Name: ____________________

Problem Set 10
Math 4281, Fall 2013
Due: Friday, November 15

Read Sections 3.3, 4.1, 5.1, and 5.2 in your textbook.

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1. Let $\mathrm{GL}_2(\mathbb{Z}_2)$ be the group of invertible 2×2 matrices with entries in $\mathbb{Z}_2$. List its elements. What is the order of the group? Find all of its subgroups.

2. a. List all of the generators of $\mathbb{Z}_{20}$.
   b. List the elements of the subgroups $\langle 3 \rangle$ and $\langle 7 \rangle$ in $U(20) = \mathbb{Z}_{20}^{\times}$.
   c. Find all subgroups of $\mathbb{Z}_{18}$ and $U(11) = \mathbb{Z}_{11}^{\times}$.

3. a. Let a be an element in a group. If $|a| = n$, show that $\langle a^k \rangle = \langle a^{\gcd(n,k)} \rangle$.
   b. Let a be an element in a group. Suppose that $|a| = 24$. Find a generator of $\langle a^{21} \rangle \cap \langle a^{10} \rangle$.

4. Given the permutations $\sigma = (1\ 2\ 4)$, $\tau = (1\ 3)(2\ 4) \in S_4$, compute the following elements:
   a. σ^{-1}
   b. $\sigma\tau$
   c. $\tau\sigma$
   d. σ^2
   e. $\sigma^2\tau$
   f. $\sigma\tau\sigma^{-1}$
   g. $\tau\sigma\tau^{-1}$

5. a. Prove that a k-cycle in S_n is an element of order k.
   b. Prove that when we represent a permutation as a product of disjoint cycles, the order of the permutation is the least common multiple of the lengths of these cycles.

6. Determine if $\sigma = (1\ 2)(1\ 3\ 4)(1\ 5\ 2), \tau = (1\ 2\ 4\ 3)(3\ 5\ 2\ 1) \in S_5$ are even or odd.

7. Prove that A_n contains an n-cycle if and only if n is odd.

Through the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ____________________