

# A life time of discontinuous schemings

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## Abstract

We present this special issue of the Journal of Scientific Computing to celebrate Bernardo Cockburn's sixtieth birthday. The theme of this issue is discontinuous Galerkin methods, a hallmark of Bernardo's distinguished professional career. This cover article provides an informal but rigorous account of what enabled Bernardo's achievements, based on the concluding presentation he gave at the workshop on July 1st, 2017 which was widely deemed as the best lecture of his career so far.

## 1 Introduction

With great enthusiasm, we are proud to present this special issue of the Journal of Scientific Computing in honor of Bernardo Cockburn on the occasion of his sixtieth birthday. It presents **XX** selected papers submitted by the participants of the special workshop entitled "Recent Advances and Challenges in Discontinuous Galerkin Methods and Related Approaches" taking place at the Institute of Mathematics and its Applications (IMA) from June 29 to July 1st. of 2017. The workshop aimed to bring together researchers at all levels and career stages, providing an opportunity to share and discuss recent progress in both theoretical and computational aspects of discontinuous Galerkin (DG) and related approaches. The list of attendees includes 111 participants from 15 countries/regions. The number and diversity of the participants, the warm and supportive atmosphere of the community at display during the workshop are testaments to the high esteem in which Bernardo is held in the international scientific computing community. It is impossible to adequately cover the extent of recent advances in theory and computation of DG and related approaches in a single issue. This issue only intends to present a fragment of the timely research topics and noteworthy new developments in the area.

A central figure in this field, Bernardo received his B.S. and M.S. degrees from the Universidad Nacional de Ingeniería, his 3rd. Cycle Doctorat from Paris VI-IX, and his Ph.D. degree from the University of Chicago, all in Mathematics or Applied Mathematics. After a year as a postdoctoral fellow at IMA, he joined the School of Mathematics at the University of Minnesota in 1987. Having spent his entire career there, he currently holds a McKnight University Professorship. Bernardo's work in the past three decades has made foundational impact in the field of discontinuous Galerkin method, in particular, and in Scientific Computing and Numerical Analysis in general. An invited speaker at the International Congress of Mathematicians in 2010, Bernardo has been listed as a Highly Cited Author in Mathematics by the ISI Web of Knowledge. In addition to his unique contributions to the field encompassing both theoretical analysis and applications, Bernardo has also had a tremendous impact in the scientific

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community by mentoring and supporting junior researchers. These include his Ph.D. students, postdocs, visiting scholars, and young colleagues from all over the world.

## 2 The *Academica Vitae* complex

In his talk, Bernardo argued that in order to understand our life as mathematicians, and give the proper meaning to the workshop, we must rely on the following, first-of-its-kind *Academica Vitae Complex* :

$$0 \longrightarrow \text{play} \xrightarrow{i} \text{school} \xrightarrow{\nabla} \text{PhD} \xrightarrow{\nabla \times} \text{Prof} \xrightarrow{\nabla \cdot} \text{Emeritus} \xrightarrow{o} 0$$

Asked to elaborate on the application of this complex, Bernardo explained that, unlike the well-known, closely related de Rham complex, the *Academica Vitae Complex* helps to describe the trajectories of our lives. Indeed, everyone of us steps out from zero to play. After a natural injection ( $i$ ) into school, most of us obtain a doctoral degree through overcoming a steep gradient ( $\nabla$ ). Our next stage of life is usually a swirling ( $\nabla \times$ ) experience in the postdoctoral years before becoming a professor. Decades of professorship, that is, of looking for nice results and avoiding falling into the abyss of irrelevance, eventually diverge ( $\nabla \cdot$ ) into becoming an emeritus professor. Finally, life comes to a full loop by bringing us back to zero ( $o$ ) for perpetuity.

So, a workshop like this is an ideal opportunity to experience a sense of belonging to a common undertaking as well as for expressing gratitude to the many people we met in this adventure.

### 2.1 A sense of belonging

Bernardo wanted “to bow seven or more times” at this special occasion to his advisors, pictured in Figure 1, Julio Ruiz-Claeyssen (University Nacional de Ingenieria), Guy Chavent (University Paris IX, INRIA), and Jim Douglas, Jr., (University of Chicago) for their guidance at various stages of his career.



Figure 1: Standing on the far left, Bernardo bows “**seven or more times**” to his advisors (from left to right) Julio Ruiz-Claeyssen, Guy Chavent, and Jim Douglas, Jr..

Julio Ruiz-Claeyssen introduced him to the world of dynamical systems, and in particular to the work of his own adviser **J. Hale**. Guy Chavent introduced him to the application of the Discontinuous Galerkin method to oil recovery simulation and so, to the pioneering work of **W.H. Reed** and **T.R. Hill**, and that of **P. Lesaint**, and **P. A. Raviart**, as well as to the work on numerical methods for nonlinear hyperbolic conservation laws of **A. Harten**, **P. Lax**, **B. van**

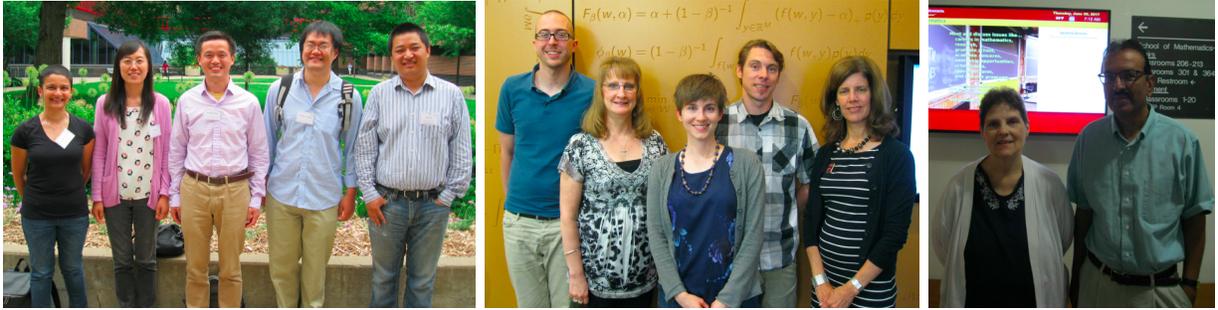


Figure 2: Bernardo’s “preliminary gratitude” to the organizers of the workshop (left: Nilima Nigam, Bo Dong, Yanlai Chen, Wujun Zhang, Ke Shi), the staff members of the IMA (middle: Samuel Richter, Georgia Kroll, Rebecca Malkovich, Chad Sullivan, Katherine Dowd), and members of the school of Mathematics at the University of Minnesota (right: Bonny Fleming, Harry Singh).



Figure 3: Bernardo’s “postliminary gratitude” to his former advisees (top left), Dominik Schötzau (top middle, who could not make the workshop because of his battle with cancer), to his one-and-only “favorite collaborator, lucid editor, and great hostess” Rosario Grau (top right), and to all participants of the workshop including those pictured above on the bottom row.

Leer, of **B. Engquist** and **S. Osher**, and to that of **S. K. Godunov**, **S. N. Kružkov**, and **N. N. Kuznetsov**. Finally, Jim Douglas, Jr., introduced him to the world of the finite element method and in particular to the work of **J. H. Bramble**, **A. H. Schatz**, **J. A. Nitsche**, and **V. Thomée**. Bernardo feels honored to have participated in the effort of trying to advance the knowledge in these areas.

## 2.2 Gratuities

Pictured in Figures 2 and 3 are those Bernardo wanted to express sincere gratitude to for the success of this workshop. In particular, he appreciates the work by the workshop organizers Nilima Nigam (of Simon Fraser University), Bo Dong, Yanlai Chen (both of the University of Massachusetts Dartmouth), Wujun Zhang (of Rutgers University), and Ke Shi (of Old Dominion University). The workshop would not have been possible without the diligent support by the staff members of the IMA Samuel Richter, Georgia Kroll, Rebecca Malkovich, Chad Sullivan, IMA Assisant Director Katherine Dowd, and members of the School of Mathematics at the University of Minnesota Bonny Fleming and Harry Singh.

Bernardo thanked Julio Ruiz-Claeyssen for his dilligent and constant nurturing during his undergraduate times which included contacting J.-L. Lions (on the occasion of the Fourth Latin-American School of Mathematics in 1978) to secure his eventual working with Guy Chavent in France. He also thanked Víctor Sánchez-Moya, for supporting his internship in Petro-Perú, the Peruvian national oil company, and for inviting Guy Chavent to give a course on DG methods for oil simulation there. Finally, he thanked Jim Douglas, Jr., for his support during (and after) his Ph.D. work.

Bernardo recalled the joy of working with all his advisees during the years. He thanked his 26 doctoral graduates and 19 postdoctoral scholars for trusting him in such crucial moment of their academic life. Bernardo acknowledged the gracious support to all postdocs by institutions like the IMA, the Mathematical Sciences Postdoctoral Research Fellowships at NSF, the School of Mathematics, and numerous foreign governments. The fact that they all chose to work with Bernardo exemplifies the appeal of his research and his welcoming personality. Finally, Bernardo was grateful to all his collaborators for having “played” with him and to all the participants for their presence in the workshop.

## 3 Algorithm: the tools of the craft

Bernardo argued that this workshop is also the right moment to share with the younger generations two sets of tools he has been counting on throughout his career. The first is the set of three so-called “Magical thinking tools of the Neolithic” (Figure 4). The first of the three is the “Hands on” approach which would be handy for when you are running out of ideas. The second is the “If Pigs Can Fly” way and is based on the dictum that “If pigs can fly, you can prove a theorem.” Rosario discovered the Flying Pig and hurried to buy the devise. It is always on Bernardo’s desk and needs only a few turns to make things happen. The “Revelation by Fortune Cookie” belief is that, if you ask a question with deep sincerity for a long, long time, chances are you will find its answer in a fortune cookie when you go to a Chinese restaurant.

Treasured as the second group are the “Rational thinking tools of the Enlightenment” (Figures 5 to 8). The best of them, as promptly pointed out by Bernardo, is to work with great collaborators – especially if they can write all the papers for you. Bernardo’s next crucial tool here is to stay focused in your direction avoiding discouragement. One fond story that Bernardo recalled is the review he received of his very first NSF proposal in 1988, not funded, on devising DG methods for nonlinear conservation laws. The reviewer concluded that “I think it is doubtful that important results will follow from this line of research”. In contrast, the world witnessed the explosive development of this exact line of research since then (Figure 6).

As to the now often-used metric, the number of citations of your articles, Bernardo jokingly pointed to his top-cited article (Figure 7) which, by that account, received more than a thousand

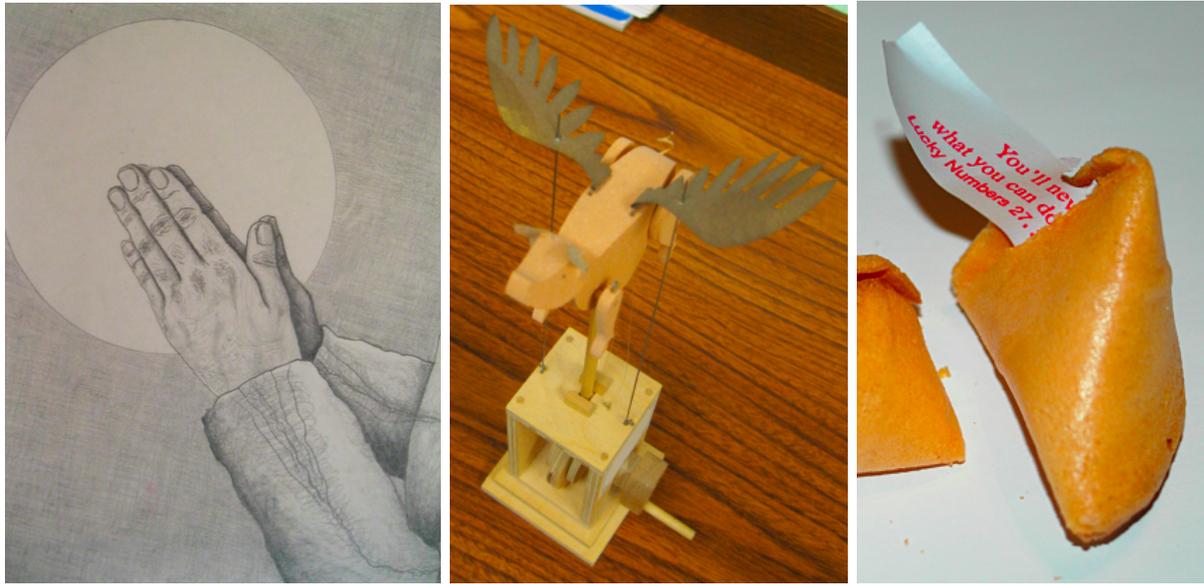


Figure 4: Bernardo's Magical thinking tools of the Neolithic: #1 The **Hands on** approach, #2 the **If Pigs Can Fly** way, and #3 the **Revelation by Fortune Cookie** belief.

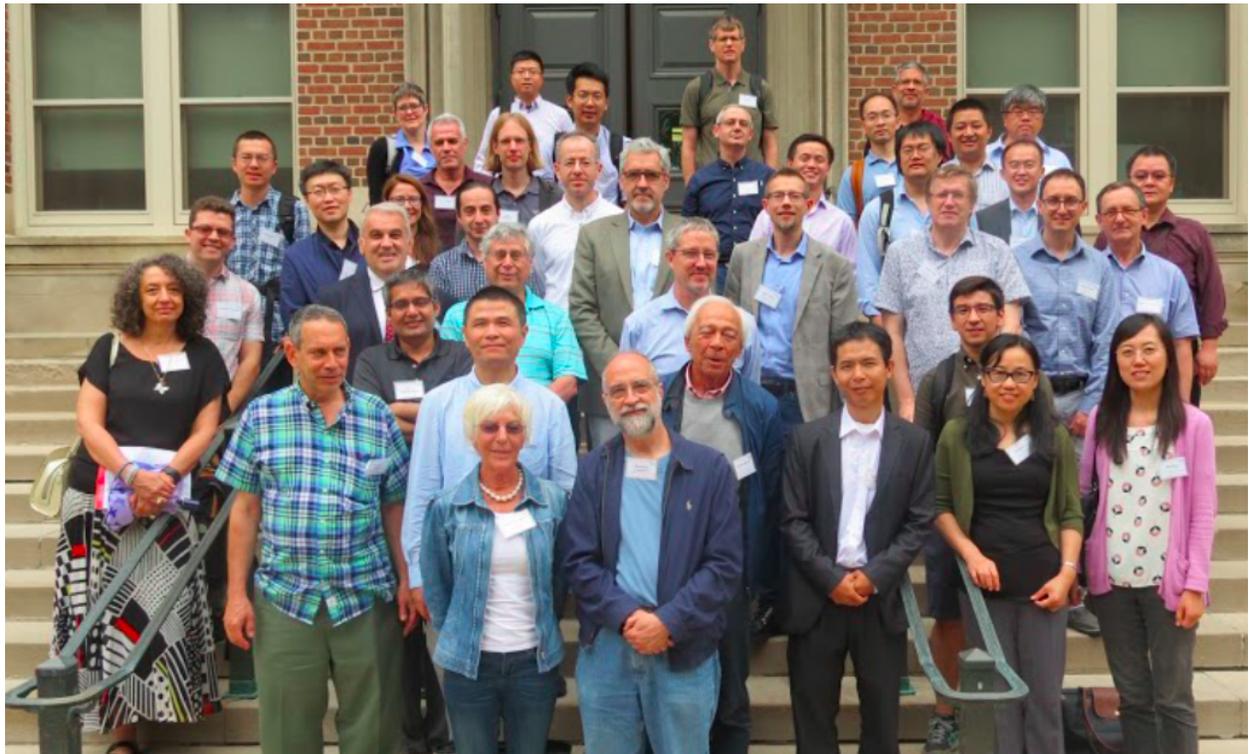


Figure 5: Bernardo's Rational thinking tool of the Enlightenment # 1: **Work with great collaborators**. Pictured are those who made the photo op.

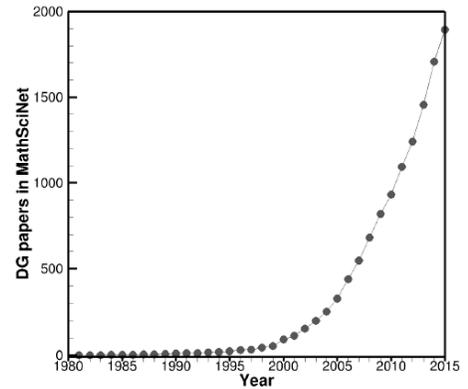
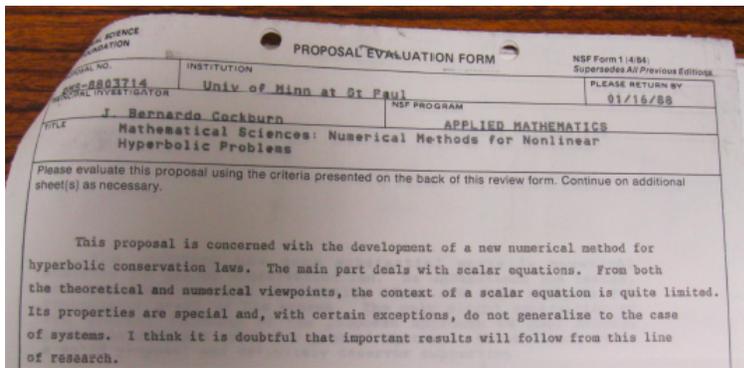


Figure 6: Bernardo’s Rational thinking tool of the Enlightenment # 2: **Do not take too seriously the NSF reviews.** On the left is one review of his 1988 NSF proposal on DG. The review concluded that “I think it is doubtful that important results will follow from this line of research”. Shown on the right is the exponential growth of the number of DG papers since then.

### Unified analysis of discontinuous Galerkin methods for elliptic problems

Arnold, D.N., Brezzi, F., Cockburn, B., Donatella Marini, L.

*SIAM Journal on Numerical Analysis*, vol. 39 (2001)

1,141	0
Citations	Readers

Figure 7: Bernardo’s Rational thinking tool of the Enlightenment # 3: **Take the number of citations with a grain of salt.** Pictured is the showing of Bernardo’s top-cited article receiving 1,141 citations at the time but 0 readers.

citations at the time but was read 0 times. He suggested to always take the number of citations with a grain of salt.

Bernardo characterized the next three Rational thinking tools of the Enlightenment (Figure 8), as being all about the efficiency of obtaining nice results. He started by warning against chasing the empty set. His favorite story was about the one and a half years he spent on looking for a bug that did not exist in his code for the very first double Mach reflection simulation by the DG method. At the same time, he did emphasize that we must be persistently iterate as many times as necessary to look for the ideal solution and its presentation even though the effort might be felt as Sisyphean. Finally, he claimed that, if you can avoid such arduous effort while still delivering, that would be even better. A technique allowing for this is when two existing results can be immediately combined to produce a new one, and called that the instant coffee theorem technique crediting his friend Bradley Lucier for introducing the concept in 1986. He urged people to use it as frequently as possible.

## 4 Numerical results

In this section, we detail the “numerical” results after years of execution of these algorithms. Listed in Figure 9 is the top of Bernardo’s Google Scholar page which, he would argue, must be taken with a grain of salt.

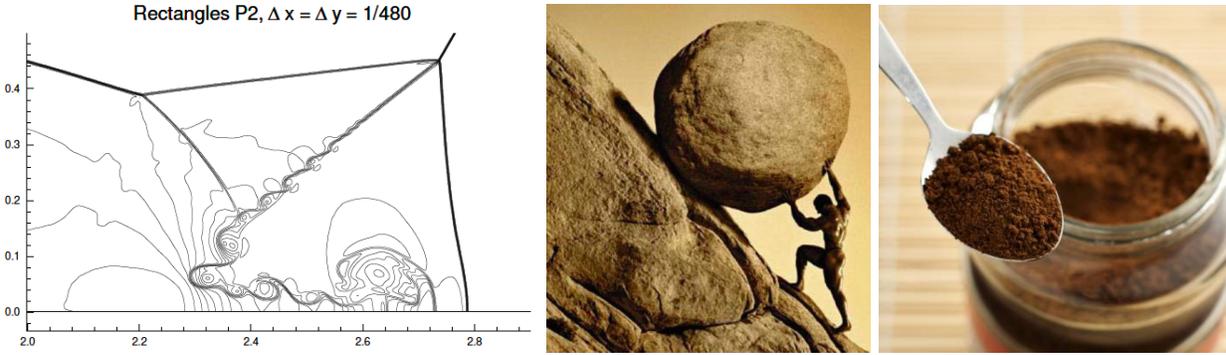


Figure 8: Bernardo’s Rational thinking tools of the Enlightenment: # 4 **Do not chase the empty set.** (Pictured on the left is the very first double Mach reflection simulation by a DG method generated by a bug-free code in which Bernardo looked for a bug.) # 5: **Iterate ad nauseam.** (Pictured in the middle is Sisyphus at work). # 6: **Use, as frequently as possible the instant coffee theorem technique** (an idea introduced by Bradley Lucier according to Bernardo).

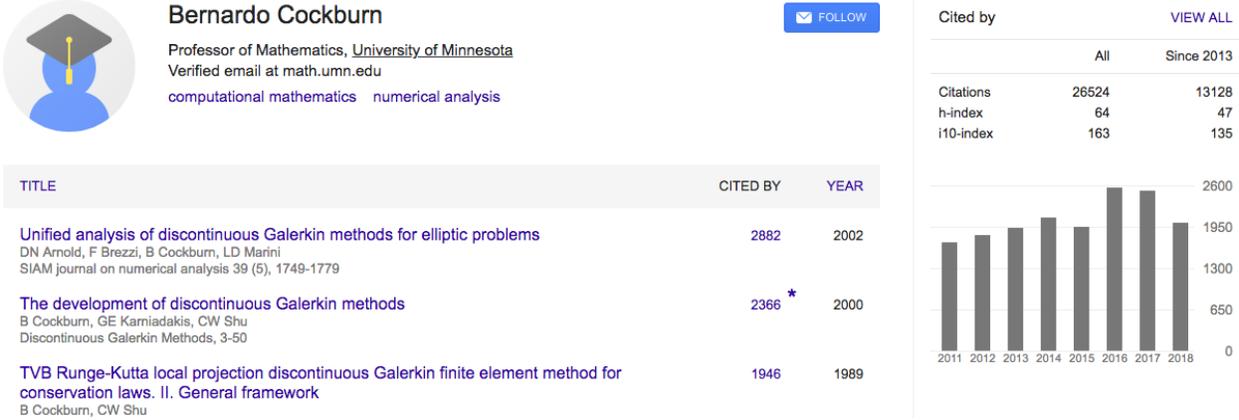


Figure 9: (Part of) Bernardo’s Google Scholar page, dated 8/6/2018.

## 5 Acknowledgement

It is the guest editors’ hope that this special issue will provide its readers an interesting glimpse into recent progress in the theory and computation of discontinuous Galerkin and related approaches. They would like to thank all the participants of the workshop, in particular the authors who submitted their papers to this issue, as well as to the reviewers for a rigorous review process and timely and constructive comments.

The role of IMA staff members Samuel Richter, Georgia Kroll, Rebecca Malkovich, Chad Sullivan, Assisant Director Katherine Dowd, and members of the School of Mathematics at the University of Minnesota Bonny Fleming and Harry Singh in the organization of the workshop can never be overstated, and their help is gratefully acknowledged. Last but not least, we express our sincere gratitude to Bernardo for the enlightening talk that inspired this article.