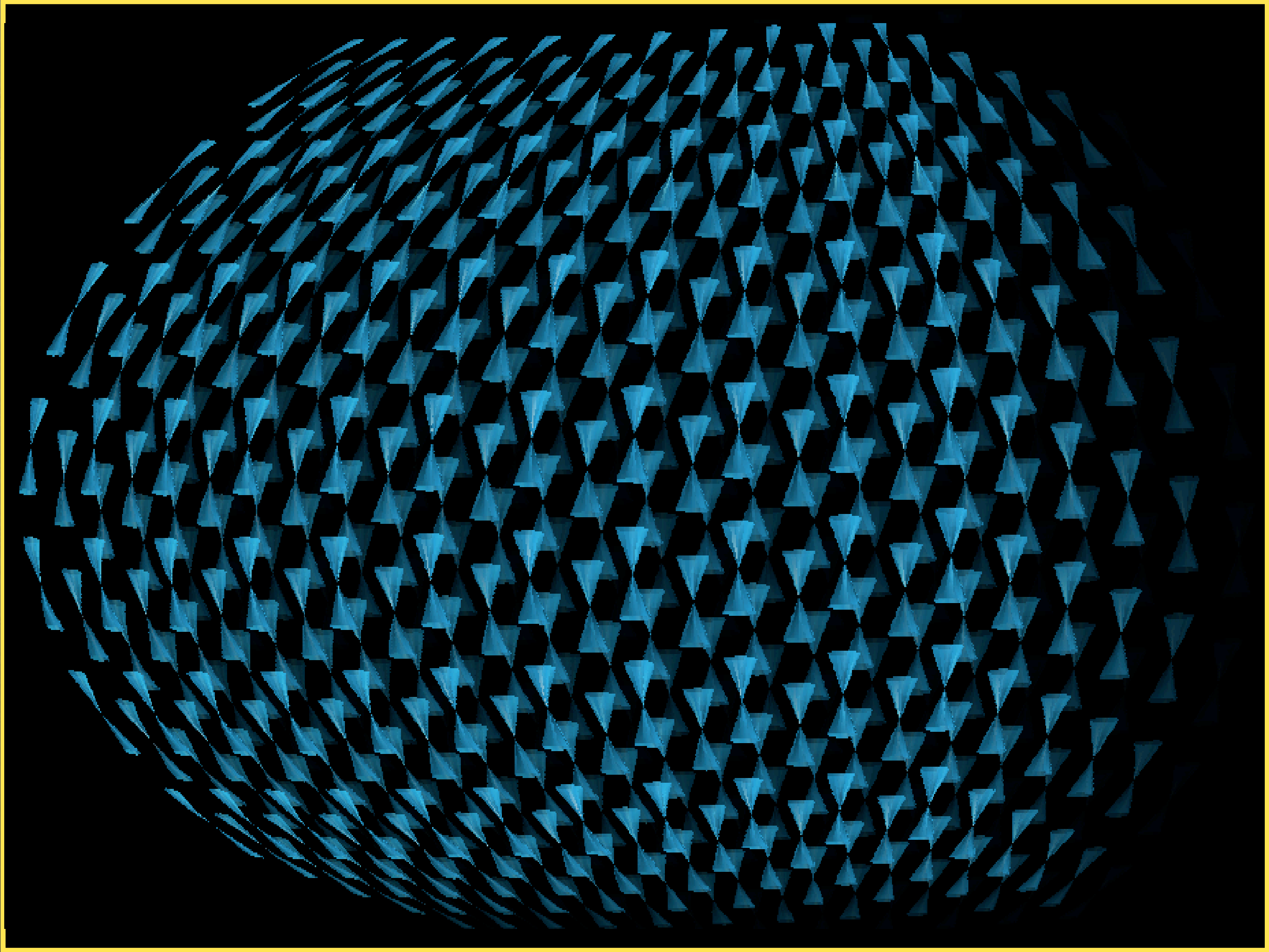
- Spacetime looks locally like Minkowski space, but globally it curves
  - Curvature arises in response to the mass and energy present



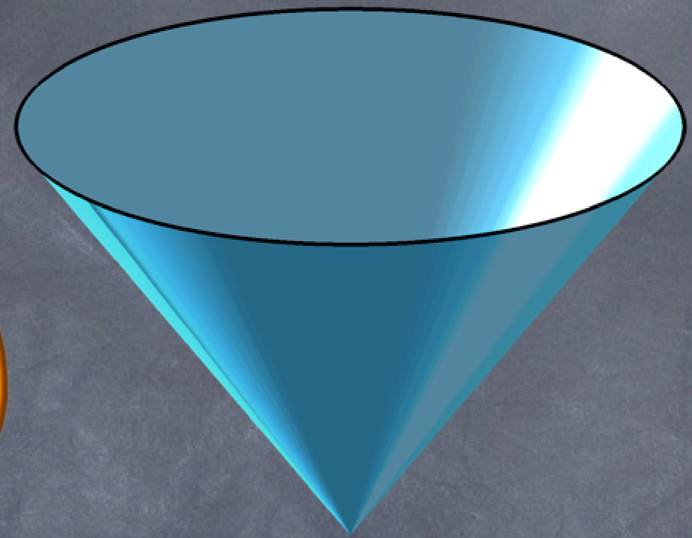
- Freely falling objects move on the straightest possible paths through curved spacetime. This is what we perceive as gravity.

Curved spacetime

Spacetime looks locally like Minkowski space, but globally it curves

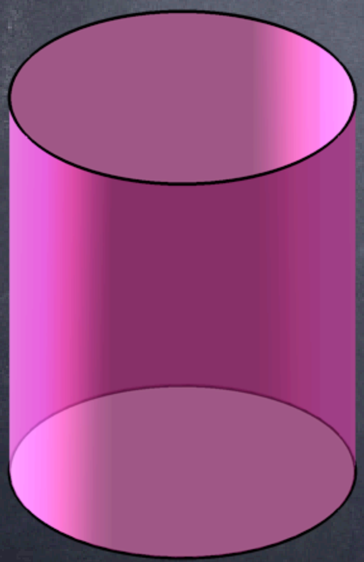


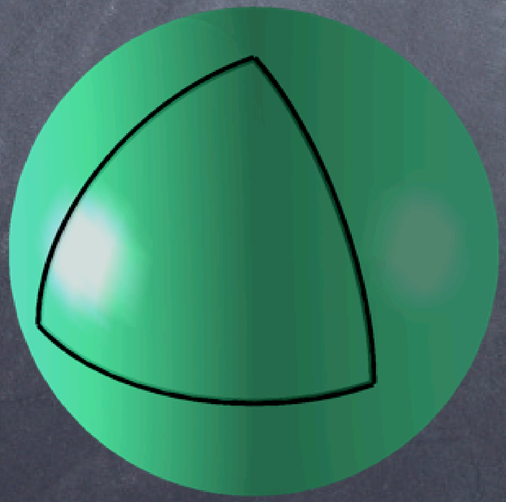
Intrinsic and extrinsic curvature



Intrinsic curvature measured by the Riemann tensor

$$R_{abc}{}^{d}$$



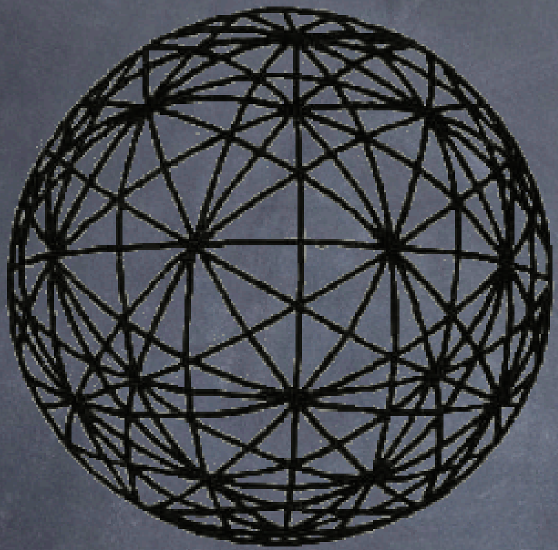


Dimension of $R_{abc}{}^{d}$

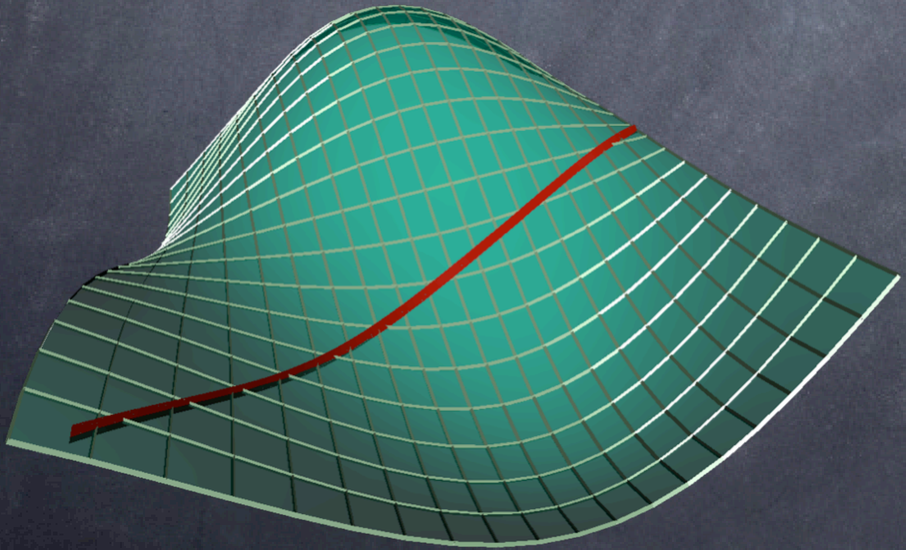
1-D	0
2-D	1
3-D	6
4-D	20

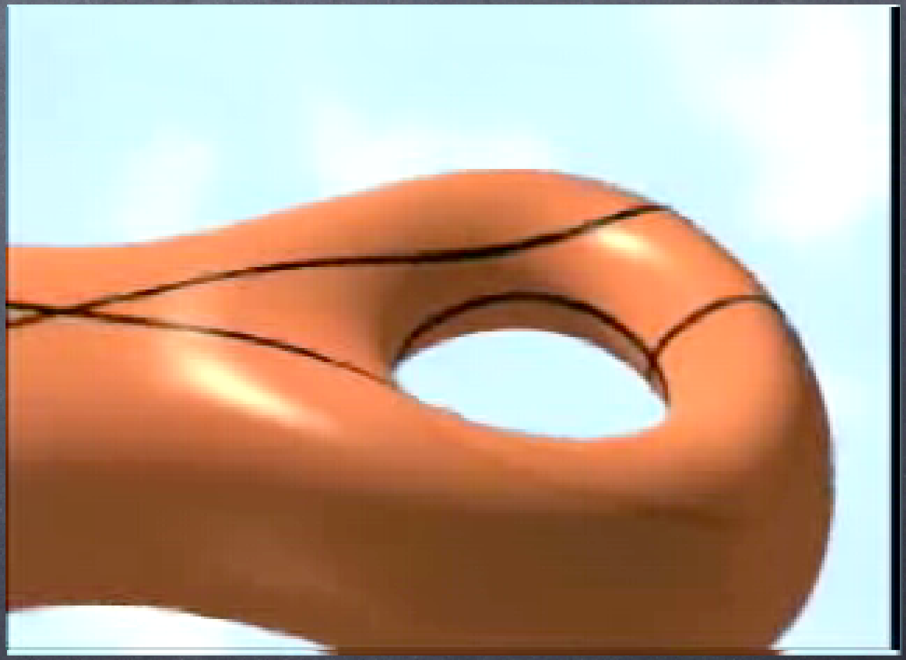
Geodesics

Straight lines through curved space



Objects in free fall move on the straightest possible paths through curved spacetime

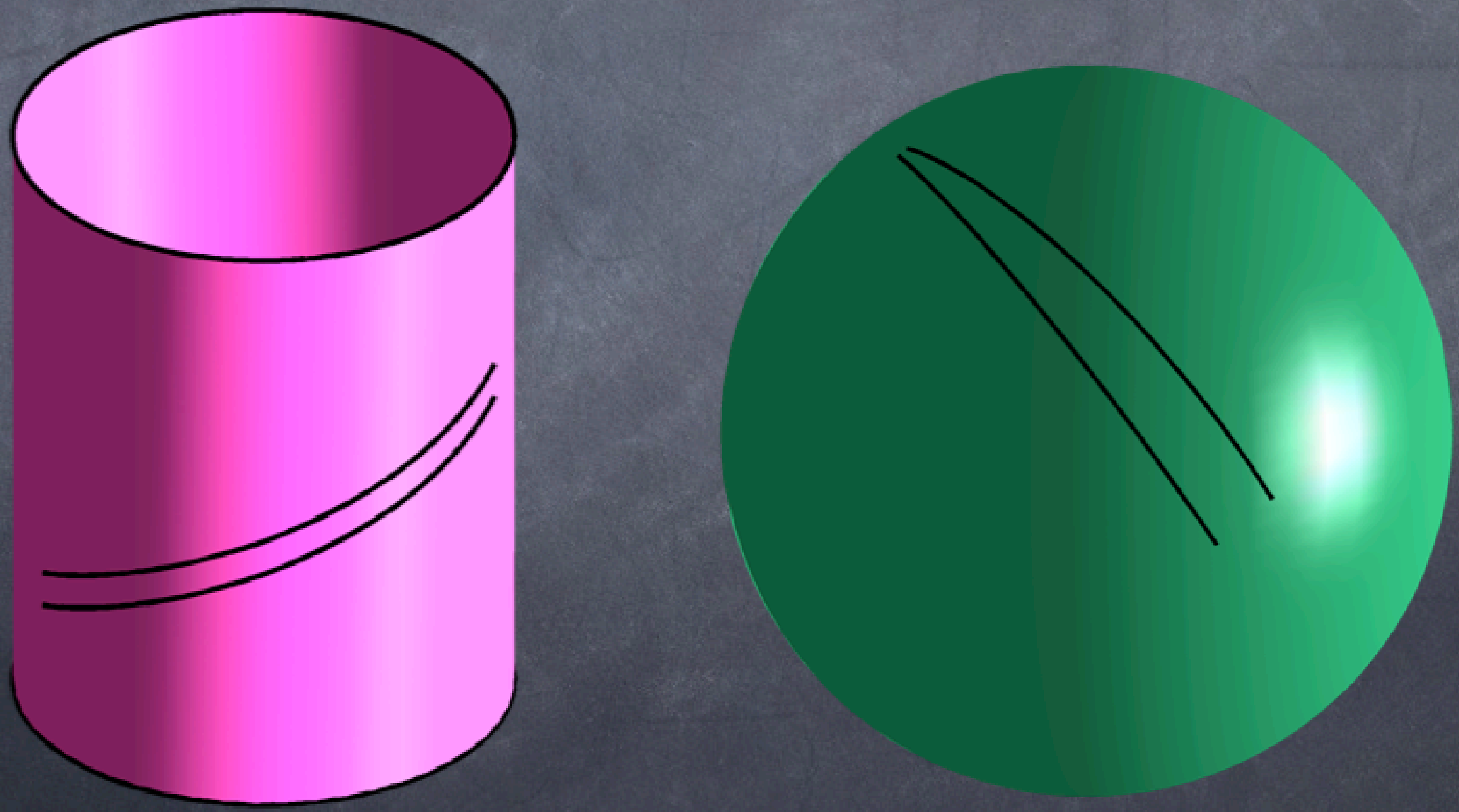




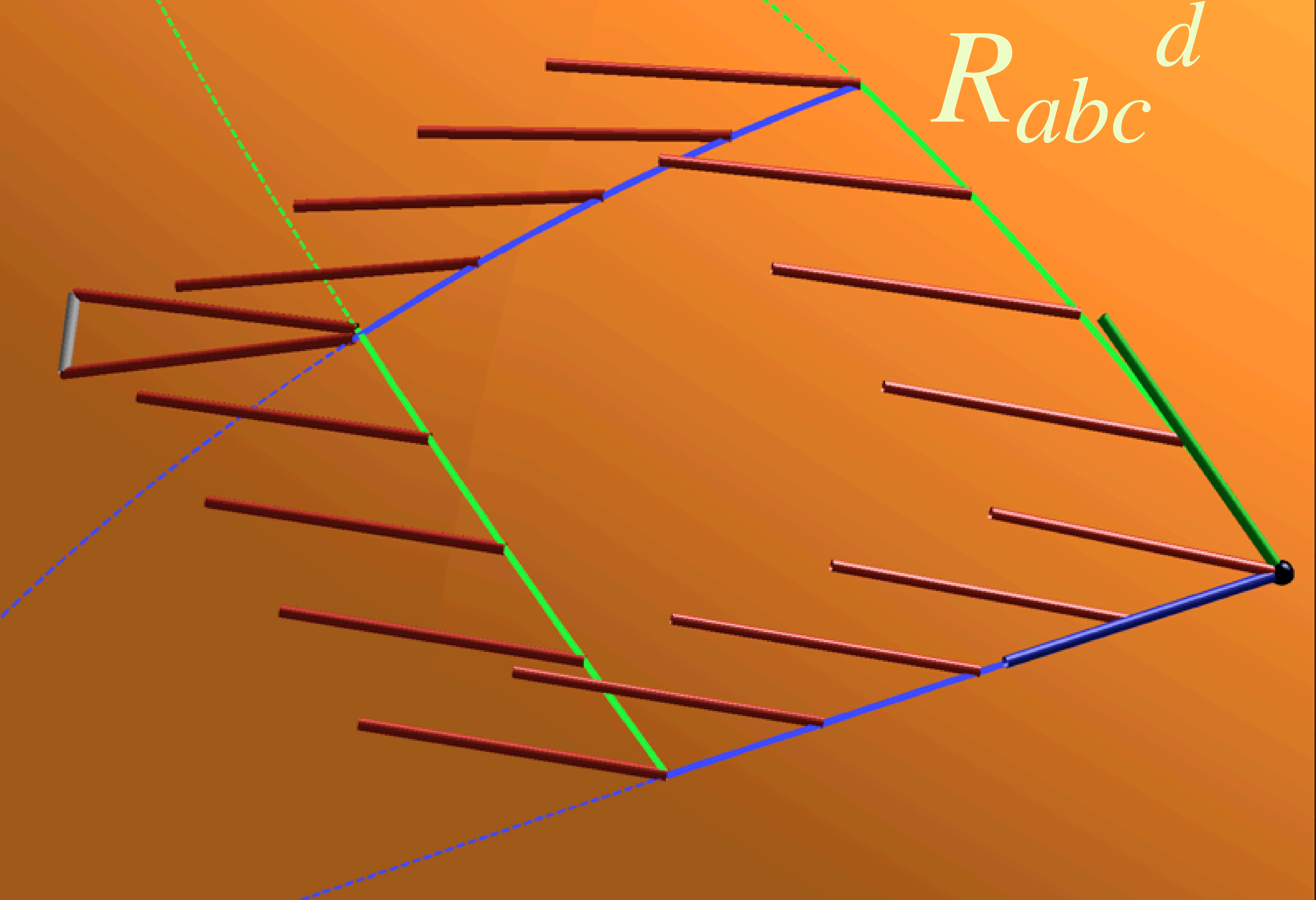
Konrad Polthier et al., ZIB

Geodesic deviation

Spreading or convergence of parallel geodesics can be used to measure intrinsic curvature.



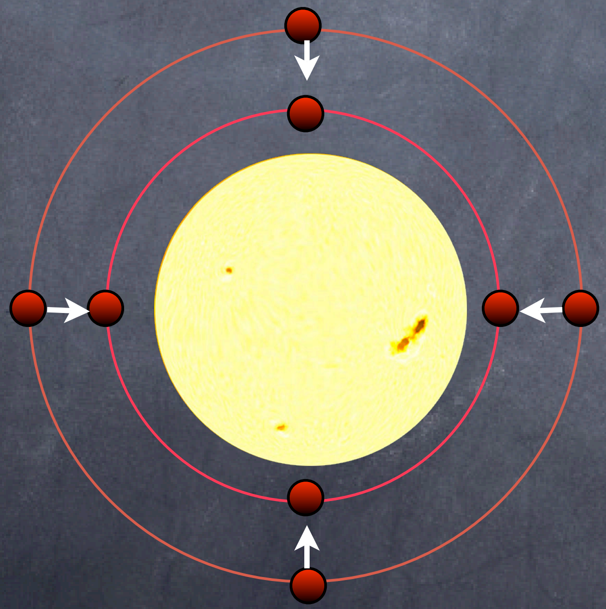
Riemann curvature tensor

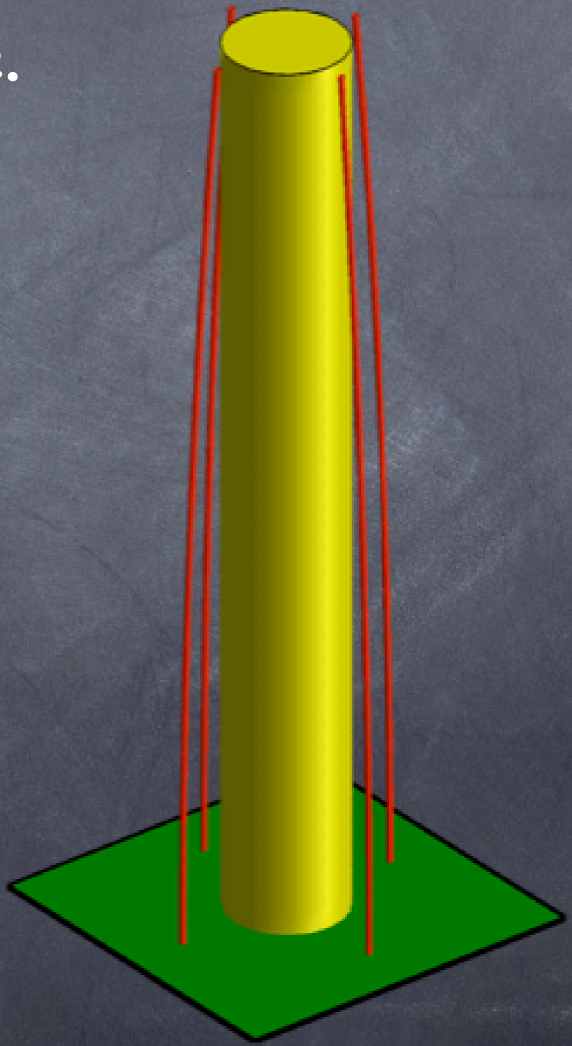


Ricci curvature

R_{ab}

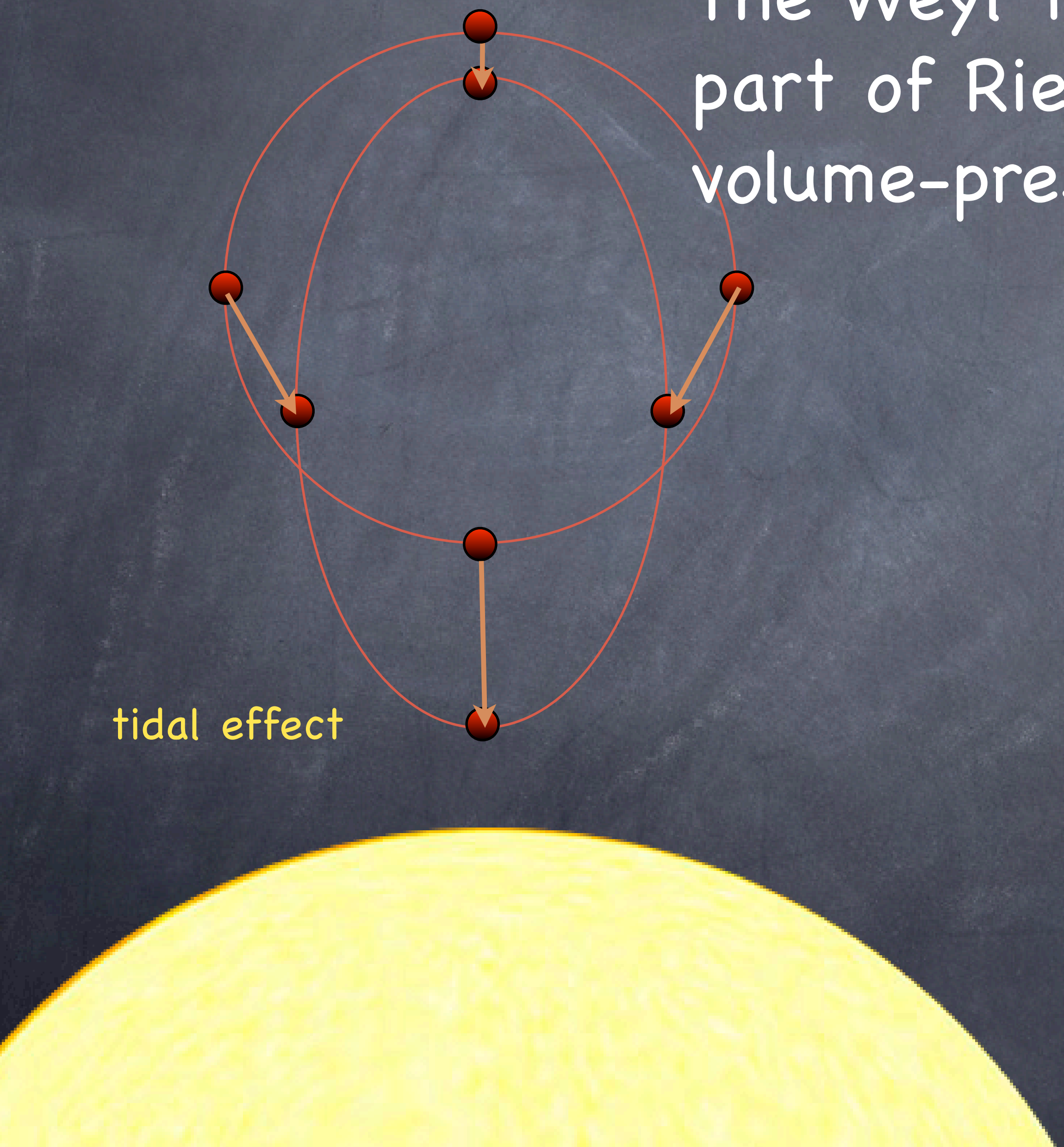
Ricci curvature is the trace of the Riemann tensor, the part responsible for volume change.

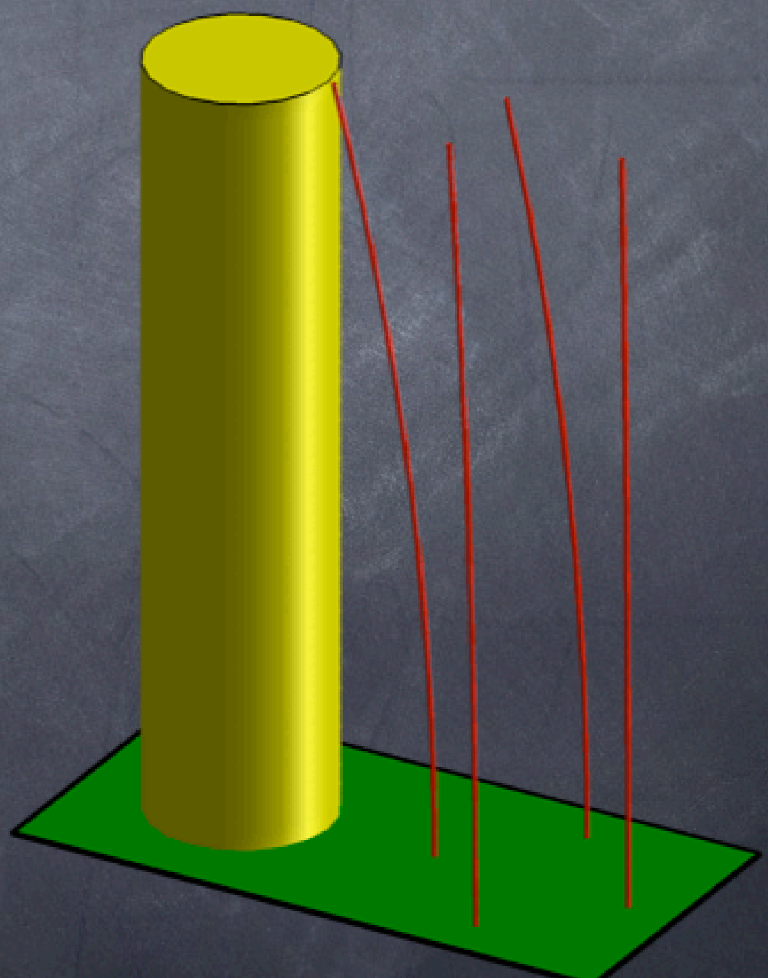




Weyl curvature

The Weyl tensor is the trace-free part of Riemann, responsible for volume-preserving shape distortion.





Riemann = Ricci + Weyl

When there is matter in a small region of space, the Ricci tensor will be non-zero there. But Ricci is zero in a vacuum.

Ricci $\propto$ Mass

Einstein's field equations

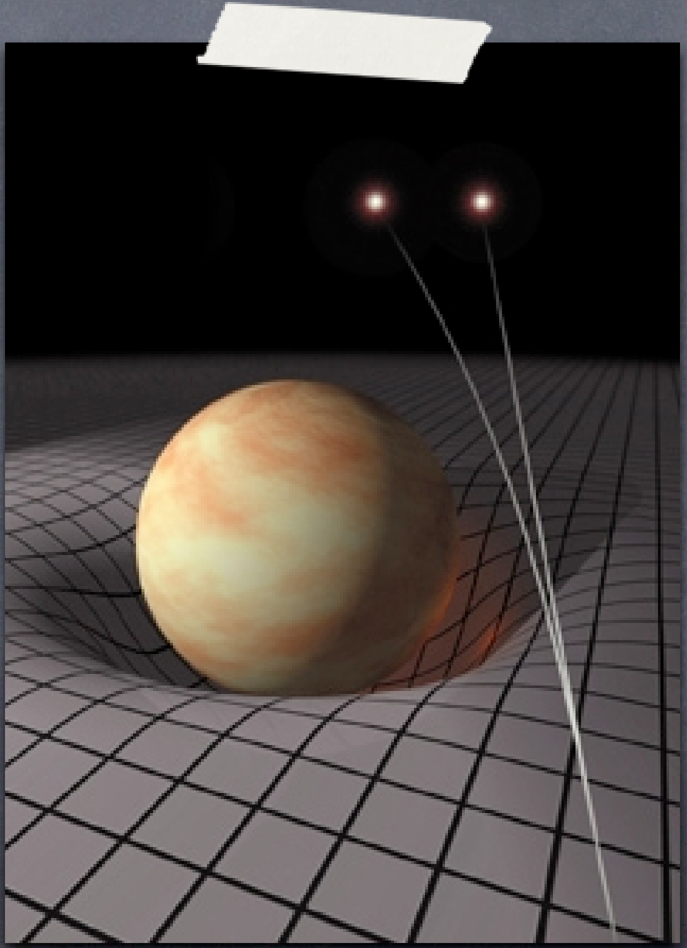
$$G_{ab} = 8\pi T_{ab}$$

Einstein tensor, small variation of Ricci tensor

Mass-energy tensor

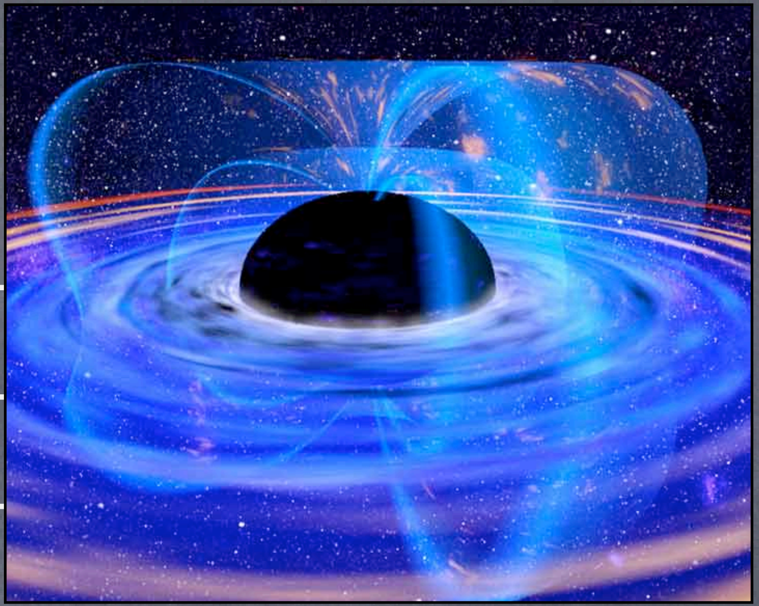
$$\frac{G}{c^4} = 8 \times 10^{-50} \sec^2/\mathrm{g\ cm}$$

Spacetime grips mass, telling it how to move; mass grips spacetime, telling it how to curve.



- John Archibald Wheeler

Black Holes

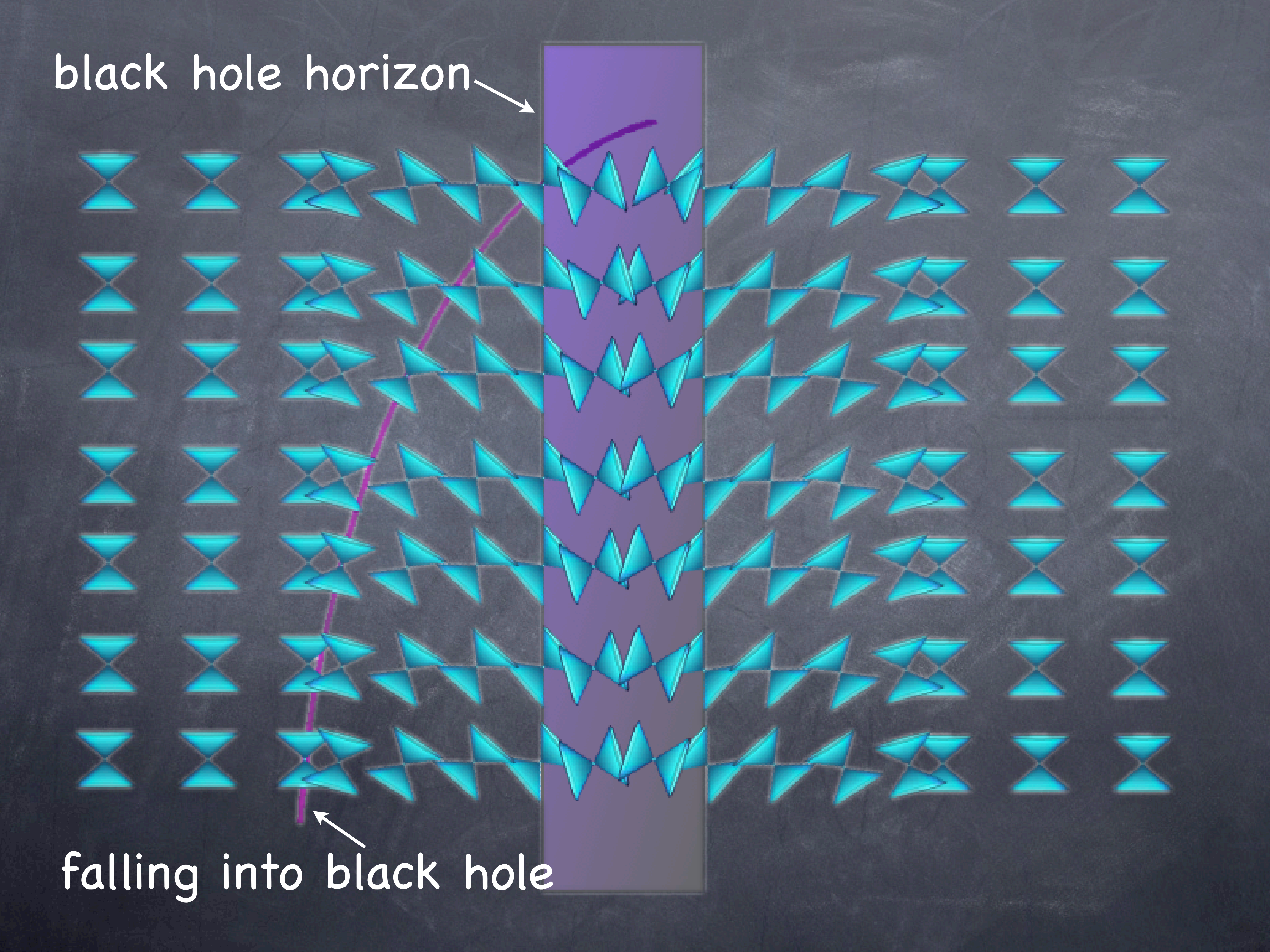


1783	John Michell
1796	Pierre Laplace
1916	Karl Schwarzschild
1930	Subrahmanyan Chandrasekhar
1939	J. Robert Oppenheimer
1968	John Wheeler

1930 Einstein still trying to prove they don't exist!

"It is therefore possible that the greatest luminous bodies in the universe are invisible."
- Laplace

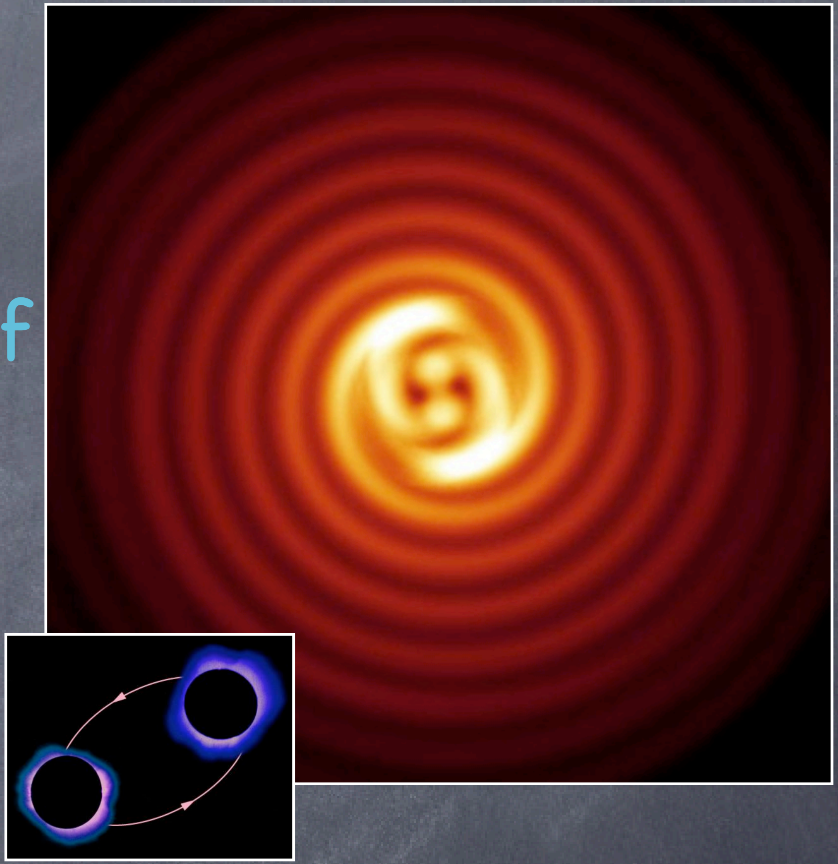
Hawking, Penrose, ...



black hole horizon
falling into black hole

Gravitational Waves

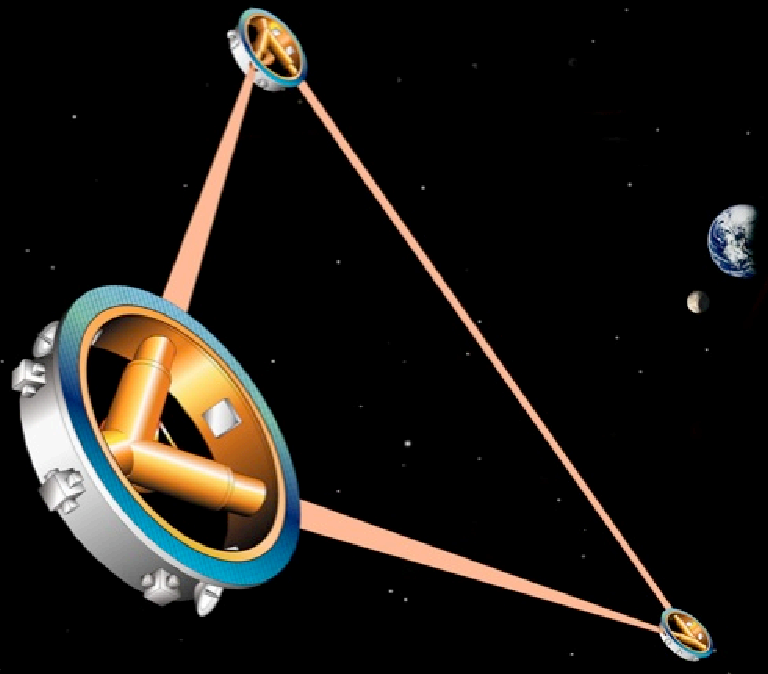
- Moving masses create ripples in spacetime
- Propagate at the speed of light, carrying energy
- Pass through matter
- Extremely weak
- Among the subtlest implication of Einstein's Field equation

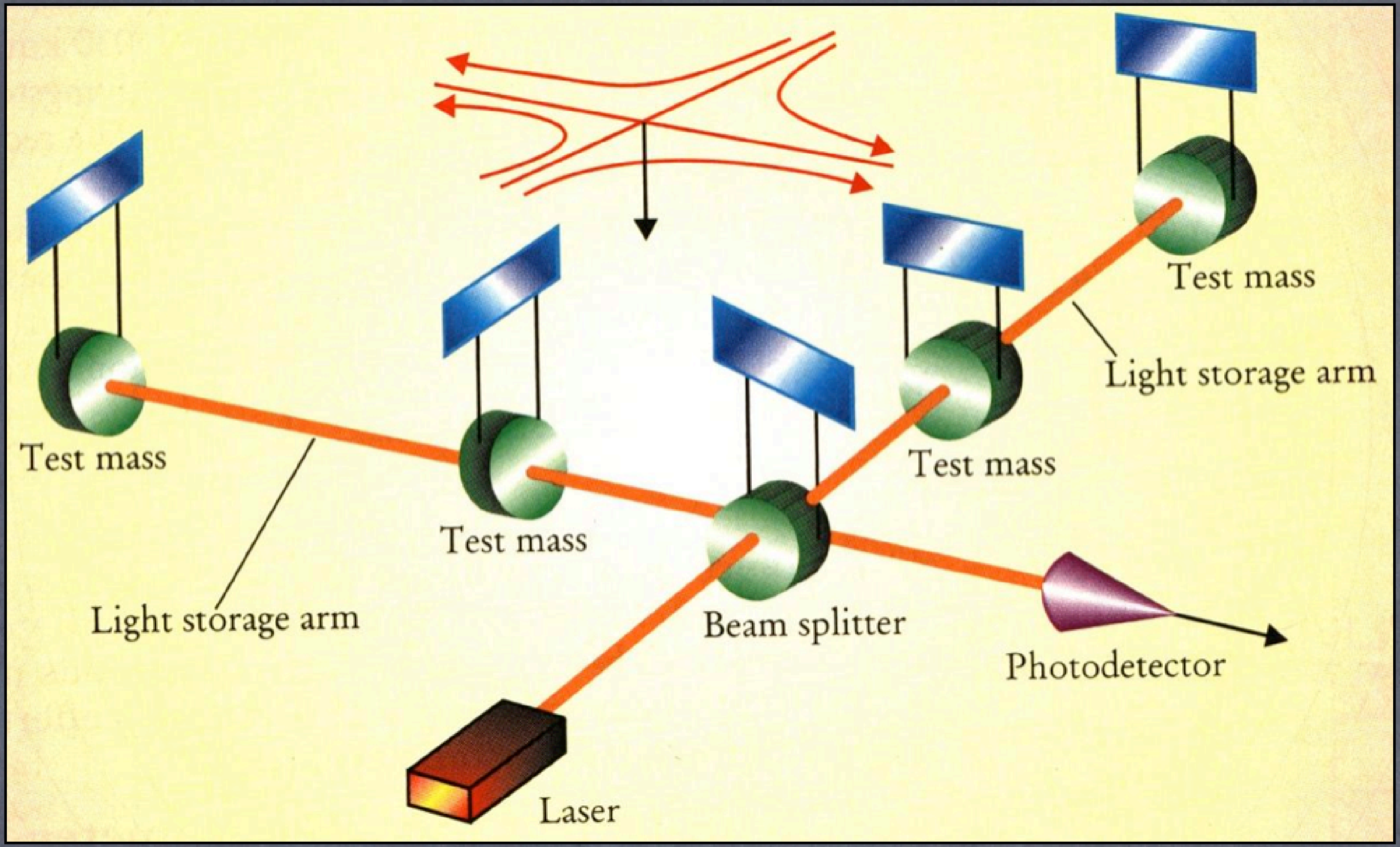


"Gravitational waves travel at the speed of imagination."
- Eddington

Gravitational Observatories

It is now believed that gravity waves from the most massive cosmic sources (BH collisions and neutron star collisions, supernovae, massive BHs) should be (barely) detectable on earth.





- To detect solar mass blackholes in nearby galaxies, we need to measure distances changes to 1 part in 10^{20}.
- LIGO can measure movements of one hundred-millionth of an atomic diameter.

Why?

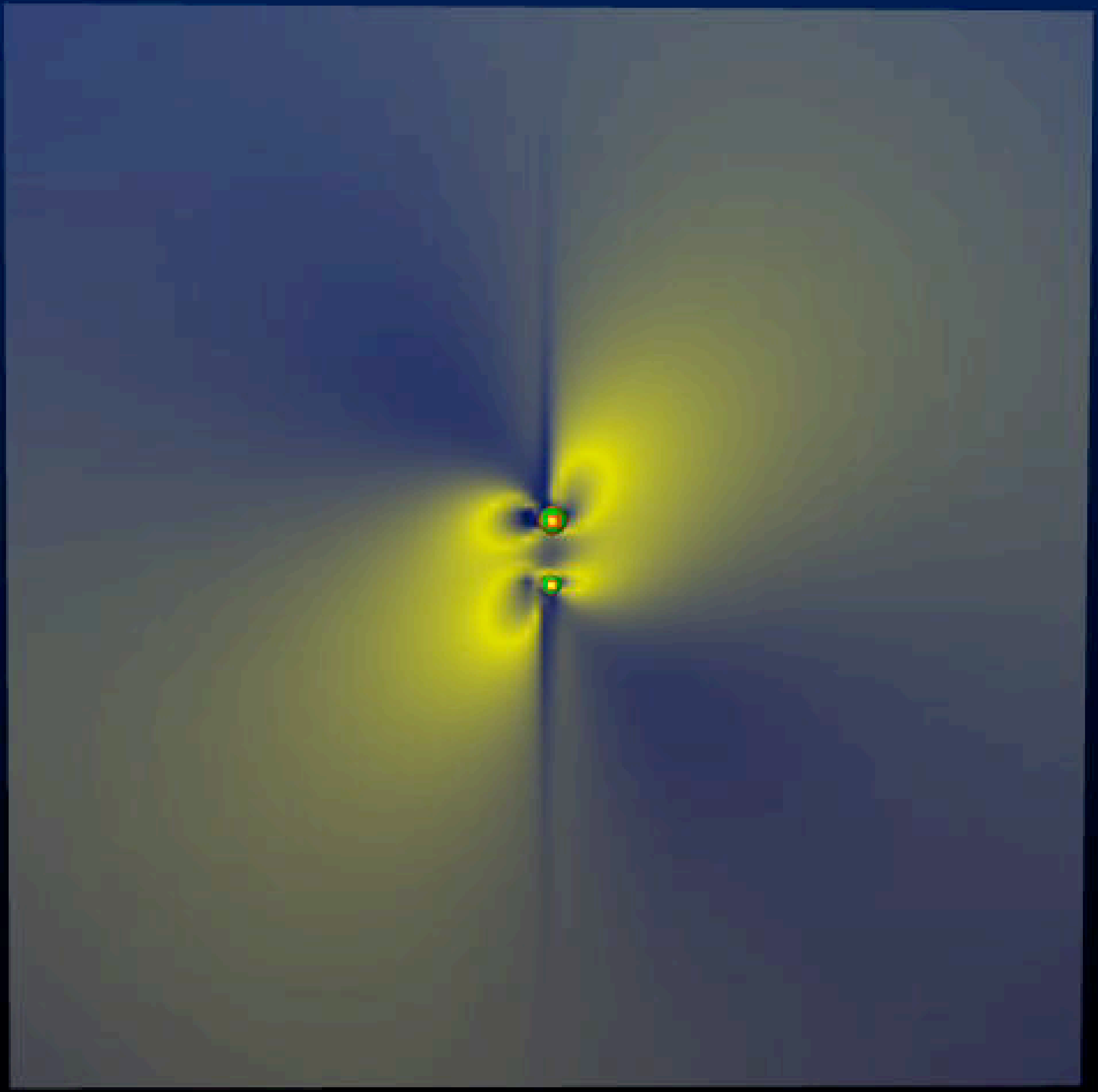
Our first window into the cosmos outside the electromagnetic spectrum.

- Settle gravity wave controversy
- Stringent verification of GR
- Direct evidence of black holes
- "See" dark matter
- Peer back to the big bang
- All sorts of stuff we can't even imagine

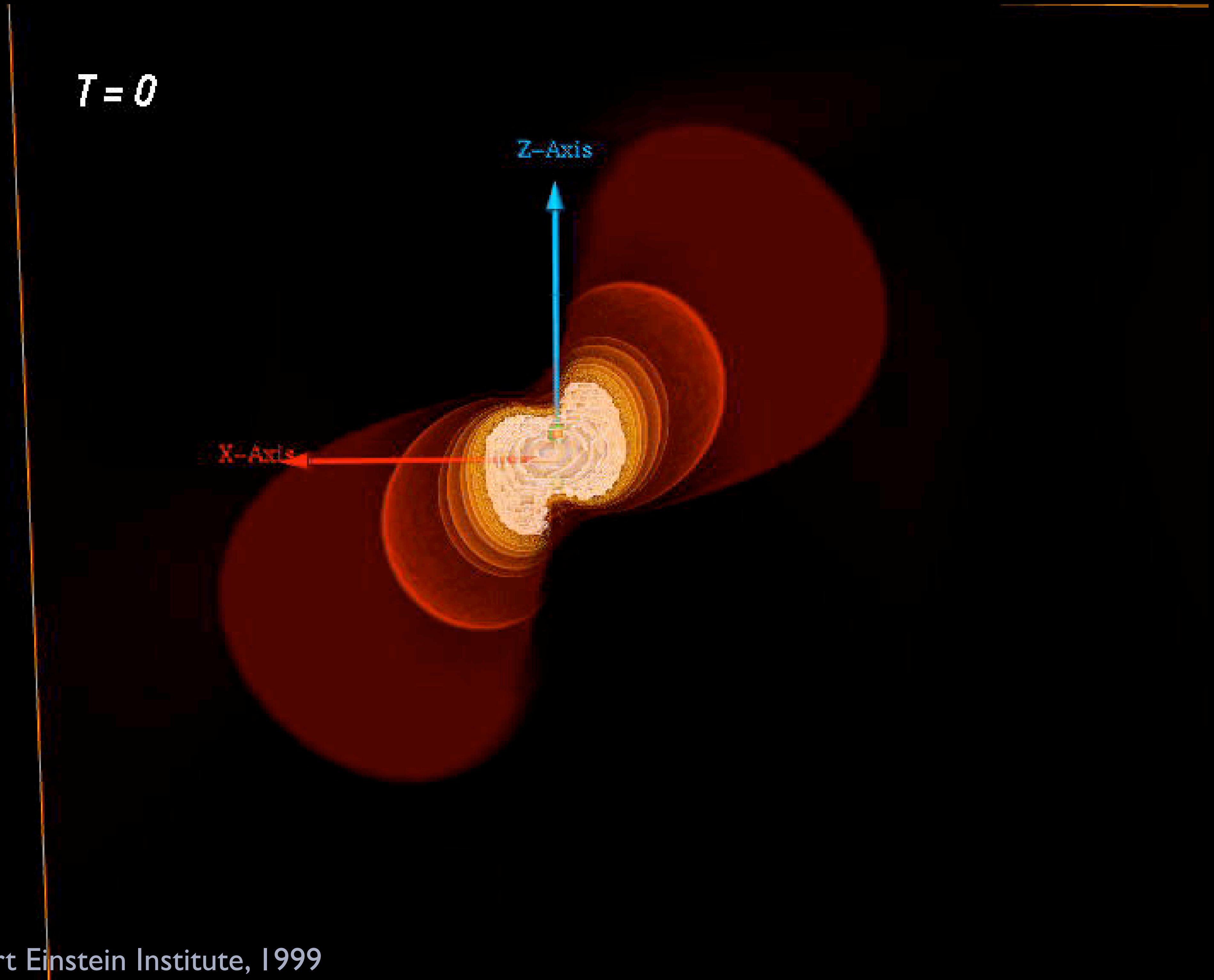
optical
radio
x-ray
gamma-ray
ultraviolet
neutrino
infrared
•
•
•
Gravity!

You gotta do the math!

- To turn a gravitational wave detector into an observatory, you have to do the math and computing to interpret the detected waveforms.
- But simulating BH collisions and gravitational radiation may be even harder than detecting them.
- We have a long way to go, but impressive first steps have been taken.



Albert Einstein Institute, 1999



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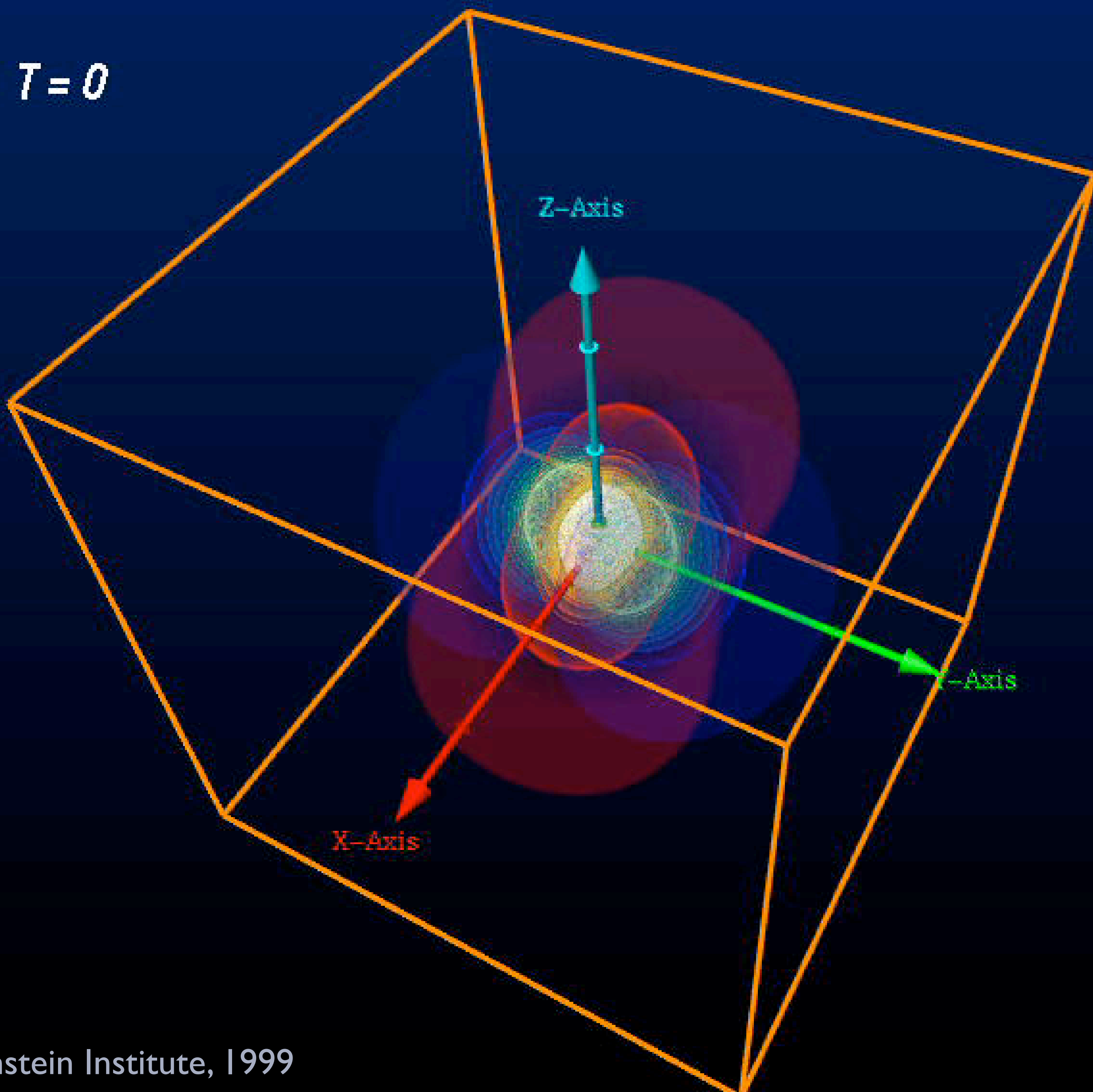
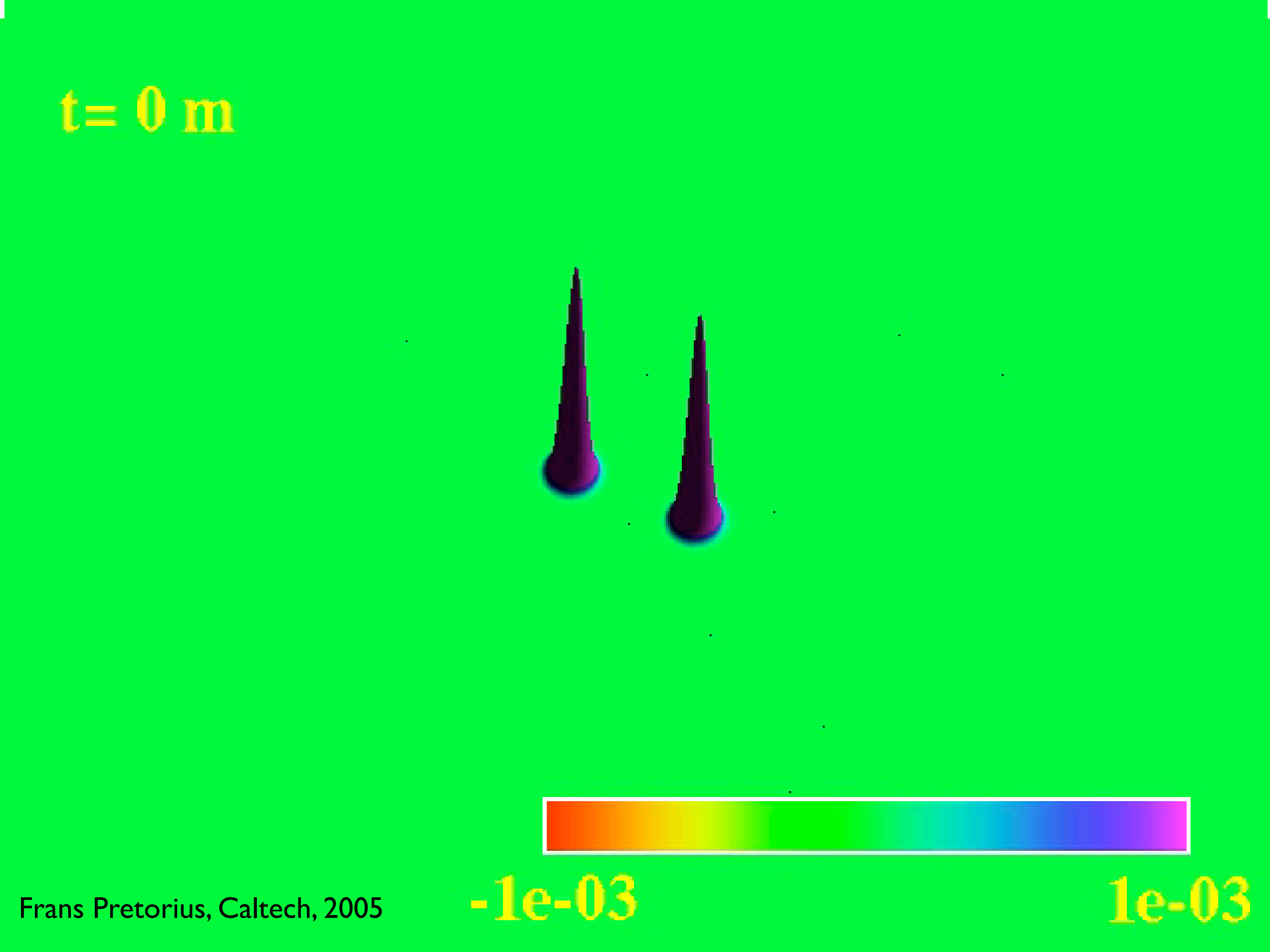


Albert Einstein Institute, 1999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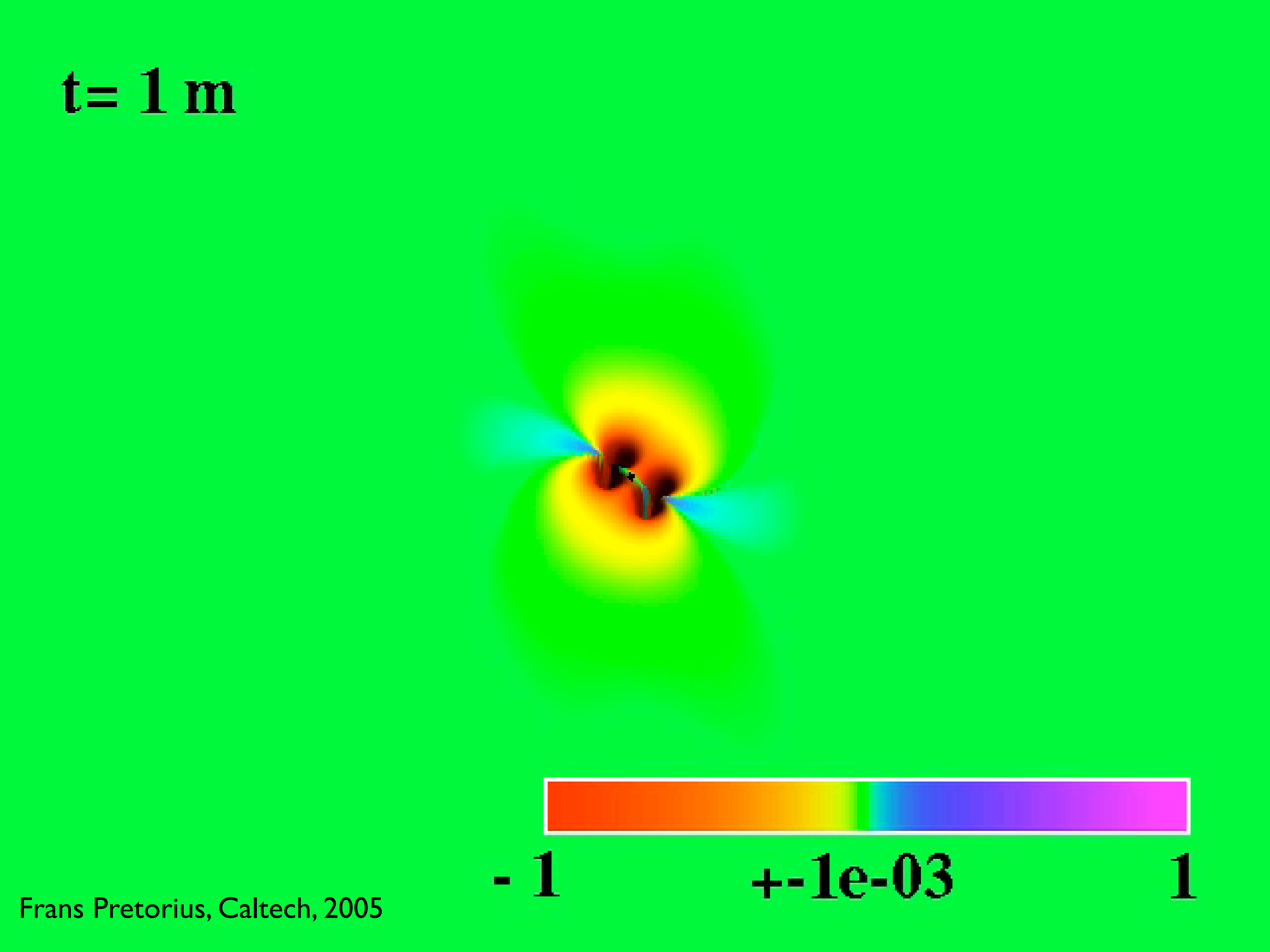
Frans Pretorius, Caltech, 2005



Frans Pretorius, Caltech, 2005

You can see a lot just by looking.

But you can see a lot more by thinking...

And computing!