Homework Assignment # 2

1. Write down a variational principle for an oscillator of frequency $\omega > 0$ subject to Robin boundary conditions:

$$u'' + \omega^2 u = 0, \qquad 0 < x < 1, \qquad u'(0) = \alpha\, u(0), \tag{1}$$

where the value of u at right hand endpoint $x = 1$ is unspecified. Discuss what boundary conditions, if any, need to be imposed at $x = 1$. Find the minimizer (critical function) for your variational problem.

2. Find all critical curves, parametrized by $y = u(x)$, $z = v(x)$, that start at the origin and end on the plane $x + y + z = 1$, for the functional

$$J[u, v] = \int_a^b (u'^2 + v'^2 + 2\,u\,v)\, dx.$$

3. Write out the general form of a second order parametric variational problem for plane curves. Is the elastica variational problem parametric?

4. A heavy uniform chain of fixed length L hangs in equilibrium under gravity between two fixed points $A = (-a, h)$, $B = (a, h)$, at the same height. The variational problem to minimize is potential energy

$$J[u] = \rho g \int_{-a}^{a} u\sqrt{1 + u'^2}\, dx,$$

where ρ is the density and g the force of gravity. Find the equilibrium configuration of the chain.

Due: Monday, November 1.