Name: ______________________________

Problem Set 8
Math 4281, Fall 2013
Due: Friday, November 1

Read Sections 20.1, 20.2, 20.3, 21.1 (after Theorem 21.3 through Example 9), and 21.3 in your textbook.

---

1. Prove that the real numbers 1 and $\sqrt{3}$ are linearly independent over $\mathbb{Q}$. Do the same for 1, $\sqrt{3}$, and $\sqrt{5}$.

2. Give a basis for each of the given vector spaces over the given field. What is the degree of each field extension?

   a. $\mathbb{Q}(\sqrt{3}, i)$ over $\mathbb{Q}$

   b. $\mathbb{Q}(\sqrt{3}, i)$ over $\mathbb{Q}(i\sqrt{3})$

   c. $\mathbb{Z}_2[x]/\langle x^3 + x + 1\rangle$ over $\mathbb{Z}_2$

   d. $\mathbb{Q}(\sqrt[5]{8})$ over $\mathbb{Q}$

3. Let F be a field. Suppose that K is a field extension of F of finite degree. Prove that if $\alpha \in K$, then there is an irreducible polynomial $f(x) \in F[x]$ having α as a root. (Hint: If $[K : F] = n$, consider $1, \alpha, \alpha^2, \ldots, \alpha^n$.)

4. Which of the following real numbers are constructible?

   a. $\sqrt[4]{5 + \sqrt{2}}$

   b. $\sqrt[6]{2}$

   c. $\frac{3}{4+\sqrt{13}}$

5. Prove that every constructible number is algebraic, i.e., is the root of some polynomial in $\mathbb{Q}[x]$.

Through the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ______________________________