AIMS Exercise Set # 5

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1. Use the power method to find the dominant eigenvalue and associated eigenvector of the following matrices: (*a*) $\begin{pmatrix} -2 & 0 & 1 \\ -3 & -2 & 0 \\ -2 & 5 & 4 \end{pmatrix}$, (*b*) $\begin{pmatrix} 4 & 1 & 0 & 1 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 1 & 0 & 1 & 4 \end{pmatrix}$.

2. Use Newton's Method to find all points of intersection of the following pairs of plane curves: $x^3 + y^3 = 3$, $x^2 - y^2 = 2$.

3. The system $x^2 + x\,z = 2$, $x\,y - z^2 = -1$, $y^2 + z^2 = 1$, has a solution $x^\star = 1$, $y^\star = 0$, $z^\star = 1$. Consider a fixed point iteration scheme with

$$\mathbf{g}(x, y, z) = \bigl(x + \alpha(x^2 + x\,z - 2), y + \alpha(x\,y - z^2 + 1), z + \alpha(y^2 + z^2 - 1) \bigr)^T,$$

where α is a constant. (*a*) For which values of α does the iterative scheme converge to the solution when the initial guess is nearby? (*b*) What is the best value of α as far as the rate of convergence goes? (*c*) For the value of α from part (*a*) (or another value of your own choosing) about how many iterations are required to approximate the solution to 5 decimal places when the initial guess is $x^{(0)} = \frac{5}{6}$, $y^{(0)} = -\frac{1}{3}$, $z^{(0)} = \frac{9}{8}$? Test your estimate by running the iteration. (*d*) Write down the Newton iteration scheme for this system. (*e*) Answer part (*c*) for the Newton scheme.