Fall 2017 Math 8211 Homework

From this list of problems, you should submit 24 problems, which will be collected together on the last day of class, December 6. You are encouraged to collaborate on the homework, but you final draft should be written up individually. Please indicate on each problem or set of problems who you collaborated with. This list of problems is subject to changes (but the final number of problems due will not be changed).

Your solutions should be typed (whenever possible), with problems statements included. Also, clearly number the problem with the numbers below, as well as their placement in Eisenbud (if applicable).

(1) Eisenbud, Exercise 1.1.
(2) Eisenbud, Exercise 1.13.
(3) Eisenbud, Exercise 1.24 (a,b) only.
(4) Eisenbud, Exercise 2.4.
(5) Eisenbud, Exercise 2.7.
(6) Eisenbud, Exercise 2.14.
(7) Eisenbud, Exercise 3.17.
(8) Eisenbud, Exercise 3.1.
(9) (a) Eisenbud, Exercise 3.6.
  (b) Eisenbud, Exercise 3.7.
  (c) Eisenbud, Exercise 3.8.
(10) Eisenbud, Exercise 3.14.
(11) (a) Show that the radical of a primary ideal is prime.
  (b) Find an example of a power of a prime ideal that is not primary.
  (c) Let p be a prime ideal of a ring R and $n \in \mathbb{N}$. The R-ideal $p^{(n)} := R \cap p^n R_p$ is called the <u>n-th symbolic power</u> of p. Show that $p^{(n)}$ is primary.
(12) Eisenbud, Exercise 4.7.
(13) Let $R \subset S \subset T$ be rings. Show that if S is integral over R and T is integral over S, then T is integral over R. Then use this to deduce that $\overline{\overline{R}^S}^S = \overline{R}^S$.
(14) Eisenbud, Exercise 4.11a (ungraded only).
(15) Eisenbud, Exercise 4.24.
(16) Eisenbud, Exercise 4.26.
(17) Eisenbud, Exercise 1.18.
(18) Eisenbud, Exercise 1.21.a.
(19) Eisenbud, Exercise 5.1.
(20) Eisenbud, Exercise 5.8.
(21) Eisenbud, Exercise 1.23.
(22) Eisenbud, Exercise 6.1.

(23) Prove Theorem 2 in the Direct and Inverse Limits section of your notes.

Thm 2. Let $\{M_i, \varphi_{i,j}\}$, $\{M'_i, \varphi'_{i,j}\}$, and $\{M''_i, \varphi''_{i,j}\}$ be three direct systems of R-modules over the same directed set $\mathcal{I}$ and suppose that we have exact sequences

$$0 \to M'_i \xrightarrow{u_i} M_i \xrightarrow{v_i} M''_i \to 0$$

where u_i and v_i are compatible with the maps in the direct system. Then the following is exact:

$$\varinjlim M'_i \xrightarrow{\varinjlim u_i} \varinjlim M_i \xrightarrow{\varinjlim v_i} \varinjlim M''_i \to 0.$$

(24) Let R be a ring, $x \in R$, M and R-module, and $\{M_i, \varphi_{i,j}\}$ the direct system of R-modules over the index set $\mathbb{N}$ given by

$$M_1 = M \xrightarrow{x} M_2 = M \xrightarrow{x} M_3 = M \xrightarrow{x} \cdots.$$

Show that there is an R-isomorphism $\varinjlim M_i \cong M_x$ mapping $\varphi_i(a)$ to $\frac{a}{x^i}$.

(25) Prove Theorem 6.a in the Direct and Inverse Limits section of your notes.

Thm 6. Let $\{M_i, \varphi_{j,i}\}$, $\{M'_i, \varphi'_{j,i}\}$, and $\{M''_i, \varphi''_{j,i}\}$ be inverse systems of R-modules over $\mathcal{I} = \mathbb{N}$. Let $u_i : M'_i \to M_i$ and $v_i : M_i \to M''_i$ be R-linear maps compatible with the maps in the inverse systems.

a. If $0 \to M'_i \xrightarrow{u_i} M_i \xrightarrow{v_i} M''_i$ are eventually exact, then

$$0 \to \varprojlim M'_i \xrightarrow{\varprojlim u_i} \varprojlim M_i \xrightarrow{\varprojlim v_i} \varprojlim M''_i$$

is exact.

(26) Eisenbud, Exercise 7.11.

(27) Let R be a Noetherian ring. Show that $R[[x_1, \ldots, x_n]]$ is a Noetherian ring.

(28) Eisenbud, Exercise A3.6.

(29) Eisenbud, Exercise A3.13.

(30) Eisenbud, Exercise A3.14.

(31) Eisenbud, Exercise A3.16.

(32) Eisenbud, Exercise A3.18.

(33) Eisenbud, Exercise A3.23.

(34) Eisenbud, Exercise A3.24.