Name: ____________________

Problem Set 5
Math 4281, Spring 2014
Due: Wednesday, February 26

The complex numbers

1. a. Evaluate $(4-5i)-\overline{(4i-4)}$.
   b. Convert $5\text{cis}\left(\frac{9\pi}{4}\right)$ to the form $a+bi$.
   c. Change $2+2i$ to polar coordinates.
   d. Calculate $(-i)^{10}$.
   e. Calculate $\left(\frac{1-i}{2}\right)^4$.

2. Find the sixth roots of $-3i$. Express your answers in the (exact) form $z=a+bi$ without trigonometric functions, and then plot them in the complex plane.

Euclidean algorithm for polynomials

3. Apply the division algorithm to the polynomials $f(x), g(x) \in \mathbb{Z}_7[x]$, where

$$f(x)=x^6+\overline{3}x^5+\overline{4}x^2-\overline{3}x+\overline{2} \quad \text{and} \quad g(x)=x^2+\overline{2}x-\overline{3}.$$

   Clearly identify $q(x)$ and $r(x)$.

4. Find the greatest common divisor $d(x)$ for the polynomials $f(x), g(x) \in \mathbb{C}[x]$, where

$$f(x)=x^2+1 \quad \text{and} \quad g(x)=x^2-i+2,$$

   and find $s(x), t(x) \in \mathbb{C}[x]$ to express $d(x)=s(x)f(x)+t(x)g(x)$.

5. Show by example that unique factorization fails in $R[x]$ when R is not an integral domain. For instance, consider $x^2+x+\overline{8} \in \mathbb{Z}_{10}[x]$.

Throughout the course of this assignment, I have followed the guidelines of the University of Minnesota Student Conduct Code.

Signed: ____________________