Name: ______________________

Problem Set 11
Math 4281, Spring 2014
Due: Wednesday, April 16

Permutation groups

1. Given the permutations $\sigma = (1\ 2\ 4)$, $\tau = (1\ 3)(2\ 4) \in S_4$, compute the following elements:
   a. σ^{-1} b. $\sigma\tau$ c. $\tau\sigma$ d. σ^2 e. $\sigma^2\tau$ f. $\sigma\tau\sigma^{-1}$ g. $\tau\sigma\tau^{-1}$

2. a. Prove that a k-cycle in S_n is an element of order k.
   b. Prove that when we represent a permutation as a product of disjoint cycles, the order of the permutation is the least common multiple of the lengths of these cycles.

3. Determine if $\sigma = (1\ 2)(1\ 3\ 4)(1\ 5\ 2), \tau = (1\ 2\ 4\ 3)(3\ 5\ 2\ 1) \in S_5$ are even or odd.

4. Prove that A_n contains an n-cycle if and only if n is odd.

Group homomorphisms and isomorphisms

5. Show that $\phi\colon \mathbb{R} \to \mathbb{C}^\times$ given by $\phi(t) = \operatorname{cis}(2\pi t)$ is a homomorphism. Show that $\mathbb{Z}$ is the kernel of ϕ and the unit circle in the complex plane is the image of ϕ.

6. Let $a \in G$ be fixed, and define $\phi\colon G \to G$ by $\phi(x) = axa^{-1}$. Prove that ϕ is a homomorphism. Under what circumstances is ϕ an isomorphism?

7. Let $\zeta = \operatorname{cis}\left(\frac{2\pi}{n}\right)$. Prove that the dihedral group D_n is isomorphic to the subgroup of $\mathrm{G}L_2(\mathbb{C})$ obtained by taking all products of the two matrices $\left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} \zeta & 0 \\ 0 & \bar{\zeta} \end{smallmatrix}\right]$ and their inverses. (In other words, by taking the subgroup generated by these two matrix elements.)

Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ______________________