Applied Linear Algebra, First Edition

by Peter J. Olver and Chehrzad Shakiban

Corrections to First Printing (2006)

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⋆⋆⋆ Page xvii ⋆⋆⋆ First bullet item "The core:"

Add the following as a footnote:

Caveat: The complete proof of Theorem 8.20 relies on material outside the core. In particular, the proof of part (a) requires the Hermitian inner product, which is covered in section 3.6. The proof of the general version of part (c), when the matrix A has multiple eigenvalues, relies on developments in Chapter 7. If (as often happens in a one semester course) time constraints have prevented one from introducing the required material, one needs to skip, or at least downplay, those aspects of the proofs.

⋆⋆⋆ Page xxii ⋆⋆⋆

Add the following paragraph after the last paragraph:

We would also like to thank Nihat Bayhan, Joe Benson, Juan Cockburn, Richard Cook, Stephen DeSalvo, Anne Dougherty, Kathleen Fuller, Mary Halloran, Stuart Hastings, Jeffrey Humpherys, Roberta Jaskolski, Tian–Jun Li, James Meiss, Willard Miller, Jr., Sean Rostami, Timo Schürg, David Tieri, and Timothy Welle for sending us their comments, suggestions, and corrections to earlier printings of this book. A particular thanks to David Hiebeler for his careful reading and corrections.

⋆⋆⋆ Page xxii ⋆⋆⋆

Change Cheri Shakiban's email address to `cshakiban@stthomas.edu`

⋆⋆⋆ Page 11 ⋆⋆⋆ Exercise 1.2.31 (d):

Change the first term in the first bilinearity formula from $c\,[\,A,B\,]$ to $c\,[\,A,C\,]$:

$$[\,cA + dB, C\,] = c\,[\,A,C\,] + d\,[\,B,C\,]$$

⋆⋆⋆ Page 43 ⋆⋆⋆ Theorem 1.29:

To avoid confusion, change "having the nonzero pivots on the diagonal" to "with nonzero diagonal entries".

⋆⋆⋆ Page 43 ⋆⋆⋆ Theorem 1.31:

Insert "with nonzero diagonal entries" after "diagonal".

⋆⋆⋆ Page 48 ⋆⋆⋆ Example 1.36:

In the first displayed formula, change the $(1, 2)$ entry of U to b:

$$U = \begin{pmatrix} a & b \\ 0 & \dfrac{ac - b^2}{a} \end{pmatrix}$$

⋆⋆⋆ Page 51 ⋆⋆⋆ second displayed equation:

Replace the summand j by $j - 1$:

$$\sum_{j=1}^{n} (j - 1) = \frac{n^2 - n}{2}$$

⋆⋆⋆ Page 57 ⋆⋆⋆ last displayed equation:

Replace 3210 by 32100:

$$10\,x + 1600\,y = 32100, \qquad x + .6\,y = 22,$$

⋆⋆⋆ Page 58 ⋆⋆⋆ first displayed equation:

Replace 3210 by 32100:

$$\left(\begin{array}{cc|c} 1600 & 10 & 32100 \\ .6 & 1 & 22 \end{array} \right)$$

⋆⋆⋆ Page 79 ⋆⋆⋆ Example 2.2:

Change $\mathbf{v} + \mathbf{v}$ to $\mathbf{v} + \mathbf{w}$ in the displayed formula:

$$\mathbf{v} + \mathbf{w} = \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \vdots \\ v_n + w_n \end{pmatrix}, \qquad c\mathbf{v} = \begin{pmatrix} cv_1 \\ cv_2 \\ \vdots \\ cv_n \end{pmatrix}, \qquad \text{whenever} \qquad \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}.$$

⋆⋆⋆ Page 88 ⋆⋆⋆ Exercise 2.2.22 (b):

Change $\mathbf{v}$ to $\mathbf{w}$:

$$W + Z = \{\, \mathbf{w} + \mathbf{z} \mid \mathbf{w} \in W, \ \mathbf{z} \in Z \,\}$$

⋆⋆⋆ Page 90 ⋆⋆⋆ Last displayed equation:

Correct last entry in third and fourth column vectors:

$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1 + 2c_2 \\ -2c_1 - 3c_2 \\ c_1 + c_2 \end{pmatrix}.$$

$\star\star\star$ Page 91 $\star\star\star$ First displayed equation:

Correct third equation:

$$c_1 + 2\,c_2 = 0, \qquad -2\,c_1 - 3\,c_2 = 1, \qquad c_1 + c_2 = -1.$$

$\star\star\star$ Page 100 $\star\star\star$ Exercise 2.3.39 (b):

Add closing bracket to $W[\,f(x), g(x)\,]$.

$\star\star\star$ Page 106 $\star\star\star$ Exercise 2.4.24 (b):

Change "Under the hypotheses of part (b)" to "Under the hypothesis of part (a)".

$\star\star\star$ Page 118 $\star\star\star$ lines 12–13:

Change "Solving the homogeneous system $\widehat{U}\mathbf{y} = \mathbf{0}$, we conclude that" to

The two nonzero rows of $\widehat{U}$ form a basis for corng A^T, and therefore

$\star\star\star$ Page 130 $\star\star\star$ line 6 and line 8:

Delete first "both"'s:

Inner products and norms lie at the heart of linear (and nonlinear) analysis, in both finite-dimensional vector spaces and infinite-dimensional function spaces. It is impossible to overemphasize their importance for theoretical developments, practical applications, and the design of numerical solution algorithms.

$\star\star\star$ Page 138 $\star\star\star$

Add the following sentence after the last displayed formula:

Also, as noted above, equality holds if and only if $\mathbf{v}$ and $\mathbf{w}$ are parallel.

$\star\star\star$ Page 143 $\star\star\star$ Exercise 3.2.31 (a):

Add ^{T} superscripts to $(\,1, 2, 3\,)^T$ and $(\,1, -1, 2\,)^T$.

$\star\star\star$ Page 161 $\star\star\star$ Formula before Proposition 3.34:

Change dt to dx.

$\star\star\star$ Page 161 $\star\star\star$ Two lines before Proposition 3.34:

Change Theorem 3.31 to Theorem 3.28.

$\star\star\star$ Page 162 $\star\star\star$ Exercise 3.4.22 (c):

Change "null vectors" to "null directions".

⋆⋆⋆ Page 162 ⋆⋆⋆ Exercise 3.4.32:

Change "null vector" to "null direction" and $K = A^T A$ to $K = A^T C A$:

Show that $\mathbf{0} \neq \mathbf{z}$ is a null direction for the quadratic form $q(\mathbf{x}) = \mathbf{x}^T K \mathbf{x}$ based on the Gram matrix $K = A^T C A$ if and only if $\mathbf{z} \in \ker K$.

⋆⋆⋆ Page 163 ⋆⋆⋆ Exercise 3.4.35 (c):

Rephrase for clarity:

Show that K is also a Gram matrix, by finding a matrix A such that $K = A^T C A$.

⋆⋆⋆ Page 167 ⋆⋆⋆ Exercise 3.5.10 (c):

Change "2×2 matrix" to "2×2 symmetric matrix".

⋆⋆⋆ Page 168 ⋆⋆⋆ Sentence after that containing (3.70):

Rephrase for clarity:

Note that M is a lower triangular matrix with all positive diagonal entries, namely the square roots of the pivots: $m_{ii} = \sqrt{d_i}$.

⋆⋆⋆ Page 177 ⋆⋆⋆ Exercise 3.6.29:

Delete part (e). ("Orthogonal" and "orthonormal" are not yet defined.)

⋆⋆⋆ Page 179 ⋆⋆⋆ Exercise 3.6.51:

For each of the following . . .

⋆⋆⋆ Page 188 ⋆⋆⋆ Formula in middle of page:

y^* and z^* should not be bold face.

⋆⋆⋆ Page 188 ⋆⋆⋆ Theorem 4.4:

Delete the words "null vector".

⋆⋆⋆ Page 191 ⋆⋆⋆ Equation (4.26):

The last equality, $c = \| \mathbf{b} \|^2$, is correct provided one uses the weighted norm. However, to avoid confusion with the Euclidean norm used in (4.25), it would be better to write this as $c = \mathbf{b}^T C \mathbf{b}$.

⋆⋆⋆ Page 194 ⋆⋆⋆ Equation (4.30):

$$\| A\mathbf{x}^\star - \mathbf{b} \| = \sqrt{\| \mathbf{b} \|^2 - \mathbf{f}^T \mathbf{x}^\star} = \sqrt{\| \mathbf{b} \|^2 - \mathbf{b}^T A (A^T A)^{-1} A^T \mathbf{b}}.$$

★★★ Page 201 ★★★ Line after Equation (4.43) :

Delete first "coefficient":

The $m \times (n+1)$ coefficient matrix ...

★★★ Page 204 ★★★ Ddisplayed equation in Proof of Theorem 4.16:

Middle term should be $y_k L_k(t_k)$:

$$p(t_k) = y_1 L_1(t_k) + \cdots + y_k L_k(t_k) + \cdots + y_{n+1} L_{n+1}(t_k) = y_k,$$

★★★ Page 221 ★★★ Exercise 5.1.11:

$$\mathbf{u}_2 = \pm \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

★★★ Page 231 ★★★ Second line:

Replace "For exercises #1–8 use " by "For Exercises #1–7 use ".

★★★ Page 236 ★★★ Example 5.20:

Change $ac + bd$ to $ab + cd$ in the second (twice), third and fifth displayed formulas:

Equivalently, the requirement

$$Q^T Q = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2 + c^2 & ab + cd \\ ab + cd & b^2 + d^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

implies that its entries must satisfy the algebraic equations

$$a^2 + c^2 = 1, \qquad ab + cd = 0, \qquad b^2 + d^2 = 1.$$

The first and last equations say that the points $(a, c)^T$ and $(b, d)^T$ lie on the unit circle in $\mathbb{R}^2$, and so

$$a = \cos\theta, \qquad c = \sin\theta, \qquad b = \cos\psi, \qquad d = \sin\psi,$$

for some choice of angles θ, ψ. The remaining orthogonality condition is

$$0 = ab + cd = \cos\theta \ \cos\psi + \sin\theta \ \sin\psi = \cos(\theta - \psi),$$

★★★ Page 239 ★★★ Exercise 5.3.19:

Delete (a), and then change (b) to (a), and (c) to (b).

★★★ Page 240 ★★★ Exercise 5.3.22:

Delete (a), and then change (b) to (a), (c) to (b), and (d) to (c).

★★★ Page 245 ★★★ First line under Figure:

Change "$\mathbf{x}$ and $\mathbf{y}$" to "$\mathbf{v}$ and $\mathbf{w}$".

★★★ Page 250 ★★★ Displayed equation after (5.47):

Change period after $P_5(t) = \frac{63}{8}\,t^5 - \frac{35}{4}\,t^3 + \frac{15}{8}\,t$, to comma.

★★★ Page 261 ★★★ First displayed equation in Example 5.41:

Change last $\mathbf{w}_1$ to $\mathbf{v}_1$:

$$\mathbf{v}_2 = \mathbf{w}_2 - \frac{\mathbf{w}_2 \cdot \mathbf{v}_1}{\|\,\mathbf{v}_1\,\|^2}\,\mathbf{v}_1 = \begin{pmatrix} \frac{5}{2} \\ -2 \\ -\frac{3}{2} \end{pmatrix},$$

★★★ Page 274 ★★★ Remark:

… solving the homogeneous adjoint system, …

★★★ Page 276 ★★★ Exercise 5.6.20 (c):

Change the sign in front of $4\,x_3$ in last equation:

$x_1 + 2\,x_2 + 3\,x_3 = b_1,\;\; x_2 + 2\,x_3 = b_2,\;\; 3\,x_1 + 5\,x_2 + 7\,x_3 = b_3,\;\; -2\,x_1 + x_2 + 4\,x_3 = b_4;$

★★★ Page 279 ★★★ Equation (5.90):

Change $e^{\,i\,k\,x_n}$ to $e^{\,i\,k\,x_{n-1}}$ in first line.

★★★ Page 280 ★★★ Equation (5.94):

Correct the value of $\zeta_3 = -\frac{1}{2} + \frac{\sqrt{3}}{2}\,i$.

★★★ Page 283 ★★★ Figure 5.13:

Capitalize "Fourier" and change $x^2 - 2\,\pi\,x$ to $2\,\pi\,x - x^2$.

★★★ Page 284 ★★★ Figure 5.14:

Change $x^2 - 2\,\pi\,x$ to $2\,\pi\,x - x^2$.

★★★ Page 285 ★★★ Line -5:

Change $n = 2^8 = 256$ to $n = 2^9 = 512$.

★★★ Page 296 ★★★ Equation (6.9):

Insert space between 1 and -1 in last row of matrix.

★★★ Page 298 ★★★ Two lines before (6.15):

Change $K\,\mathbf{x} = \mathbf{f}$ to $K\,\mathbf{u} = \mathbf{f}$.

$\star\star\star$ Page 298 $\star\star\star$ Four lines after (6.15):

Change $\mathbf{y} = A^{-1}\mathbf{f}$ to $\mathbf{y} = A^{-T}\mathbf{f}$.

$\star\star\star$ Page 311 $\star\star\star$ Exercise 6.2.1 (b):
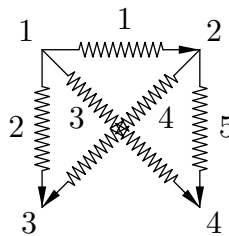
Change last row of matrix:

$$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

$\star\star\star$ Page 311 $\star\star\star$ Exercise 6.2.2:

Add labels to the wires:

$\star\star\star$ Page 312 $\star\star\star$ Exercise 6.2.12 (a):

Start the exercise with:

 Assuming all wires have unit resistance, find the voltage . . .

$\star\star\star$ Page 312 $\star\star\star$ Exercise 6.2.15 (a):

Change "a battery" to "batteries".

$\star\star\star$ Page 312 $\star\star\star$ Exercise 6.2.17, add new part:

(c) Write down a formula for the power in the network.

$\star\star\star$ Page 313 $\star\star\star$ Line 19:

Delete "the" before "Section 6.1".

$\star\star\star$ Page 317 $\star\star\star$ Displayed equation above (6.51):

Change 0 to $\mathbf{0}$.

$\star\star\star$ Page 319 $\star\star\star$ Equation (6.58):

Change the last formula to

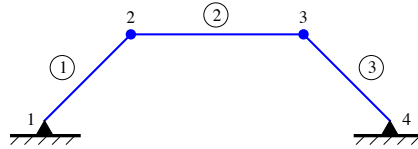$$\mathbf{z}_3 \cdot \mathbf{f} = -\tfrac{\sqrt{3}}{2}\, f_1 + \tfrac{1}{2}\, g_1 + g_2 = 0.$$

$\star\star\star$ Page 319 $\star\star\star$ Last line:

Change "first node" to "third node".

$\star\star\star$ Page 320 $\star\star\star$ Two lines before displayed equation for $A^{\star\star}$:

This serves to also eliminate ...

$\star\star\star$ Page 321 $\star\star\star$ Figure 6.13:

Label the bars in the figure:

$\star\star\star$ Page 322 $\star\star\star$ Example 6.7:

Change the last displayed equation and subsequent text to:

$$\mathbf{y} = \mathbf{e} = A^\star \, \mathbf{u}^\star = \left(\ \sqrt{2}, \ 1, \ \sqrt{2} \ \right)^T,$$

are well-defined, indicating that, under our stabilizing uniform vertical (upwards) force, all three bars are elongated, with the two diagonals experiencing 41.4% more elongation than the horizontal bar.

$\star\star\star$ Page 323 $\star\star\star$ Figure 6.16:

Label the bars in the figure:
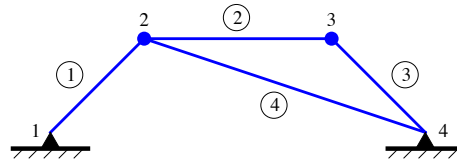
$\star\star\star$ Page 324 $\star\star\star$ First displayed formula:

Change final period to comma.

$\star\star\star$ Page 324 $\star\star\star$ Figure 6.17:

Label the bars in the figure:
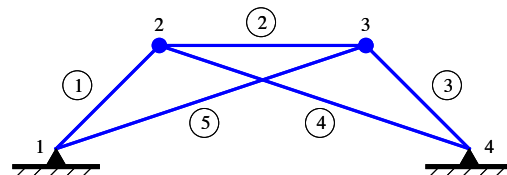
$\star\star\star$ Page 325 $\star\star\star$ Line -5:

Change "three bars" to "five bars".

$\star\star\star$ Page 327 $\star\star\star$ Exercise 6.3.7:

Change A to $A^\star$ (3 times).

$\star\star\star$ Page 339 $\star\star\star$ Equation (7.12):

Add period at end of equation.

★★★ Page 343 ★★★ Exercise 7.1.47, last line:

Change "part (b)" to "part (c)".

★★★ Page 362 ★★★ Exercise 7.3.16:

Add $\diamondsuit$ before exercise number.

★★★ Page 377 ★★★ Definition 7.46 is incomplete. Here is a corrected version::

Definition 7.46. A complex vector space V is called *conjugated* if it admits an operation of *complex conjugation* taking $\mathbf{u} \in V$ to $\overline{\mathbf{u}} \in V$ with the following properties: (a) conjugating twice returns one to the original vector: $\overline{\overline{\mathbf{u}}} = \mathbf{u}$; (b) compatibility with vector addition: $\overline{\mathbf{u} + \mathbf{v}} = \overline{\mathbf{u}} + \overline{\mathbf{v}}$; (c) compatibility with scalar multiplication: $\overline{\lambda \mathbf{u}} = \overline{\lambda}\, \overline{\mathbf{u}}$, for all $\lambda \in \mathbb{C}$ and $\mathbf{u}, \mathbf{v} \in V$.

★★★ Page 382 ★★★ second displayed formula:

The inner product should have double angle brackets:

$$\langle\!\langle\, \mathbf{v}\, , \widetilde{\mathbf{v}}\, \rangle\!\rangle \qquad \text{to denote the inner product between} \qquad \mathbf{v}, \widetilde{\mathbf{v}} \in V.$$

★★★ Page 384 ★★★ Exercise 7.5.1 (b):

$$\langle\, \mathbf{v}\, , \mathbf{w}\, \rangle = 2\, v_1\, w_1 + 3\, v_2\, w_2$$

★★★ Page 399 ★★★ Final displayed equation:

The bar on the second term should extend over both A and $\mathbf{v}$:

$$\overline{A}\,\overline{\mathbf{v}} = \overline{A\,\mathbf{v}} = \overline{\lambda\,\mathbf{v}} = \overline{\lambda}\,\overline{\mathbf{v}}.$$

★★★ Page 400 ★★★ Two lines before Remark:

Change "combinations of the real eigenvalues" to " combinations of the real eigenvectors".

★★★ Page 400 ★★★ Line 3 of item (c):

Insert identity matrix in equation after $\widetilde{\lambda}$:

$$(A - \widetilde{\lambda}\,\mathrm{I})\mathbf{v} = \mathbf{0}$$

★★★ Page 410 ★★★ line 16:

Delete ^{T} on formula for $S = (\, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\,)$.

★★★ Page 411 ★★★ Last line:

Delete "the" before "Section 8.6".

★★★ Page 428 ★★★ Line 9 from bottom::

Change "orthogonal stretching transformations" to "stretching transformation along the coordinate axes".

★★★ Page 432 ★★★ Change lines 7–11 to:

and, as you can verify

$$A = P\Sigma Q^T = \begin{pmatrix} -.2180 & -.7869 \\ .7869 & -.2180 \\ -.5024 & -.2845 \\ .2845 & -.5024 \end{pmatrix} \begin{pmatrix} 6.4415 & 0 \\ 0 & 2.5509 \end{pmatrix} \begin{pmatrix} .4990 & -.8344 & .2340 \\ -.8472 & -.4128 & .3345 \end{pmatrix}$$

is the singular value decomposition of our coefficient matrix. Its pseudoinverse is immediately computed:

$$A^+ = Q\,\Sigma^{-1}P^T = \begin{pmatrix} .2444 & .1333 & .0556 & .1889 \\ .1556 & -.0667 & .1111 & .0444 \\ -.1111 & 0 & -.0556 & -.0556 \end{pmatrix}.$$

★★★ Page 438 ★★★ **Definition 8.46**, first line:

  Change $\mathbf{w}_1, \ldots, \mathbf{w}_j \in \mathbb{C}^m$ to $\mathbf{w}_1, \ldots, \mathbf{w}_j \in \mathbb{C}^n$.

★★★ Page 438 ★★★ line -2:

  Change "Thus, $\mathbf{w}_2$ a generalized …" to "Thus, $\mathbf{w}_2$ is a generalized …"

★★★ Page 448 ★★★ Equation (9.8) and line 19:

  Change $\dot{u} = A\mathbf{u}$ to $\dot{\mathbf{u}} = A\mathbf{u}$.

★★★ Page 455 ★★★ Exercise 9.1.22:

  Change $\dot{u} = A\mathbf{u}$ to $\dot{\mathbf{u}} = A\mathbf{u}$.

★★★ Page 458 ★★★ Theorem 9.13:

  Change $\dot{u} = A\mathbf{u}$ to $\dot{\mathbf{u}} = A\mathbf{u}$.

★★★ Page 465 ★★★ Exercise 9.2.18:

  Change $\dot{u} = -\nabla H$ to $\dot{\mathbf{u}} = -\nabla H$.

★★★ Page 468 ★★★ Equation (9.31):

  Change the sign in front of the last term:

$$\mathbf{u}(t) = r\,e^{\mu t}\,[\cos(\nu t - \sigma)\,\mathbf{w} - \sin(\nu t - \sigma)\,\mathbf{z}], \tag{9.31}$$

★★★ Page 477 ★★★ Exercise 9.4.15:

  Change $\mathbf{v} = A^T\mathbf{v}$ to $\dot{\mathbf{v}} = A^T\mathbf{v}$.

★★★ Page 481 ★★★ Exercise 9.4.34:

  Change $\dot{u} = A\mathbf{u} + e^{\lambda t}\,\mathbf{v}$ to $\dot{\mathbf{u}} = A\mathbf{u} + e^{\lambda t}\,\mathbf{v}$.

⋆⋆⋆ Page 483 ⋆⋆⋆ Equation (9.55):

Correct final formula:

$$e^{t\,A_z} = \begin{pmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

⋆⋆⋆ Page 488 ⋆⋆⋆ line after (9.70):

Change $r_i > 0$ to $r_i \geq 0$.

⋆⋆⋆ Page 501 ⋆⋆⋆ Lines 2–4 after (9.96):

Switch "first" and "second":
... — the second, vibrating with frequency ω, represents the internal or natural vibrations of the system, while the first, with frequency η, represents the response ...

⋆⋆⋆ Page 509 ⋆⋆⋆ Exercise 9.6.21:

Capitalize "The" in part (a). Add semicolons at the end of each line in parts (a–e). Add a period at the end of part (f).

⋆⋆⋆ Page 517 ⋆⋆⋆ First line of equation (10.15):

$$\mathbf{u}^{(k)} = \begin{pmatrix} u^{(k)} \\ u^{(k+1)} \end{pmatrix}$$

⋆⋆⋆ Page 523 ⋆⋆⋆ Exercise 10.1.41:

Change $x_0, x_1, \ldots$ to $u^{(0)}, u^{(1)}, \ldots$

⋆⋆⋆ Page 526 ⋆⋆⋆ Line 4:

Change $-\frac{2}{3}$ to $\frac{2}{3}$: $\mathbf{u}^{(k)} \approx c_1 \left(\frac{2}{3}\right)^k \mathbf{v}_1$

⋆⋆⋆ Page 526 ⋆⋆⋆ Line 5:

Change $\lambda_1 = -\frac{2}{3}$ to $\lambda_1 = \frac{2}{3}$:

⋆⋆⋆ Page 526 ⋆⋆⋆ Line 8:

Change "... the first ten iterates are" to "... iterates $\mathbf{u}^{(11)}, \ldots, \mathbf{u}^{(20)}$ are"

⋆⋆⋆ Page 533 ⋆⋆⋆ Line 9 from bottom:

Change the sign of the $(2, 1)$ entry of the matrix:

$$A = \begin{pmatrix} \frac{1}{2} & -\frac{3}{5} \\ -\frac{3}{5} & \frac{1}{4} \end{pmatrix} \qquad \text{has matrix norm} \qquad \|A\|_\infty = \tfrac{11}{10} > 1.$$

Note: The matrix in the text also illustrates the point: it has eigenvalues $(15 \pm i\sqrt{551})/40$ and, hence, spectral radius $\rho(A) = .6964$.

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★★★ Page 556 ★★★ Exercise 10.5.12:

In parts (e,g), change T to $\widetilde{T}$ and $\mathbf{c}$ to $\widetilde{\mathbf{c}}$:

(e) Write the Gauss–Seidel iteration in the form $\mathbf{x}^{(k+1)} = \widetilde{T}\mathbf{x}^{(k)} + \widetilde{\mathbf{c}}$. Find the 3×3 matrix $\widetilde{T}$ and the vector $\widetilde{\mathbf{c}}$ explicitly.

(g) Determine the spectral radius of the Gauss–Seidel matrix $\widetilde{T}$. Which method converges faster?

★★★ Page 564 ★★★ Lines 15–17:

Change
the parameter t_1 so that the corresponding residual vector

$$\mathbf{r}_1 = \mathbf{f} - K\mathbf{u}_1 = \mathbf{r}_0 - t_1 K\mathbf{v}_1 \tag{10.91}$$

is as close to $\mathbf{0}$ (in the Euclidean norm) as possible. This occurs when $\mathbf{r}_1$ is orthogonal to $\mathbf{r}_0$ (why?), and so we require

$$0 = \mathbf{r}_0^T\mathbf{r}_1 = \|\mathbf{r}_0\|^2 - t_1\,\mathbf{r}_0^T K\mathbf{v}_1 = \|\mathbf{r}_0\|^2 - t_1\,\langle\!\langle\,\mathbf{r}_0\,,\mathbf{v}_1\,\rangle\!\rangle = \|\mathbf{r}_0\|^2 - t_1\,\langle\!\langle\,\mathbf{v}_1\,,\mathbf{v}_1\,\rangle\!\rangle. \tag{10.92}$$

to
the parameter t_1 that minimizes

$$p(\mathbf{u}_1) = p(t_1\mathbf{v}_1) = \tfrac{1}{2}t_1^2\mathbf{v}_1^T K\mathbf{v}_1 - t_1\,\mathbf{v}_1^T\mathbf{f} = \tfrac{1}{2}t_1^2\,\langle\!\langle\,\mathbf{v}_1\,,\mathbf{v}_1\,\rangle\!\rangle - t_1\|\mathbf{r}_1\|^2. \tag{10.91}$$

★★★ Page 564 ★★★ Line 7 from bottom:

Correct second and third terms in displayed formula:

$$0 = \langle\!\langle\,\mathbf{v}_2\,,\mathbf{v}_1\,\rangle\!\rangle = \langle\!\langle\,\mathbf{r}_1 + s_1\,\mathbf{v}_1\,,\mathbf{v}_1\,\rangle\!\rangle = \langle\!\langle\,\mathbf{r}_1\,,\mathbf{v}_1\,\rangle\!\rangle + s_1\,\langle\!\langle\,\mathbf{v}_1\,,\mathbf{v}_1\,\rangle\!\rangle,$$

★★★ Page 565 ★★★ Line 7:

Delete "as small as possible, which is accomplished by requiring it to"

★★★ Page 572 ★★★ Displayed formula before (10.102):

The subscripts on R and Q are wrong: $A_2 = R_1 Q_1$.

★★★ Page 572 ★★★ change final sentence::

For each eigenvalue, the computation of the corresponding eigenvector can be most efficiently accomplished by applying the shifted inverse power method of Exercise 10.6.7 with parameter μ chosen near the computed eigenvalue.

★★★ Page 573 ★★★ Second displayed formula in Example 10.54:

Switch the minus sign in the matrix Q_0 from the $(2,1)$ to the $(1,2)$ entry:

$$Q_0 = \begin{pmatrix} .7071 & -.7071 \\ .7071 & .7071 \end{pmatrix}, \qquad R_0 = \begin{pmatrix} 2.8284 & 2.8284 \\ 0 & 1.4142 \end{pmatrix}.$$

★★★ Page 575 ★★★ Change equations (10.106) and (10.107) to:

$$A^k = \left(Q_0 \, Q_1 \cdots Q_{k-1} \right) \left(R_{k-1} \cdots R_1 \, R_0 \right). \tag{10.106}$$

$$\begin{aligned} S_k &= Q_0 \, Q_1 \cdots Q_{k-1} = S_{k-1} \, Q_{k-1}, \\ P_k &= R_{k-1} \cdots R_1 \, R_0 = R_{k-1} \, P_{k-1}. \end{aligned} \tag{10.107}$$

★★★ Page 577 ★★★ Replace the paragraph after Theorem 10.57 by the following::

The last remaining item is a proof of Lemma 10.56. We write

$$S = \left(\mathbf{u}_1 \ \mathbf{u}_2 \ \ldots \ \mathbf{u}_n \right), \qquad S_k = (\mathbf{u}_1^{(k)}, \ldots, \mathbf{u}_n^{(k)})$$

in columnar form. Let $t_{ij}^{(k)}$ denote the entries of the positive upper triangular matrix T_k. The first column of the limiting equation $S_k \, T_k \to S$ reads

$$t_{11}^{(k)} \, \mathbf{u}_1^{(k)} \ \longrightarrow \ \mathbf{u}_1.$$

Since both $\mathbf{u}_1^{(k)}$ and $\mathbf{u}_1$ are unit vectors, and $t_{11}^{(k)} > 0$,

$$\| \, t_{11}^{(k)} \, \mathbf{u}_1^{(k)} \, \| = t_{11}^{(k)} \ \longrightarrow \ \| \, \mathbf{u}_1 \, \| = 1, \quad \text{and hence} \quad \mathbf{u}_1^{(k)} \ \longrightarrow \ \mathbf{u}_1.$$

The second column reads
$$t_{12}^{(k)} \, \mathbf{u}_1^{(k)} + t_{22}^{(k)} \, \mathbf{u}_2^{(k)} \ \longrightarrow \ \mathbf{u}_2.$$

Taking the inner product with $\mathbf{u}_1^{(k)} \to \mathbf{u}_1$ and using orthonormality, we deduce $t_{12}^{(k)} \to 0$, and so $t_{22}^{(k)} \, \mathbf{u}_2^{(k)} \to \mathbf{u}_2$, which, by the previous reasoning, implies $t_{22}^{(k)} \to 1$ and $\mathbf{u}_2^{(k)} \to \mathbf{u}_2$. The proof is completed by working in order through the remaining columns, employing a similar argument at each step. Details are left to the interested reader.

★★★ Page 582 ★★★ Exercise 10.6.25:

(a): "…a generic $n \times n$ symmetric matrix…"
(c): "…eigenvalues of a symmetric matrix?"

★★★ Page 591 ★★★ Equation (11.21):

Insert minus sign before integral:

$$u'(\ell) = - \int_0^\ell f(x) \, dx = 0, \tag{11.21}$$

★★★ Page 594 ★★★ Line before (11.28):

Change "to satisfy" to "satisfy".

★★★ Page 598 ★★★ 3 lines after (11.40):

Change $L[u] = u(y)$ to $L_y[u] = u(y)$.

$\star\star\star$ Page 607 $\star\star\star$ Equation (11.60):

Missing factor of c in differential equation:

$$-cu'' = f(x), \qquad u(0) = 0 = u(1),$$

$\star\star\star$ Page 607 $\star\star\star$ Equation (11.59):

The middle expression is missing a c in the denominator:

$$G(x, y) = \frac{(1-y)x - \rho(x-y)}{c} = \begin{cases} x(1-y)/c, & x \le y, \\ y(1-x)/c, & x \ge y, \end{cases} \tag{11.59}$$

$\star\star\star$ Page 608 $\star\star\star$ Lines 10 and 7 from bottom:

Two missing factors of c:

$$c\,\frac{du}{dx} = (1-x)\,x\,f(x) + \int_0^x [-y\,f(y)]\,dy - x\,(1-x)\,f(x) + \int_x^1 (1-y)\,f(y)\,dy$$

$$= -\int_0^1 y\,f(y)\,dy + \int_x^1 f(y)\,dy.$$

Differentiating again, we conclude that $c\,\dfrac{d^2u}{dx^2} = -f(x)$, as claimed.

$\star\star\star$ Page 619 $\star\star\star$ Exercise 11.3.16 (b):

Delete "is" after $K = L^* \circ L$.

$\star\star\star$ Page 640 $\star\star\star$ Section 11.6, middle of second paragraph:

Change "Chapter 4.1" to "Section 4.1".

$\star\star\star$ Page 653 $\star\star\star$ Solution 1.2.4 (d):

$$A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & -1 & 3 \\ 3 & 0 & -2 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix};$$

$\star\star\star$ Page 653 $\star\star\star$ Solution 1.2.4 (f):

$$\mathbf{b} = \begin{pmatrix} -3 \\ -5 \\ 2 \\ 1 \end{pmatrix}.$$

$\star\star\star$ Page 654 $\star\star\star$ Solution 1.3.12:

Reverse the inequalities:

$$\text{Set } l_{ij} = \begin{cases} a_{ij}, & i > j, \\ 0, & i \le j, \end{cases} \quad u_{ij} = \begin{cases} a_{ij}, & i < j, \\ 0, & i \ge j, \end{cases} \quad \ldots$$

$\star\star\star$ Page 655 $\star\star\star$ Solution 1.4.15 (a):

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

$\star\star\star$ Page 658 $\star\star\star$ Solution 1.8.4:

(*i*) $a \neq b$ and $b \neq 0$; (*ii*) $a = b \neq 0$, or $a = -2$, $b = 0$; (*iii*) $a \neq -2$, $b = 0$.

$\star\star\star$ Page 659 $\star\star\star$ Solution 1.8.23 (e):

$(0, 0, 0)^T$;

$\star\star\star$ Page 660 $\star\star\star$ Solution 2.5.5 (b):

$$\mathbf{x}^\star = (1, -1, 0)^T, \quad \mathbf{z} = z \left(-\tfrac{2}{7}, -\tfrac{1}{7}, 1 \right)^T;$$

$\star\star\star$ Page 661 $\star\star\star$ Solution 2.5.42:

True. If $\ker A = \ker B \subset \mathbb{R}^n$, then both matrices have n columns, and so $n - \operatorname{rank} A = \dim \ker A = \dim \ker B = n - \operatorname{rank} B$.

$\star\star\star$ Page 665 $\star\star\star$ Solution 3.4.22 (v):

Change "null vectors" to "null directions".

$\star\star\star$ Page 665 $\star\star\star$ Solution 3.4.32:

Change all $\mathbf{x}$'s to $\mathbf{z}$:

$0 = \mathbf{z}^T K \mathbf{z} = \mathbf{z}^T A^T C A \mathbf{z} = \mathbf{y}^T C \mathbf{y}$, where $\mathbf{y} = A \mathbf{z}$. Since $C > 0$, this implies $\mathbf{y} = \mathbf{0}$, and hence $\mathbf{z} \in \ker A = \ker K$.

$\star\star\star$ Page 666 $\star\star\star$ Solution 4.2.9:

Change (??) to (4.12)

$\star\star\star$ Page 667 $\star\star\star$

Change 4.4.5 to 4.4.6 and change 4.4.7 to 4.4.5.

$\star\star\star$ Page 667 $\star\star\star$ Solution 4.4.27 (a):

Change "the interpolating polynomial" to "an interpolating polynomial".

$\star\star\star$ Page 667 $\star\star\star$ Solution 4.4.52 (b):

$z = \tfrac{3}{5} (x - y)$.

(The solution given in the text is for the square $S = \{ 0 \leq x \leq 1, \ 0 \leq y \leq 1 \}$.)

★★★ Page 668 ★★★ Solution 5.1.14 (a):

$$\mathbf{v}_2 = \pm \left(-\sin\theta, \tfrac{1}{\sqrt{2}}\cos\theta \right)^T$$

★★★ Page 668 ★★★ Solution 5.1.21:

Insert (c) before final formula.

★★★ Page 668 ★★★ Solution 5.3.19:

Insert (a) before solution. (Also note change on p. 239.)

★★★ Page 670 ★★★ Solution 5.4.15:

$$p_0(x) = 1, \qquad p_1(x) = x, \qquad p_2(x) = x^2 - \tfrac{1}{3}, \qquad p_3(x) = x^3 - \tfrac{9}{10}x.$$

(The solution given in the text is for the interval $[0,1]$, not $[-1,1]$.)

★★★ Page 670 ★★★ Solution 5.5.6 (ii) (c):

$$\left(\tfrac{23}{43}, \tfrac{19}{43}, -\tfrac{1}{43} \right)^T.$$

★★★ Page 672 ★★★ Solution 5.6.20 (c):

The solution corresponds to the revised exercise — see correction on page 276. For the given system, the solution is $-3b_1 + b_2 + b_3 = 0$.

★★★ Page 672 ★★★ Solution 5.7.2 (a) (i):

To avoid any confusion, delete the superfluous last sample value in the first equation:
$$f_0 = 2.$$

★★★ Page 672 ★★★ Solution 5.7.2 (c) (i):

To avoid any confusion, delete the superfluous last sample value in the first equation:
$$f_0 = 6.$$

★★★ Page 673 ★★★ Solution 5.7.16:

Change first (??) to (5.103) and second (??) to (5.104):

True for the odd case (5.103), but false for the even case (5.104).

★★★ Page 674 ★★★

The page layout is a bit strange. The top of the second column (before Solution 6.2.1) is the solution to Exercise 6.1.16(c). Also, the solution to Exercise 6.2.10 spans across both columns.

★★★ Page 674 ★★★ Solution 6.2.1 (b):

The solution corresponds to the revised exercise — see correction on page 311.

For the given matrix, the solution is
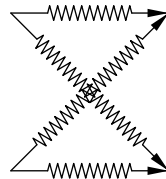
★★★ Page 674 ★★★ Solution 6.2.12 & 6.2.13:

Change all **e**'s to **y**'s.

★★★ Page 674 ★★★ Solution 6.3.5 (b):

$$\begin{aligned}
\tfrac{3}{2}\,u_1 - \tfrac{1}{2}\,v_1 - u_2 &= f_1, \\
-\tfrac{1}{2}\,u_1 + \tfrac{3}{2}\,v_1 &= g_1, \\
-u_1 + \tfrac{3}{2}u_2 + \tfrac{1}{2}\,v_2 &= f_2, \\
\tfrac{1}{2}u_2 + \tfrac{3}{2}\,v_2 &= g_2.
\end{aligned}$$

★★★ Page 677 ★★★ Solution 7.1.47:

Change (b) to (c), and (d) to (e).

★★★ Page 678 ★★★ Solution 7.2.19:

Change $\det I_2$ to $\det(-I_2)$ and $\det I_3$ to $\det(-I_3)$.

★★★ Page 681 ★★★ Solution 8.2.45:

Change Theorem ?? to Theorem 3.28

★★★ Page 682 ★★★ Solution 8.3.21 (a):

$$\begin{pmatrix} \tfrac{5}{3} & \tfrac{4}{3} \\ \tfrac{8}{3} & \tfrac{1}{3} \end{pmatrix}$$

★★★ Page 683 ★★★ Solution 8.4.27:

Insert (iii) before second matrix.

★★★ Page 683 ★★★ Solution 8.5.1 (a):

$$\sqrt{3 \pm \sqrt{5}}$$

★★★ Page 684 ★★★ Solution 8.5.26:

Change (b) to (c).

$\star\star\star$ Page 685 $\star\star\star$ Solution 9.1.28 (g):

Change $\dot{u} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \mathbf{u}$ to $\dot{\mathbf{u}} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \mathbf{u}$.

$\star\star\star$ Page 688 $\star\star\star$ Solution 9.5.31:

Change "four unstable modes" to "two unstable modes".

$\star\star\star$ Page 690 $\star\star\star$ Solution 10.2.18:

Delete (a).

$\star\star\star$ Page 691 $\star\star\star$ Solution 10.3.24 (e):

Change $-2.69805 \pm .806289$ to $-2.69805 \pm .806289\,\mathrm{i}$.

$\star\star\star$ Page 692 $\star\star\star$

Change 10.5.21 to 10.5.22.

$\star\star\star$ Page 694 $\star\star\star$ Solution 11.2.8 (d):

$$f'(x) = 4\,\delta(x+2) + 4\,\delta(x-2) + \begin{cases} 1, & |x| > 2, \\ -1, & |x| < 2, \end{cases}$$
$$= 4\,\delta(x+2) + 4\,\delta(x-2) + 1 - 2\,\sigma(x+2) + 2\,\sigma(x-2),$$
$$f''(x) = 4\,\delta'(x+2) + 4\,\delta'(x-2) - 2\,\delta(x+2) + 2\,\delta(x-2).$$

$\star\star\star$ Page 694 $\star\star\star$ Solution 11.2.31 (a):

$$u_n(x) = \begin{cases} x\,(1-y), & 0 \le x \le y - \frac{1}{n}, \\ -\frac{1}{4}\,n\,x^2 + \left(\frac{1}{2}\,n - 1\right)x\,y - \frac{1}{4}\,n\,y^2 + \frac{1}{2}\,y + \frac{1}{2}\,x - \frac{1}{4n}, & |x-y| \le \frac{1}{n}, \\ y\,(1-x), & y + \frac{1}{n} \le x \le 1. \end{cases}$$

$\star\star\star$ Page 694 $\star\star\star$ Solution 11.3.3 (c):

(i) $u_\star(x) = \frac{1}{2}\,x^2 - \frac{5}{2} + x^{-1}$,

(ii) $\mathcal{P}[u] = \displaystyle\int_1^2 \left[\,\frac{1}{2}\,x^2\,(u')^2 + 3\,x^2\,u\,\right] dx, \quad u'(1) = u(2) = 0,$

(iii) $\mathcal{P}[u_\star] = -\frac{37}{20} = -1.85,$

(iv) $\mathcal{P}[x^2 - 2\,x] = -\frac{11}{6} = -1.83333, \quad \mathcal{P}\left[-\sin\frac{1}{2}\,\pi x\right] = -1.84534.$

$\star\star\star$ Page 695 $\star\star\star$ Solution 11.3.17:

Add (iv) at beginning of last line:

(iv) integrating factor $\rho(x) = x^2$, so $-(x^3\,u')' = x^3$.

$\star\star\star$ Page 696 $\star\star\star$ Solution 11.5.7 (b):

$$\lambda = -\omega^2 < 0, \quad G(x,y) = \begin{cases} \dfrac{\sinh\omega\,(y-1)\,\sinh\omega\,x}{\omega\sinh\omega}, & x < y, \\[2mm] \dfrac{\sinh\omega\,(x-1)\,\sinh\omega\,y}{\omega\sinh\omega}, & x > y; \end{cases}$$

$$\lambda = 0, \quad G(x,y) = \begin{cases} x(y-1), & x < y, \\ y(x-1), & x > y; \end{cases}$$

$$\lambda = \omega^2 \neq n^2\pi^2 > 0, \quad G(x,y) = \begin{cases} \dfrac{\sin\omega\,(y-1)\,\sin\omega\,x}{\omega\sin\omega}, & x < y, \\[2mm] \dfrac{\sin\omega\,(x-1)\,\sin\omega\,y}{\omega\sin\omega}, & x > y. \end{cases}$$