Allen Tannenbaum and Computer Vision

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Rutgers, February 2024

Allen Tannenbaum

Allen Robert Tannenbaum (January 25, 1953 - December 28, 2023) was an American applied mathematician who was a Distinguished Professor of Computer Science and Applied Mathematics & Statistics at the State University of New York at Stony Brook. He was also Visiting Investigator of Medical Physics at Memorial Sloan Kettering Cancer Center in New York City. He had held a number of other positions in the United States, Israel, and Canada including the Bunn Professorship of Electrical and Computer Engineering and Interim Chair, and Senior Scientist at the Comprehensive Cancer Center at the University of Alabama, Birmingham. He received his B.A. from Columbia University in 1973 and Ph.D. with thesis advisor Heisuke Hironaka at the Harvard University in 1976.[1][2]

Tannenbaum had done research in numerous areas including robust control, computer vision, and biomedical imaging, having almost 500 publications. He pioneered the field of robust control with the solution of the gain margin and phase margin problems using techniques from Nevanlinna–Pick interpolation theory, which was the first H-infinity type control problem solved. Tannenbaum used techniques from elliptic curves to show that the reachability does not imply pole assignability for systems defined over polynomial rings in two or more variables over an arbitrary field. He pioneered the use of partial differential equations in computer vision and biomedical imaging co-inventing with Guillermo Sapiro an affine-invariant heat equation for image enhancement. Tannenbaum further formulated a new approach to optimal mass transport (Monge-Kantorovich) theory in joint work with Steven Haker and Sigurd Angenent. In recent work, he had developed techniques using graph curvature ideas for analyzing the robustness of complex networks.

His work had won several awards including IEEE Fellow[3] in 2008, O. Hugo Schuck Award[4] of the American Automatic Control Council in 2007 (shared with S. Dambreville and Y. Rathi), and the George Taylor Award for Distinguished Research[5] from the University of Minnesota in 1997. He has given numerous plenary talks at major conferences including the Society for Industrial and Applied Mathematics (SIAM) Conference on Control in 1998, IEEE Conference on Decision and Control of the IEEE Control Systems Society in 2000, and the International Symposium on the Mathematical Theory of Networks and Systems (MTNS)[6] in 2012. He is also well known as one of the authors of the textbook *Feedback Control Theory* (with John Doyle and Bruce Francis), which is currently a standard introduction to robust control at the graduate level.

His wife Rina Tannenbaum is a chemist and his son Emmanuel David Tannenbaum was a biophysicist and applied mathematician.

☰ Google Scholar

Allen Tannenbaum

FOLLOW

Distinguished Professor, Computer Science/Applied Mathematics, Stony Brook University
Verified email at stonybrook.edu - Homepage

Medical Imaging   Systems and Control   Computer Vision   Applied Mathematics   Image Processing

TITLE	CITED BY	YEAR
Feedback control theory JC Doyle, BA Francis, AR Tannenbaum Courier Corporation	5269	2013
Localizing region-based active contours S Lankton, A Tannenbaum IEEE transactions on image processing 17 (11), 2029-2039	1512	2008
Gradient flows and geometric active contour models S Kichenassamy, A Kumar, P Olver, A Tannenbaum, A Yezzi Proceedings of IEEE International Conference on Computer Vision, 810-815	1005	1995
A geometric snake model for segmentation of medical imagery A Yezzi, S Kichenassamy, A Kumar, P Olver, A Tannenbaum IEEE Transactions on medical imaging 16 (2), 199-209	926	1997
Shapes, shocks, and deformations I: the components of two-dimensional shape and the reaction-diffusion space BB Kimia, AR Tannenbaum, SW Zucker International journal of computer vision 15, 189-224	855	1995
Robust control of linear time-invariant plants using periodic compensation P Khargonekar, K Poolla, A Tannenbaum IEEE Transactions on Automatic Control 30 (11), 1088-1096	807	1985

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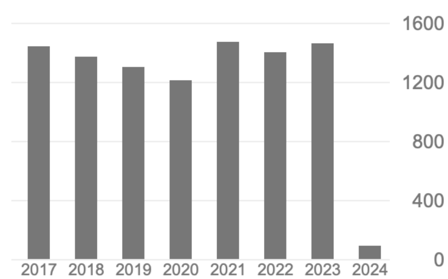

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not available   available

Based on funding mandates

Allen in Benin

Sydney, December 2000

Peter, Tryphon, Allen

Sarah, Rina, Allen, Manny

Where it began …

Harvard University Science Center

Allen Robert Tannenbaum

MathSciNet

Ph.D. Harvard University 1976

Dissertation: *Deformations of I-Cycles and the Chow Scheme*

Advisor 1: Heisuke Hironaka

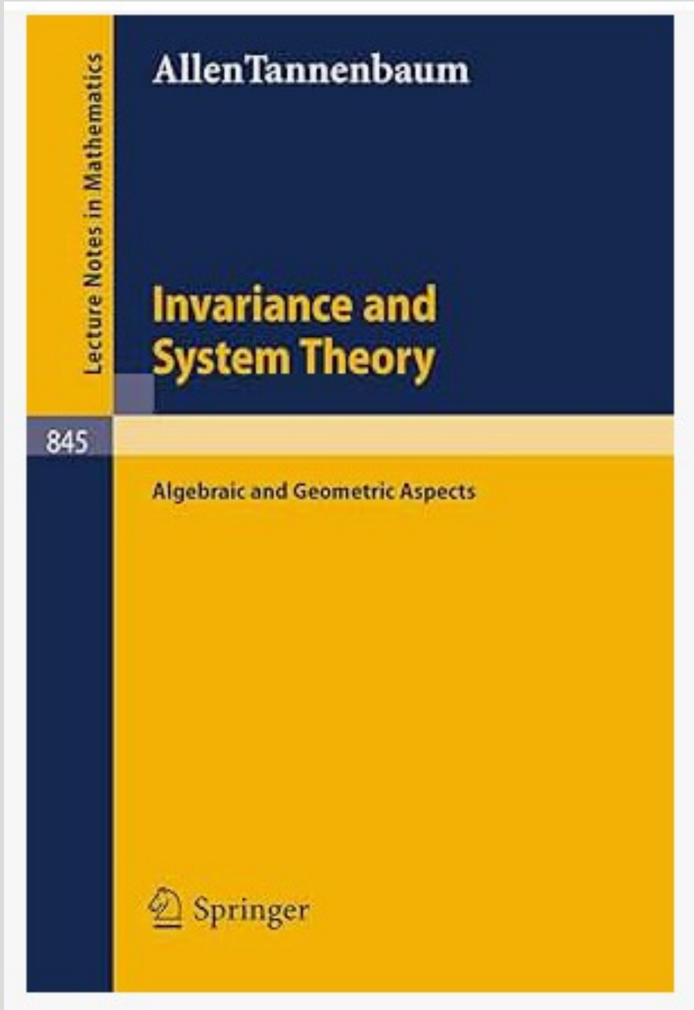
Students:
Click here to see the students listed in chronological order.

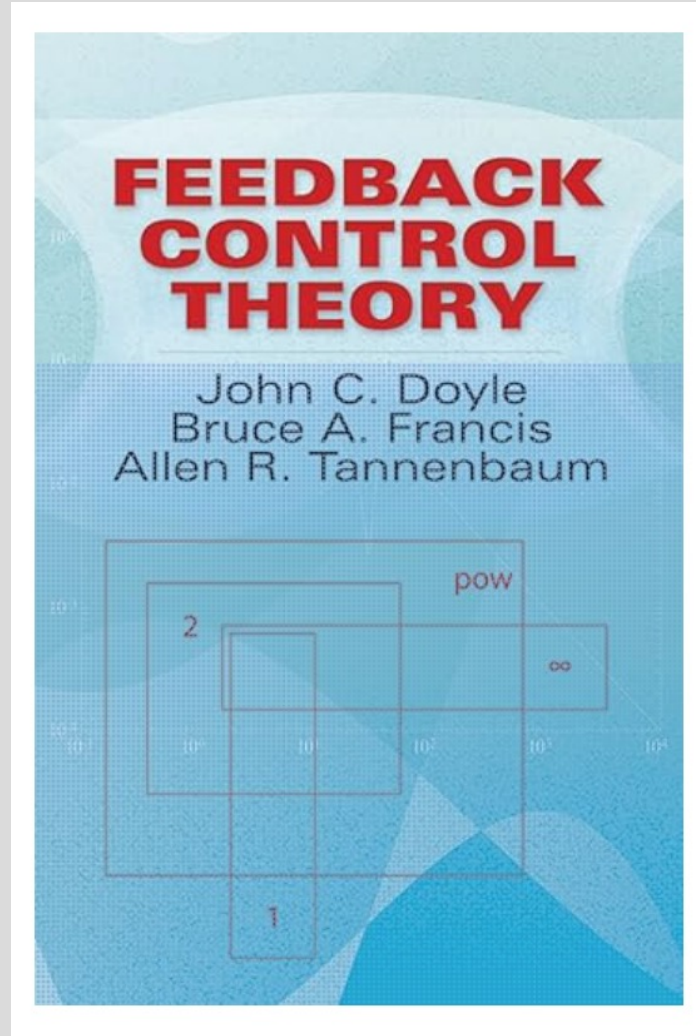
Name	School	Year	Descendants
Cockburn, Juan	University of Minnesota - Twin Cities	1994	
Curry, Cecilia	Georgia Institute of Technology	2002	
Elgersma, Michael	University of Minnesota - Twin Cities	1988	
Fer, Huseyin	University of Minnesota - Twin Cities	1997	
Gholami, Behnood	Georgia Institute of Technology	2010	
Haker, Steven	University of Minnesota - Twin Cities	1999	
Montminy, Matthew	University of Minnesota - Twin Cities	2001	
Nakhmani, Arie	Technion-Israel Institute of Technology	2011	3
Ozbay, Hitay	University of Minnesota - Twin Cities	1989	2
Sandhu, Romeil	Georgia Institute of Technology	2010	1
Sapiro, Guillermo	Technion-Israel Institute of Technology	1993	12
Stein, Joseph	Weizmann Institute of Science	1980	
Yezzi, Jr., Anthony	University of Minnesota - Twin Cities	1997	2

According to our current on-line database, Allen Tannenbaum has 13 students and 33 descendants.
We welcome any additional information.

If you have additional information or corrections regarding this mathematician, please use the update form. To submit students of this mathematician, please use the new data form, noting this mathematician's MGP ID of 19358 for the advisor ID.

1986:
Allen
arrives in
Minnesota





- Harvard University, 1973 - 1976.
- Weizmann Institute of Science, 1976 - 1978, 1980 - 1983.
- Institut des Hautes Études Scientifiques, 1978.
- E.T.H., Zurich, 1978 - 1980.
- University of Florida, 1982 - 1984.
- Ben-Gurion University, 1984 - 1986.
- McGill University, 1985 - 1986.
- University of Minnesota, 1986 - 2002.
- Technion, 1989 - 1992, 2005 - 2010.
- Georgia Tech, 1999 - 2011.
- University of Alabama, 2012 - 2013.
- Stony Brook University, 2013 – 2023.
- Memorial Sloan Kettering Cancer Center, 2015 - 2023

Allen Tannenbaum

In Memoriam - Distinguished Professor

It is with great sadness that the Department of Computer Science reports the loss of Dr. Allen Tannenbaum, Professor Emeritus of Computer Science.

Bye, my friend.

INTERESTS

Computational computer vision, image processing, medical imaging, computer graphics, control, mathematical systems theory, control of semiconductor fabrication processes, robotics, operator theory, functional analysis, algebraic geometry, di�erential geometry, invariant theory, and partial differential equations.

BIOGRAPHY

Allen Tannenbaum was affiliated with the Department of Applied Mathematics & Statistics. He obtained his Ph.D. from Harvard University.

RESEARCH

Allen Tannenbaum and Computer Vision

Basic Issues in Computer Vision

- multi-scale resolution
- denoising/smoothing
- image enhancement
- edge detection
- segmentation
- geometric attributes
  lengths, areas, volumes,
  relative positions, etc.
- object recognition
- invariant signatures
- occlusion

Medical Image Processing Applications

- ultrasound
- magnetic resonance imaging
- CT scans
- x-ray tomography

  - breast tumors
  - heart
  - brain
  - fetus
  - etc., etc., etc.

Evolutionary Smoothing

Multi-scale resolution provided by evolutionary partial differential equation

$$
\begin{aligned}
\Phi_t &= F(\mathbf{x}, \Phi, \nabla\Phi, \nabla^2\Phi, \ldots) \\
\Phi(\mathbf{x}, 0) &= I(\mathbf{x}) \\
\mathbf{x} &= \text{spatial position} \\
t &= \text{scale parameter} \\
&= \text{degree of smoothing} \\
I(\mathbf{x}) &= \text{raw gray-scale image} \\
\Phi(\mathbf{x}, t) &= \text{smoothed image}
\end{aligned}
$$

Gaussian Smoothing

$\Longrightarrow$ *Simplest model*

Heat equation = Gaussian convolution

$$\Phi_t = \Delta\Phi$$

$$\Phi(\mathbf{x}, 0) = I(\mathbf{x}).$$

$$\Phi(\mathbf{x}, t) = \mathcal{G}(\mathbf{x}, t) * I(\mathbf{x})$$

Problems:

- Smooths out both noise and relevant features indiscriminantly
- Isotropic process

$\Longrightarrow$ Need an anisotropic (nonlinear) diffusion process which eliminates noise but retains edges and other features.

Figure 11.1. Smoothing a gray scale image.

Level Set Evolution

Idea:

Use geometric diffusion to smooth

Evolve individual level sets

Theorem. The level sets

$$C_k(t) = \{ \, (x, y) \mid \Phi(x, y, t) = k \, \}$$

evolve according to the normal flow

$$C_t = -\alpha \, \mathbf{N}$$

if and only if Φ satisfies the evolution equation

$$\Phi_t = \alpha \, \| \nabla \Phi \|$$

Osher–Sethian

$\mathbf{N}$ — outward normal to level set

$$\Phi_t = \alpha(\Phi, \nabla\Phi, \nabla^2\Phi)\,\|\nabla\Phi\|$$

- Smoothing of level sets *only*
- Level sets move independently of each other
- Can continue after crossing/ separation/singularities
- Readily implementable in both 2D and 3D

$\Longrightarrow$ *Concentrate on 2D images from now on.*

Curve Evolution

$C(q,t)$ — parametrized family of (closed) curves in $\mathbb{R}^2$

$\mathbf{T}$ — unit tangent

$\mathbf{N}$ — unit (outward) normal

General curve evolution

$$\frac{dC}{dt} = \alpha\,\mathbf{N} + \beta\,\mathbf{T}$$

By reparametrizing, can assume

$$\beta = 0$$

No tangential component:

$$\frac{dC}{dt} = \alpha\,\mathbf{N}$$

Basic idea:

Since symmetry appears to be an essential attribute of human vision, let us incorporate the relevant symmetries in our image processing algorithms.

Why are humans so attuned to symmetry?

Mathematically …

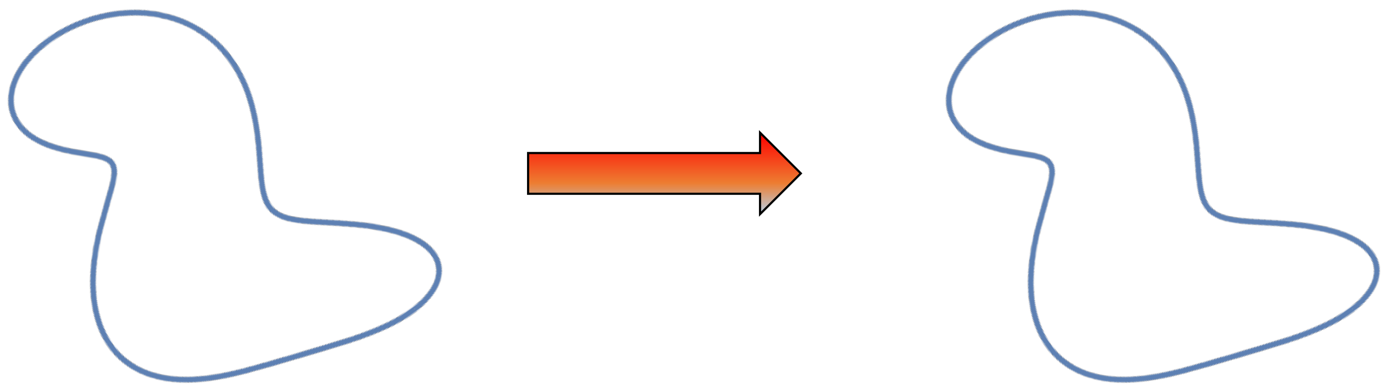
Symmetry → *Group Theory*

Next to the concept of a function, which is the most important concept pervading the whole of mathematics, the concept of a group is of the greatest significance in the various branches of mathematics and its applications.

— P.S. Alexandroff

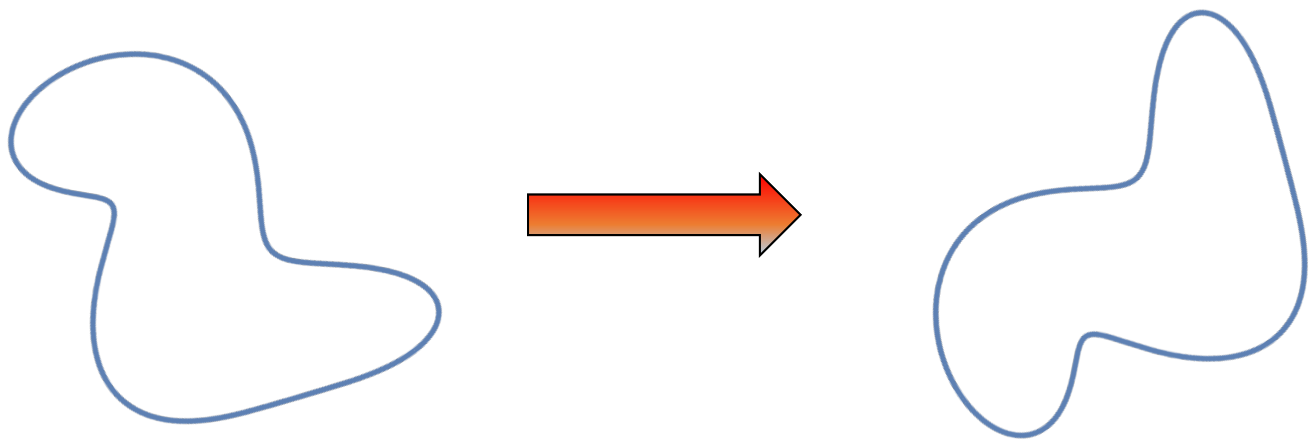
Geometric transformation groups

Translations

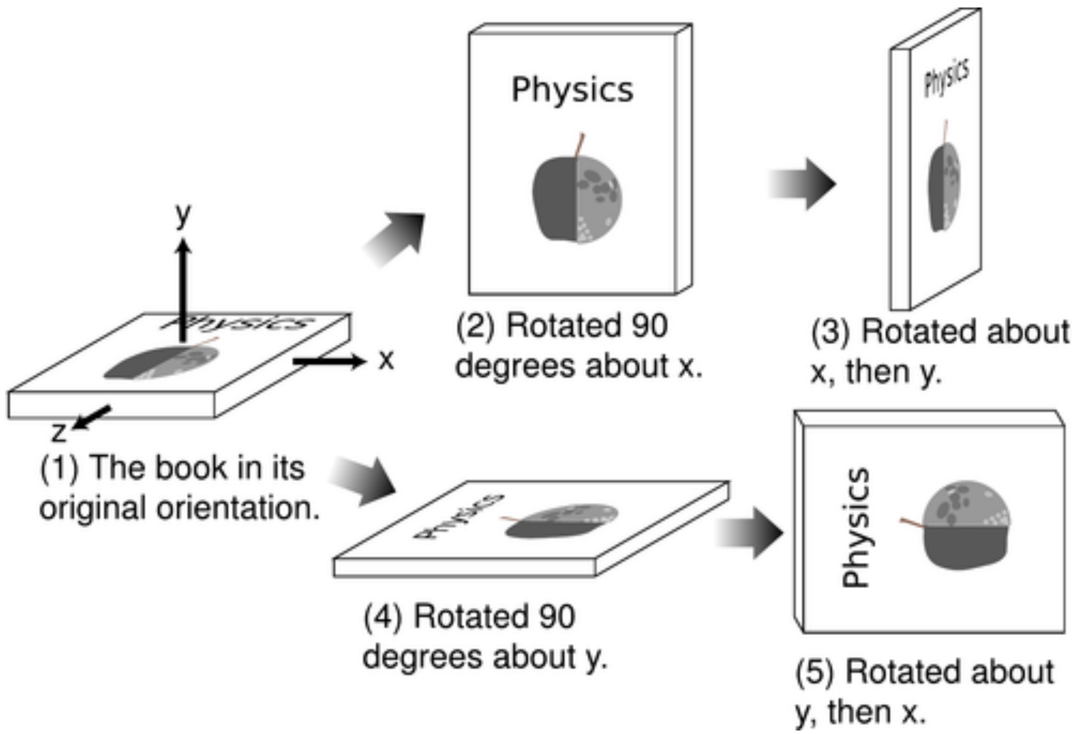


Geometric transformation groups

Rotations

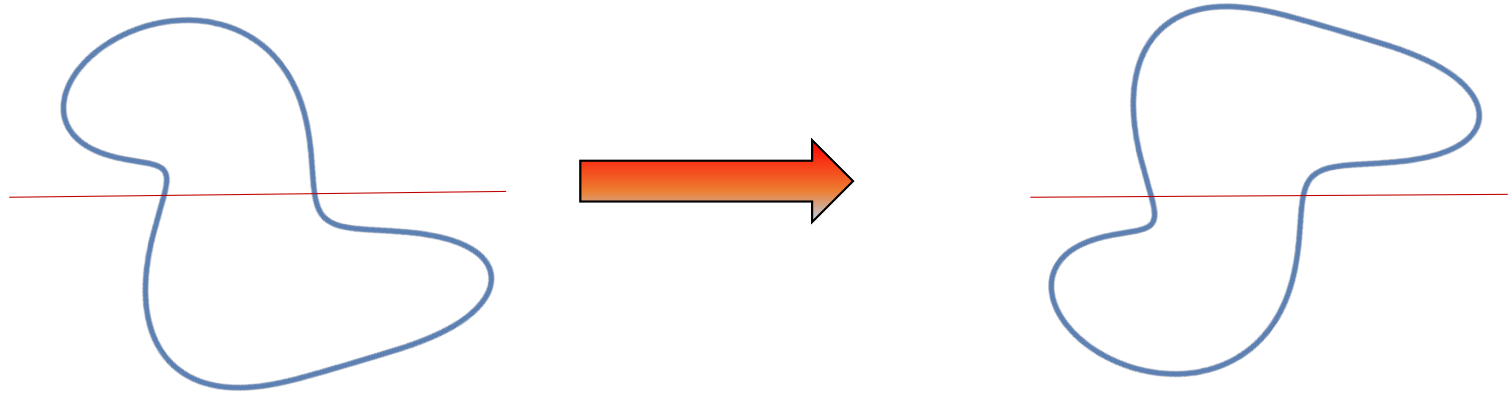


Noncommutativity of 3D rotations — order matters!



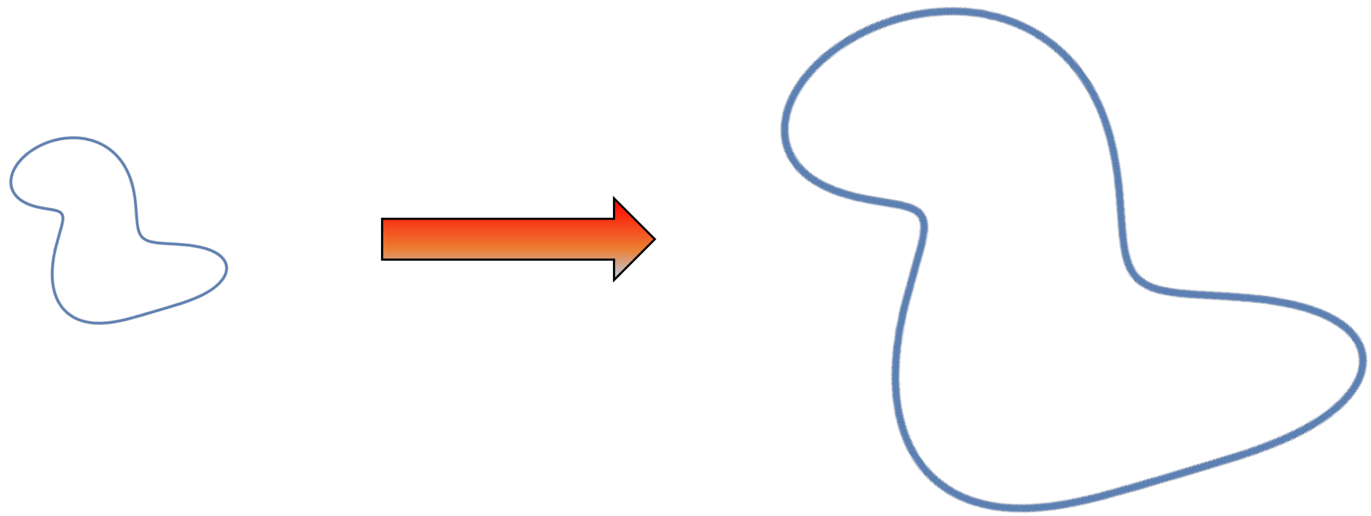
Geometric transformation groups

Reflections



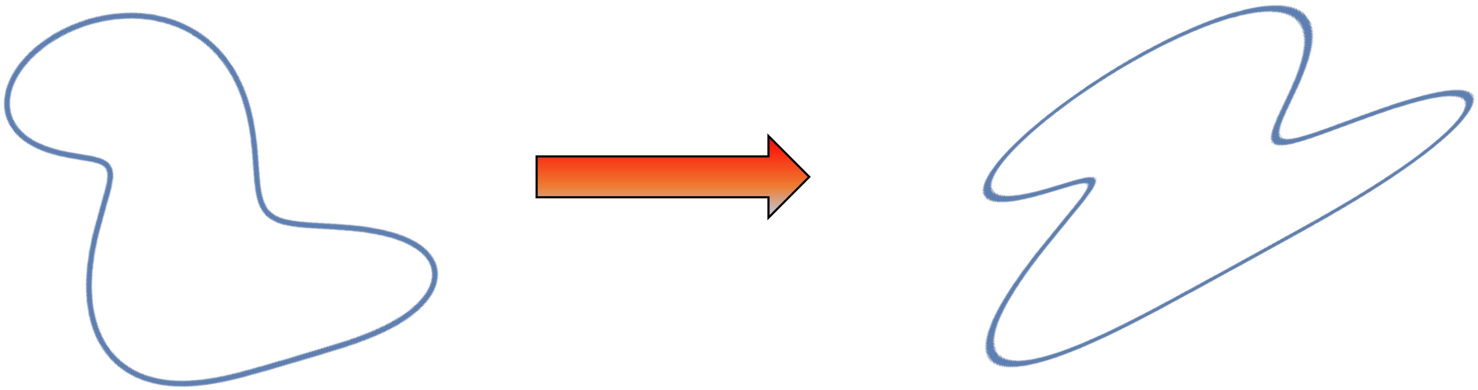
Geometric transformation groups

Scaling (similarity)



Geometric transformation groups

Projective and Equiaffine Transformations



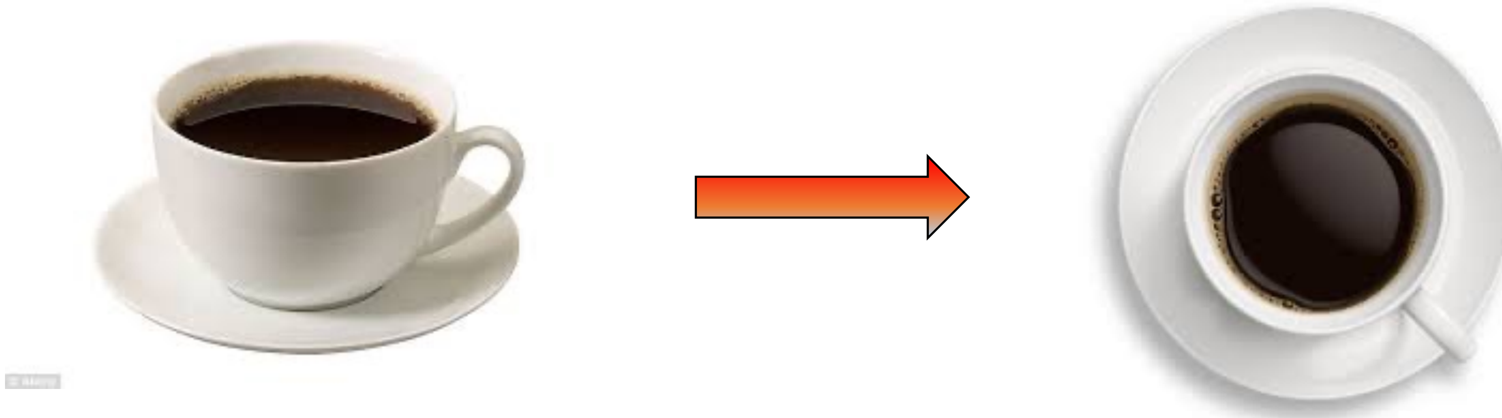
Geometric transformation groups

Projective Transformation

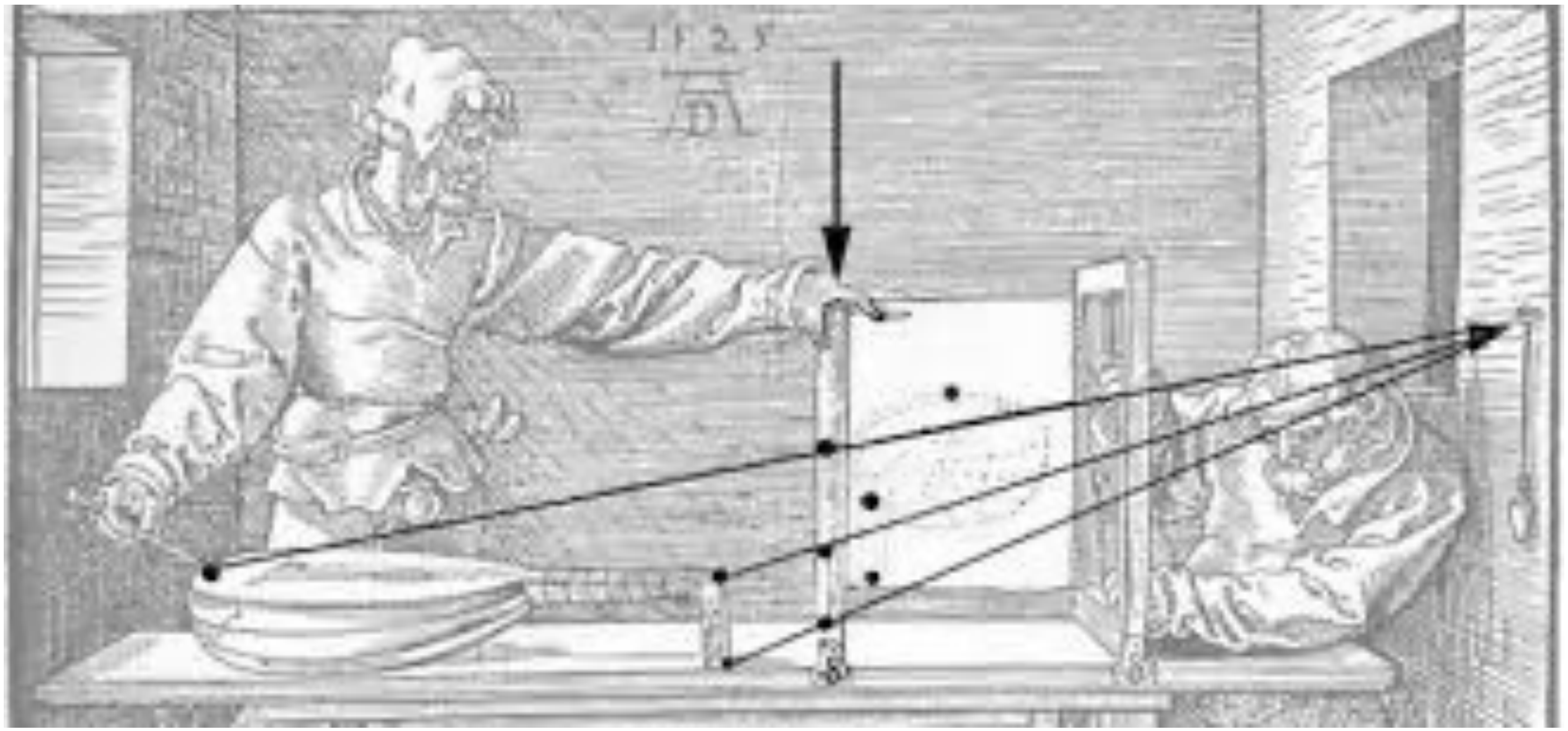
→ ?

Geometric transformation groups

Projective Transformation



Projective transformations in art and photography



Albrecht Durer — 1500

Symmetry Groups

Euclidean *Length–preserving*

- Translations
- Rotations
- Reflections

Similarity *Preserves length ratios*

- Euclidean + Scaling

Symmetry Groups

Equi-affine *Area–preserving*

Translations

Unimodular linear: $\det A = 1$

Affine *Preserves volume ratios*

$A\,\mathbf{x} + \mathbf{b}$

Equiaffine + Scaling

Projective *Preserves cross-ratios*

$$\left(\frac{ax+by+c}{gx+hy+j}, \frac{dx+ey+f}{gx+hy+j}\right)$$

Invariant Curve Flows

Assume C is a graph: $y = u(x,t)$

Grassfire flow (Hamilton-Jacobi)

$$C_t = -\mathbf{N}$$

$$u_t = -\sqrt{1 + u_x^2}$$

$$\Phi_t = \|\nabla \Phi\| = \sqrt{\Phi_x^2 + \Phi_y^2}$$

- Simplest Euclidean invariant flow
- Formation of caustics

Euclidean Curve Shortening

$$C_t = -\kappa\,\mathbf{N}$$

$$u_t = -\frac{u_{xx}}{1+u_x^2}$$

$$\Phi_t = \frac{\Phi_y^2\Phi_{xx} - 2\Phi_x\Phi_y\Phi_{xy} + \Phi_x^2\Phi_{yy}}{\Phi_x^2+\Phi_y^2}$$

$$= \|\nabla\Phi\|\ \mathrm{div}\ \frac{\nabla\Phi}{\|\nabla\Phi\|}$$

- Euclidean invariant flow
- Shortens Euclidean perimeter as rapidly as possible
- $\nabla\Phi$ — characteristic
- Nonconvex curves convexify
- Convex curves shrink to round points

Grayson–Gage–Hamilton

Equi-affine Curve Shortening

kappa שליש

$$C_t = -\sqrt[3]{\kappa}\,\mathbf{N}$$

$$u_t = \sqrt[3]{u_{xx}}$$

$$\Phi_t = (\Phi_y^2 \Phi_{xx} - 2\Phi_x \Phi_y \Phi_{xy} + \Phi_x^2 \Phi_{yy})^{1/3}$$

- Simplest equi-affine invariant flow
- Equi-affine grassfire flow,
  in direction of affine normal
- Shortens equi-affine arc length
  as rapidly as possible
- Nonconvex curves shrink to points
- Convex curves shrink to elliptical points

Angenent–Sapiro–Tannenbaum

Projective Curve Flow

$$u_t = \frac{u_{xx}^3}{(9u_{xx}^2 u_{xxxxx} - 45u_{xx}u_{xxx}u_{xxxx} + 40u_{xxx}^3)^{2/3}}$$

- Simplest projective invariant flow
- In direction of projective normal
- Shortens projective arc length as rapidly as possible
- Curves can become singular
- Involves higher order derivatives; existence/uniqueness???

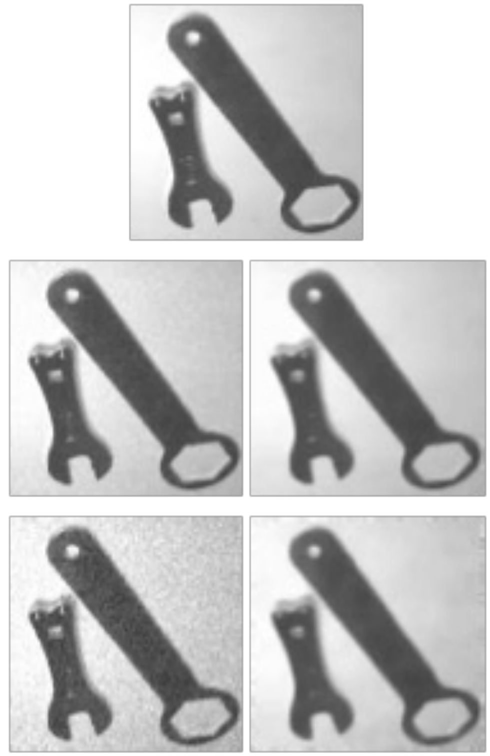
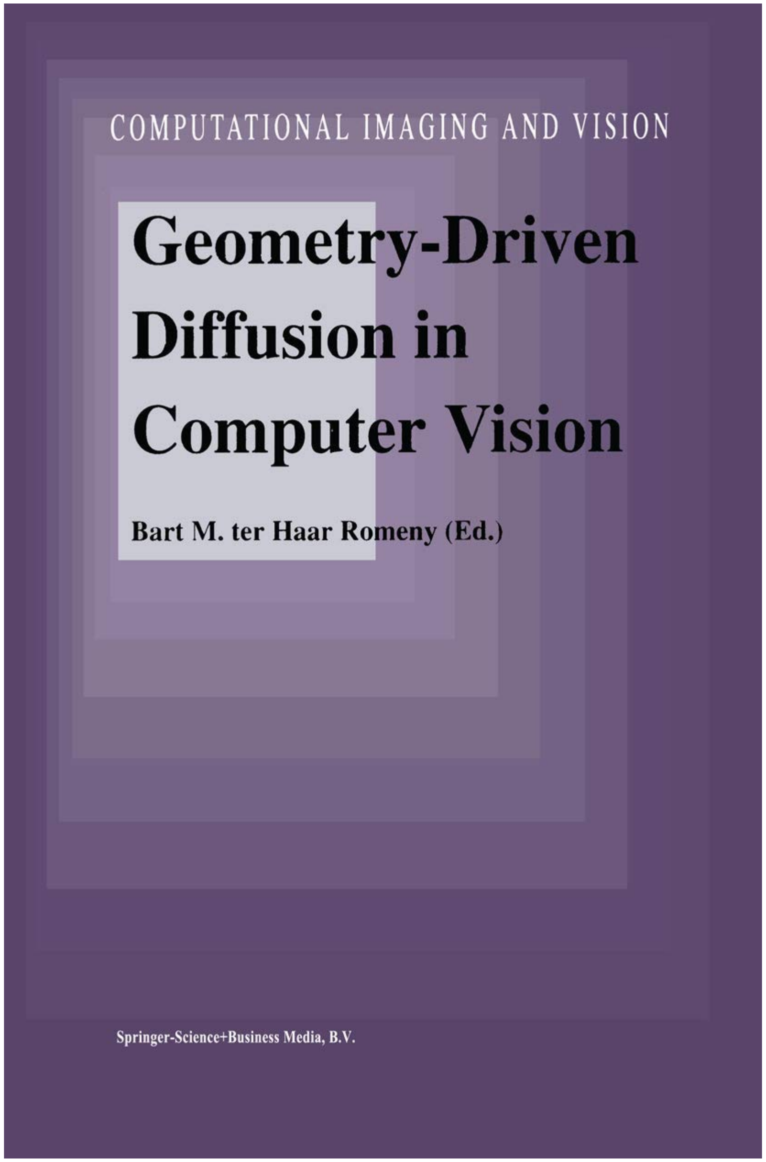


Figure 5: Examples of the affine invariant image flow for image denoising and simplification. The original image is presented on the top row. Two different noise levels are given on the left at the second and last row, and the corresponding results of the affine invariant flow on the right.



Olver, P.J.,
Sapiro, G.,
Tannenbaum, A.

Differential invariant signatures and flows in computer vision: a symmetry group approach

Edge-detection and Segmentation

Earlier detectors:

Search for:

- Max. of $\|\nabla I\|$?

  $\Longrightarrow$ needs smoothing

- Zeros of $\Delta(I * \text{Gaussian})$?

  $\Longrightarrow$ smoothing blurs!

Snakes — Active Contours

Idea:

Use a geometric curve flow to "capture" the edge

Modify curve shortening so that the "snake" is trapped by features of interest — instead of disappearing to a point

$\Longrightarrow$ *Kass, Witkin, Terzopolous*

Euclidean Snakes

Observation:

Euclidean curve shortening flow

$$C_t = -\kappa\,\mathbf{N}$$

is the gradient flow for the Euclidean length functional

$$\mathcal{L}[C] = \int_C ds = \int_C \sqrt{dx^2 + dy^2}$$

In other words, the flow decreases the length of the curve as rapidly as possible.

Idea:

Modify the Euclidean length functional by a conformal factor

$$\widehat{\mathcal{L}}[C] = \int_C d\hat{s} = \int_C \sigma(x,y)\sqrt{dx^2 + dy^2}$$

$0 < \sigma$ — *Stopping term*

$|\sigma| \ll 1$ near features of interest

Idea:

Modify the Euclidean length functional by a conformal factor

$$\widehat{\mathcal{L}}[C] = \int_C d\hat{s} = \int_C \sigma(x,y)\sqrt{dx^2 + dy^2}$$

$0 < \sigma$ — *Stopping term*

$|\sigma| \ll 1$ near features of interest

Edge = Curve of large $\|\nabla I(\mathbf{x})\|$

$$\sigma = (1 + \|\nabla I\|^2)^{-1}$$

$\Longrightarrow$ Replace I by smoothed version I^* obtained by Gaussian, Euclidean or affine smoothing.

$\Longrightarrow$ Can use color, texture, or other stopping terms

Kichenassamy–Kumar–PJO–Tannenbaum–Yezzi

Conformal Snakes

Minimize

$$\widehat{\mathcal{L}}[C] = \int_C \sigma(x,y)\sqrt{dx^2 + dy^2}$$

Curve evolution:

$$C_t = -\sigma\,\kappa\,\mathbf{N} - \nabla\sigma$$

Level set formulation:

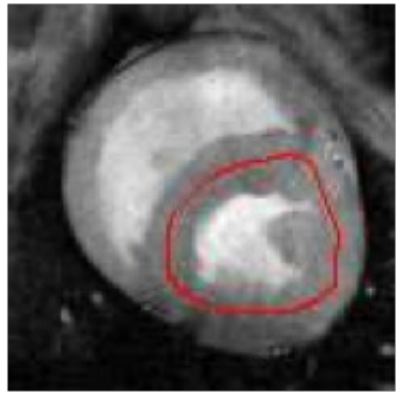
$$\Phi_t = \sigma\|\nabla\Phi\|\,\mathrm{div}\left(\frac{\nabla\Phi}{\|\nabla\Phi\|}\right) + \nabla\sigma\cdot\nabla\Phi$$

Last term:

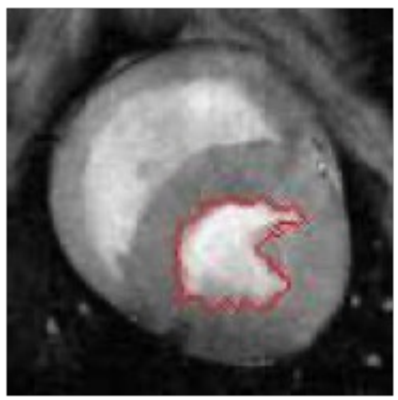
- Difficult to guess *a priori*
- Points towards contour
- Captures fine features

[confirmed by comp.]

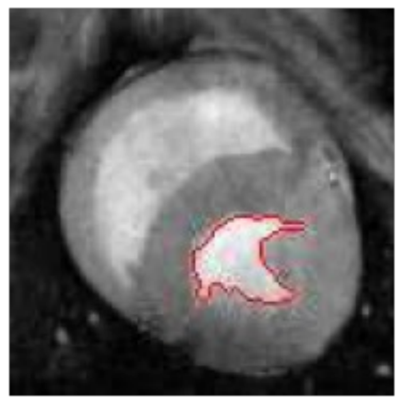
Analysis: Viscosity solutions.



(a)



(b)



(c)

Inflating Snakes = Balloons

$$C_t = -\sigma\,\kappa\,\mathbf{N} - \nabla\sigma$$

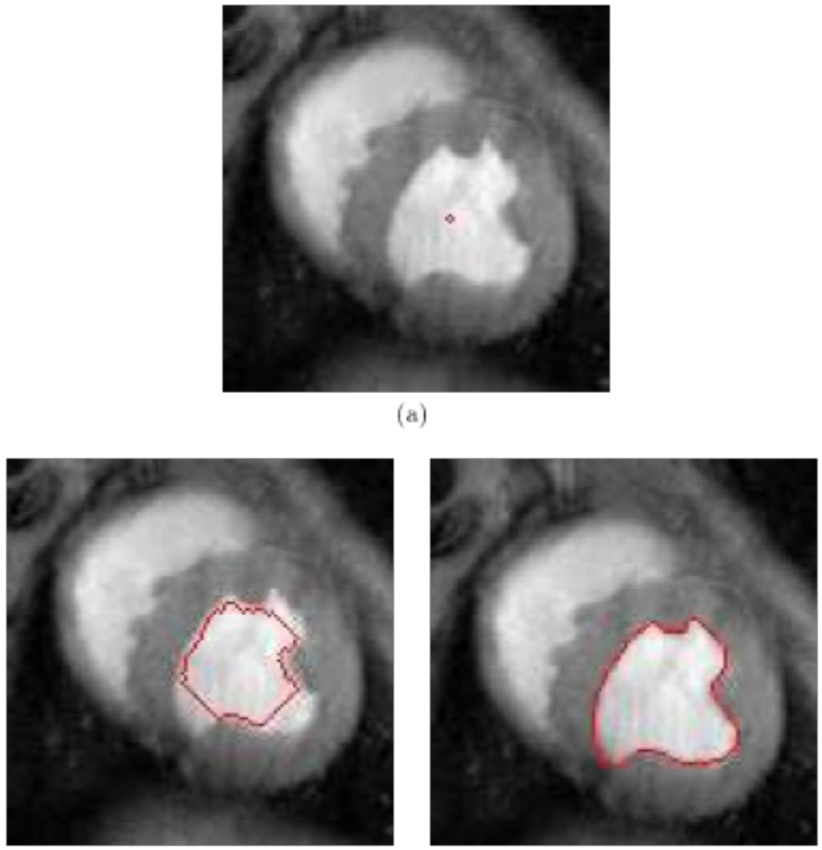
Add Inflation:

$$C_t = -\sigma \cdot (\kappa + \nu)\,\mathbf{N} - \nabla\sigma$$

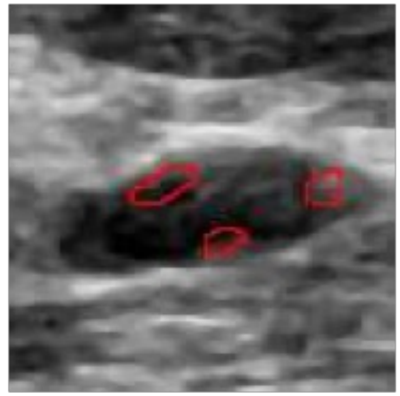
- $\nu > 0$ — inflation constant
- quick start up
- speeds up capture of edges

Level set formulation:

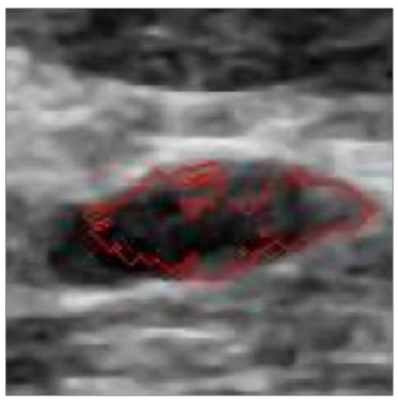
$$\Phi_t = \sigma \|\nabla\Phi\| \left(\operatorname{div} \frac{\nabla\Phi}{\|\nabla\Phi\|} + \nu \right) + \nabla\sigma \cdot \nabla\Phi$$



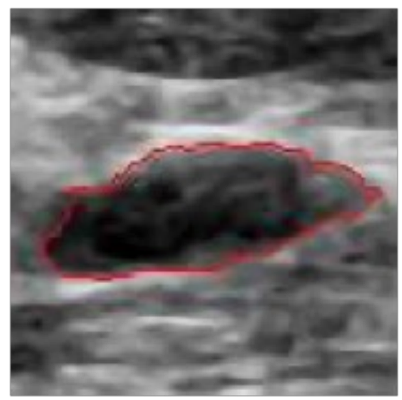
(a) (b) (c)



(a)



(b)



(c)

(a)

(b)

(c)

Object Recognition

Signature Curves

Definition. Given an (ordinary) planar action of a Lie group G, the *signature curve* $\Sigma \subset \mathbb{R}^2$ of a plane curve $\mathcal{C} \subset \mathbb{R}^2$ is parametrized by the two lowest order differential invariants

$$\chi : \mathcal{C} \longrightarrow \Sigma = \left\{ \left(\kappa \, , \, \frac{d\kappa}{ds} \right) \right\} \subset \mathbb{R}^2$$

$\Longrightarrow$ *Calabi, PJO, Shakiban, Tannenbaum, Haker*

Theorem. Two regular curves $\mathcal{C}$ and $\overline{\mathcal{C}}$ are (locally) equivalent:

$$\overline{\mathcal{C}} = g \cdot \mathcal{C}$$

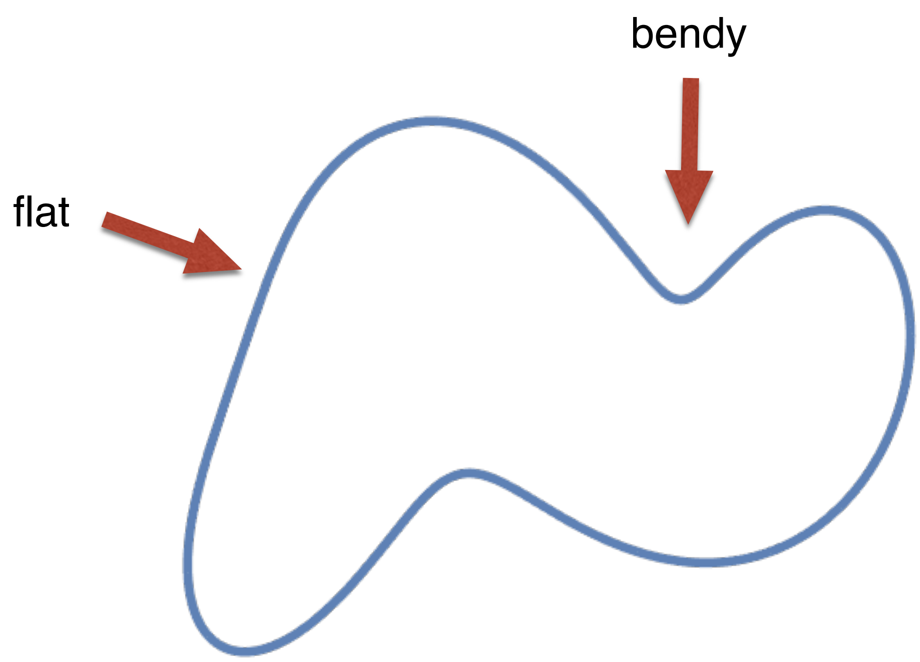
if and only if their signature curves are identical:

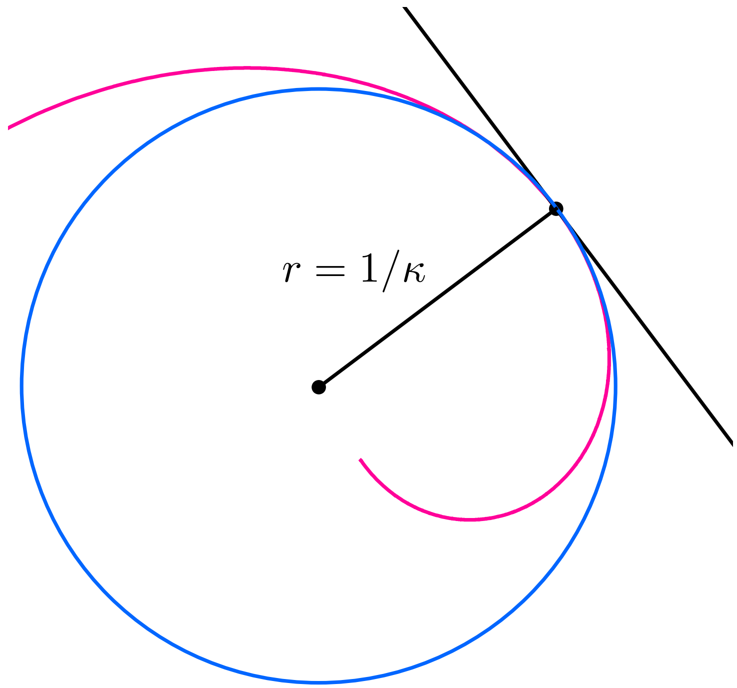
$$\overline{\Sigma} = \Sigma$$

$\Longrightarrow$ regular: $(\kappa_s, \kappa_{ss}) \neq 0$.

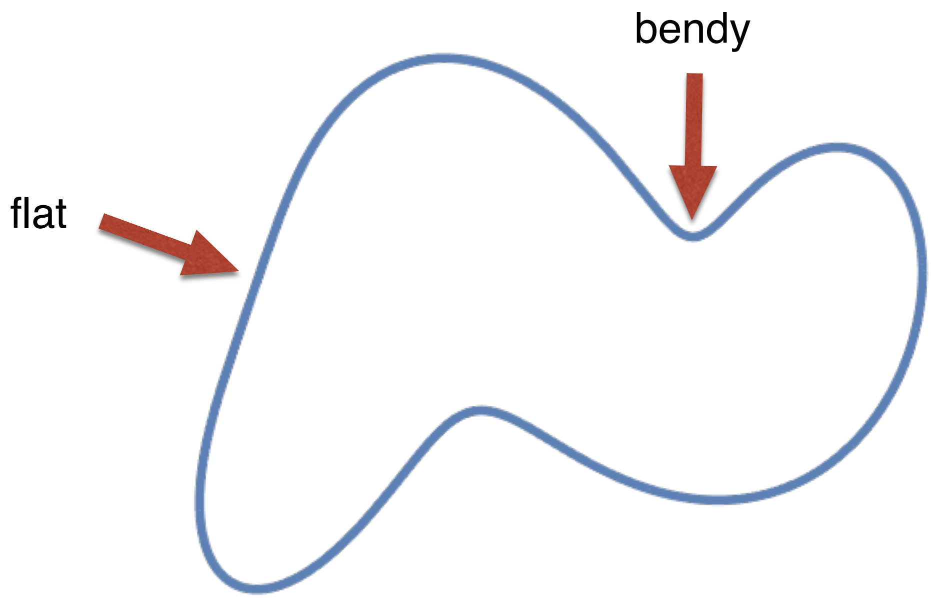
Euclidean Curvature

is a measure of "bendiness".

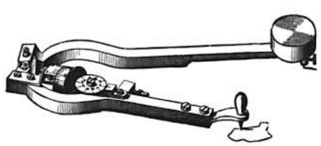


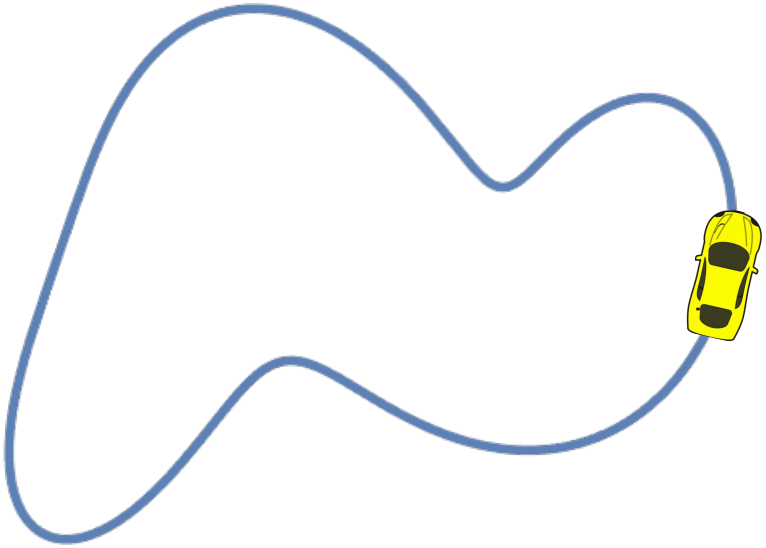


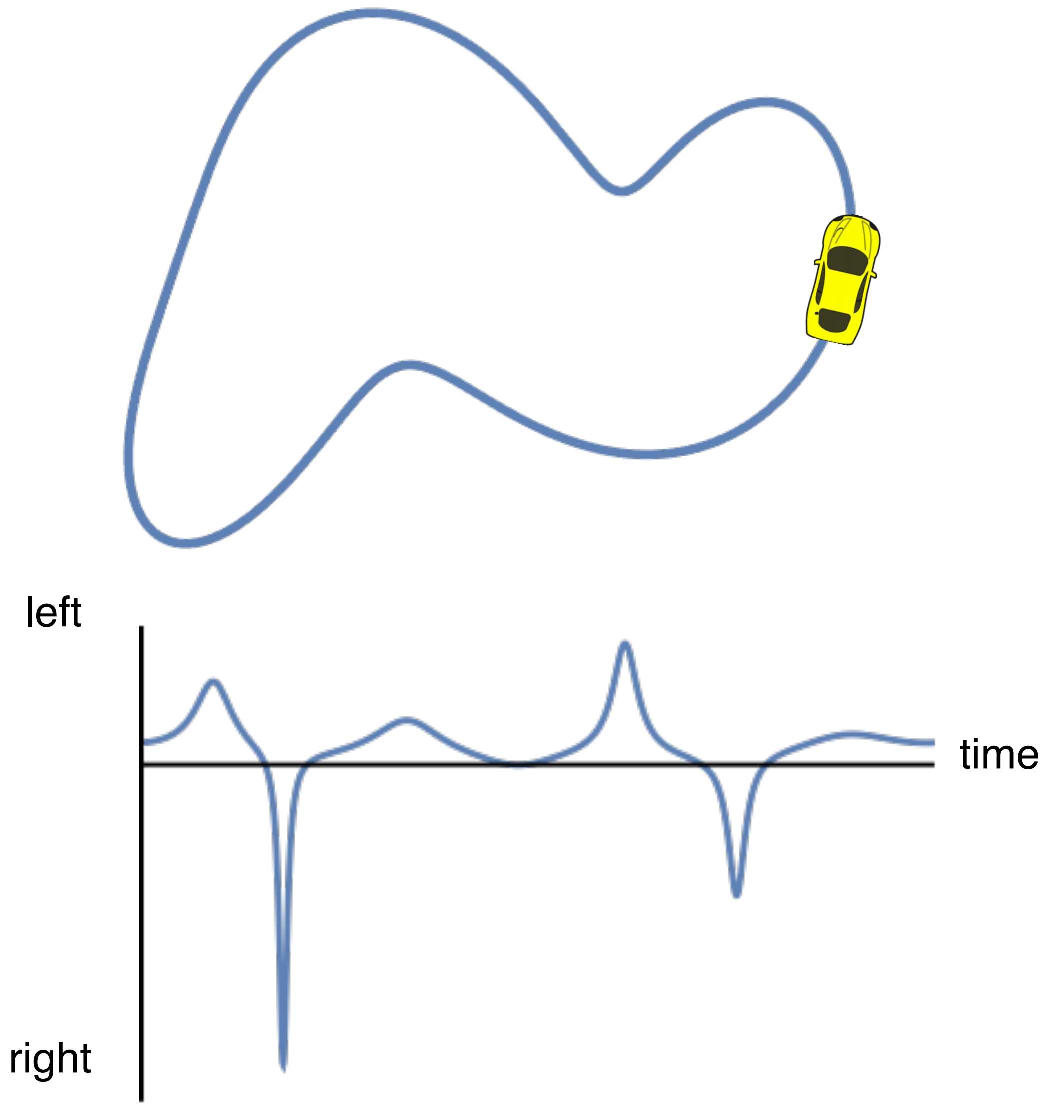
Curvature = reciprocal of radius of osculating circle



What everyday device can measure curvature?

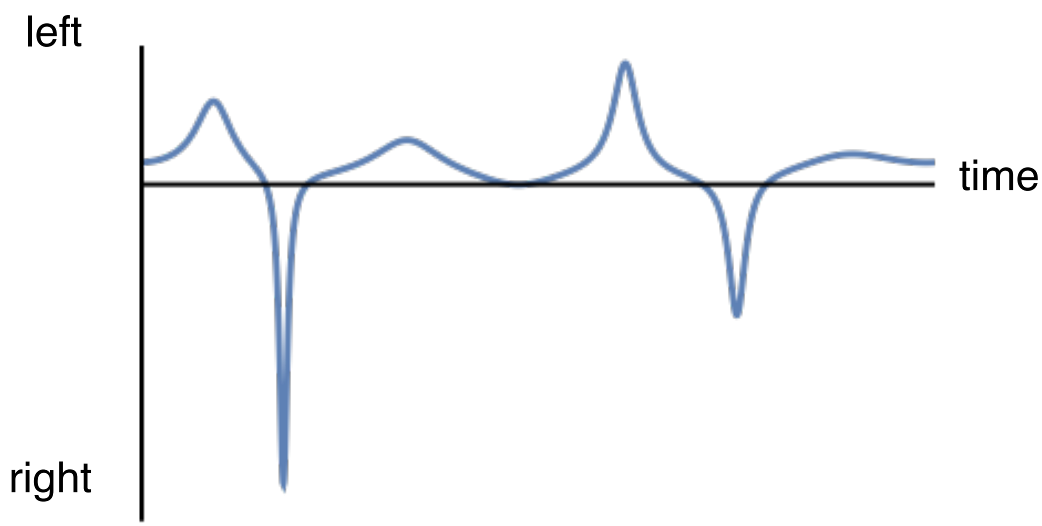




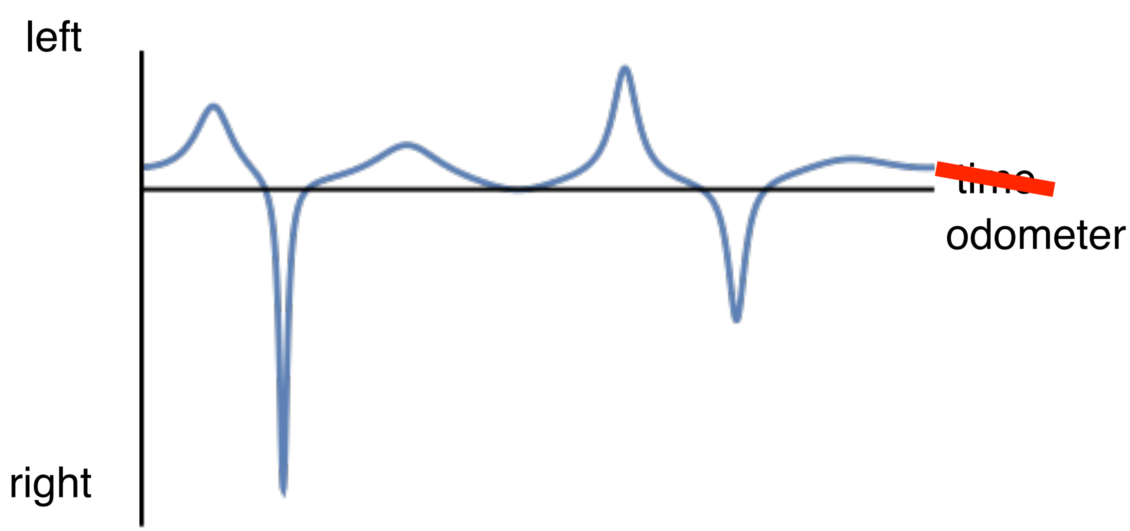


left
time
right

Can you reconstruct the racetrack?

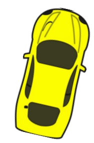


Can you reconstruct the racetrack?

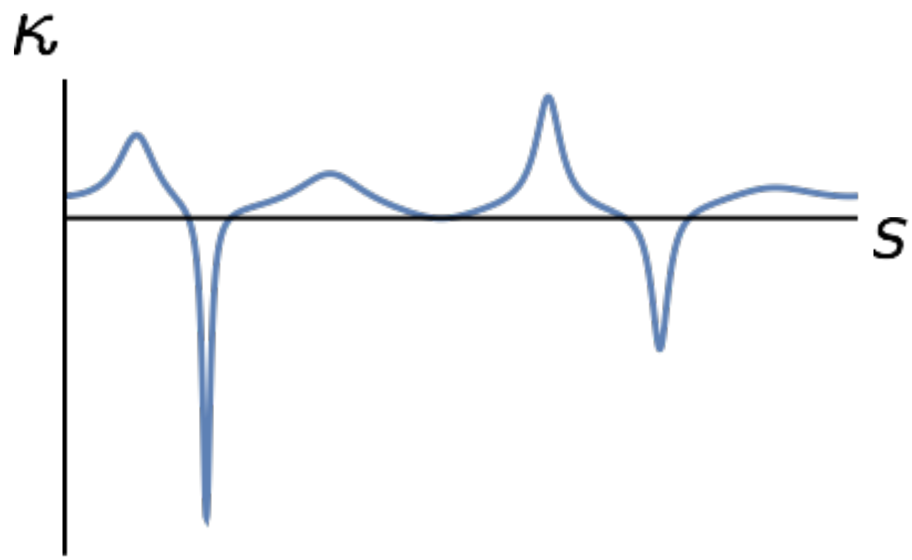


Can you reconstruct the racetrack?

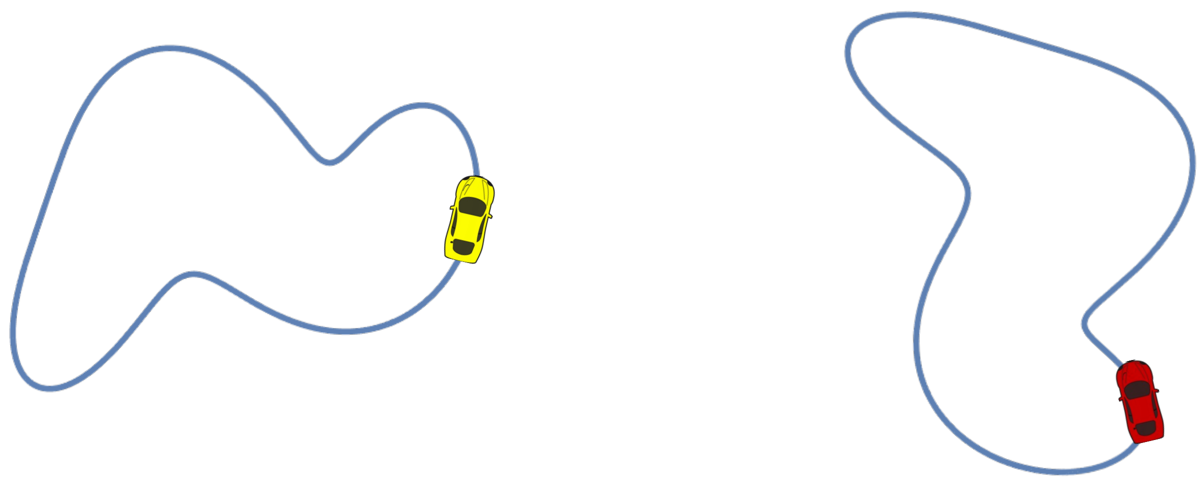
κ is (Euclidean) curvature



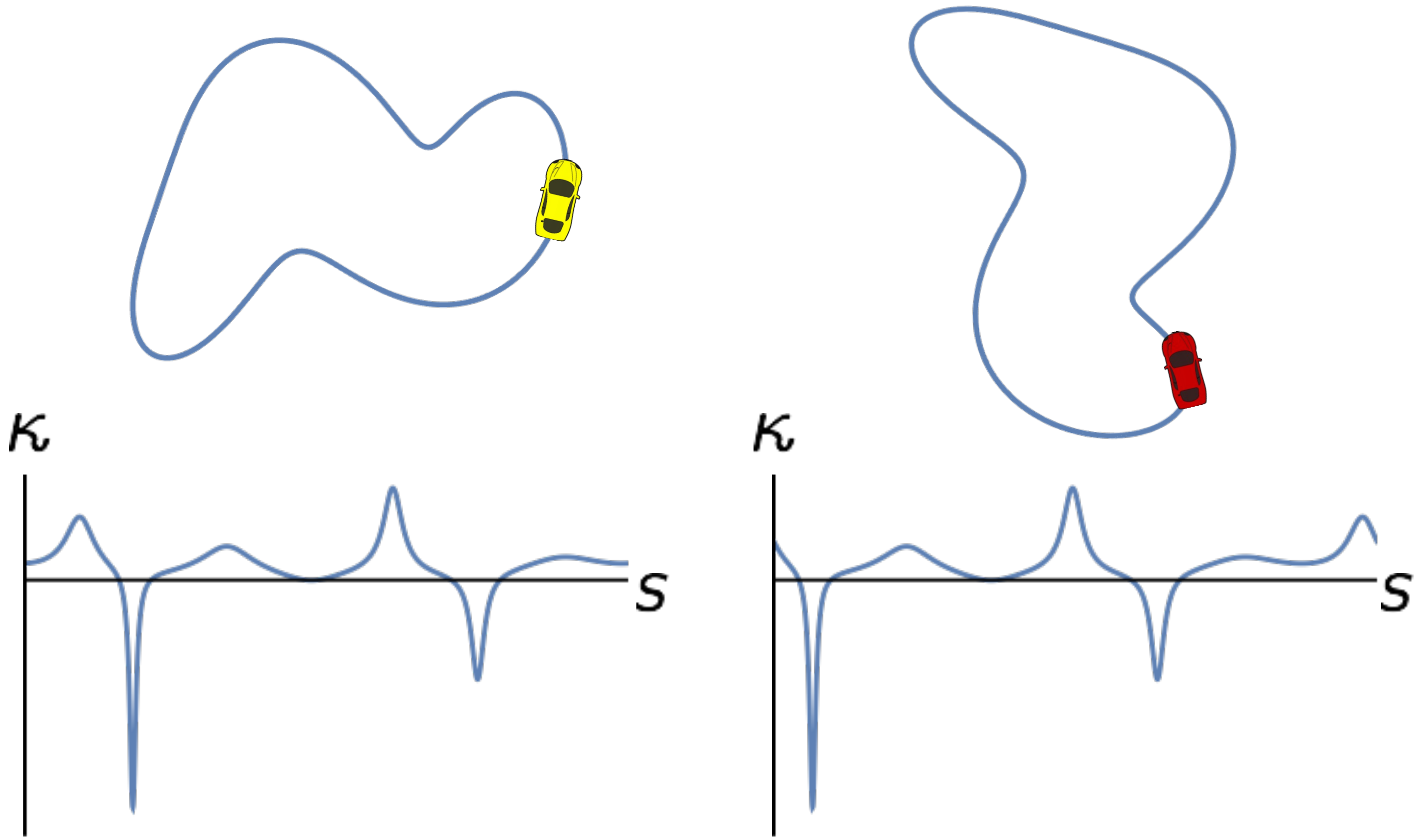
s is (Euclidean) arclength



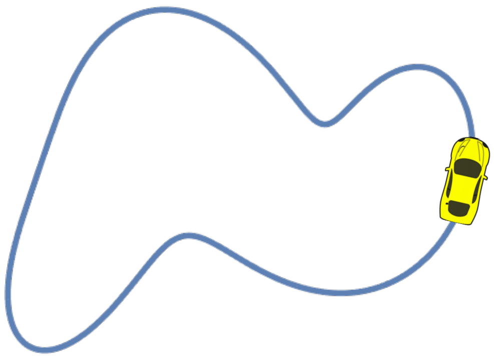
Racetrack comparison problem

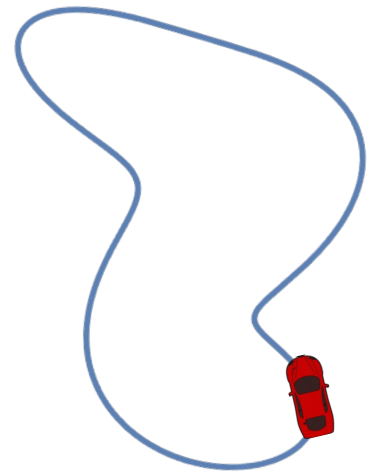


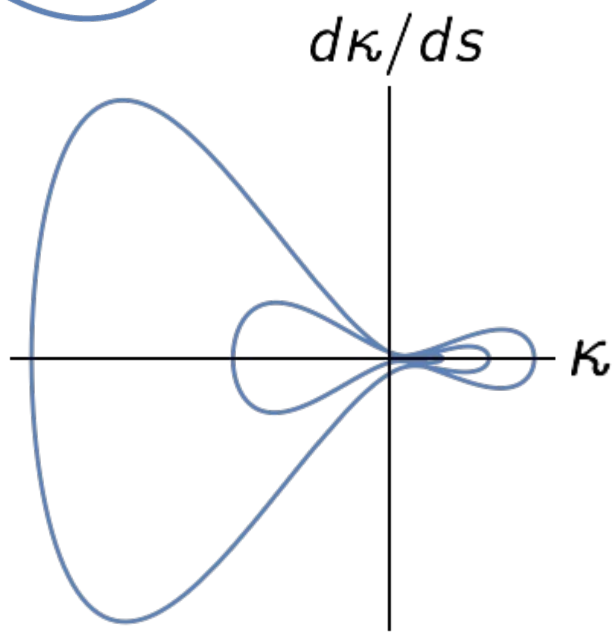
Racetrack comparison problem



Racetrack comparison problem





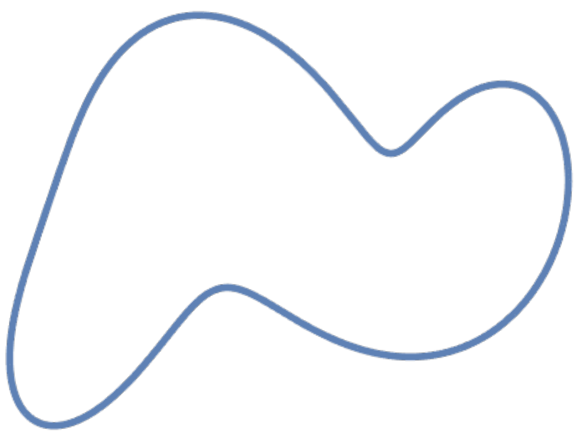


$d\kappa/ds$

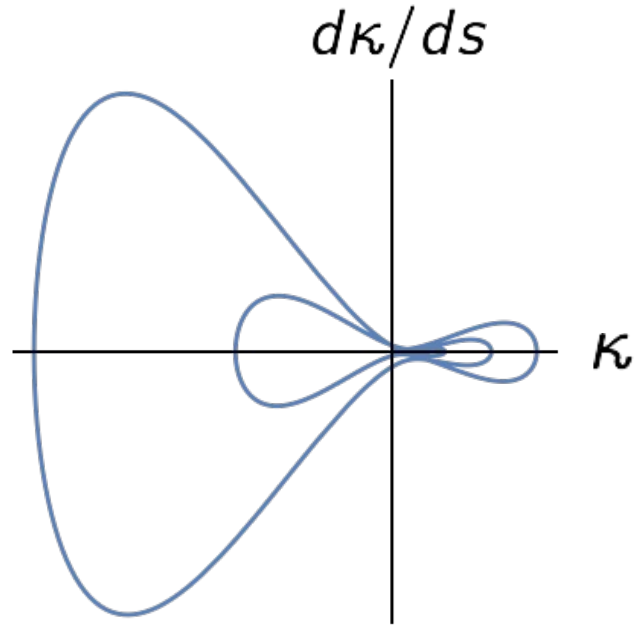
= rate of change (derivative) of curvature

The Invariant Signature

The invariant signature of a planar curve is the set traced out by curvature and the rate of change of curvature (its arclength derivative).



original curve

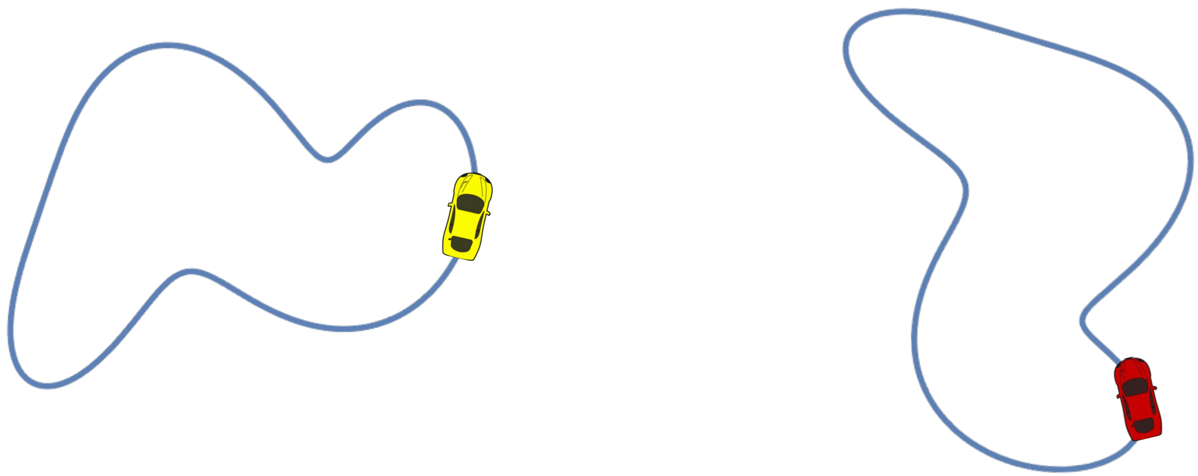


invariant signature

The invariant signature

Theorem

Two regular curves are related by a group transformation if and only if they have the same invariant signatures.



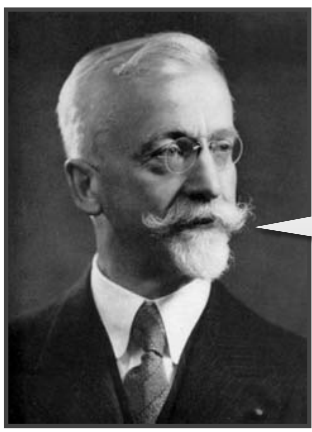
(Calabi, Haker, Olver, Shakiban, Tannenbaum 1998)

The invariant signature

Theorem

Two regular curves are related by a group transformation if and only if they have the same invariant signatures.

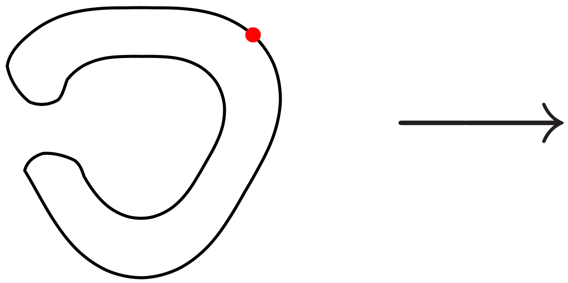
Proof idea



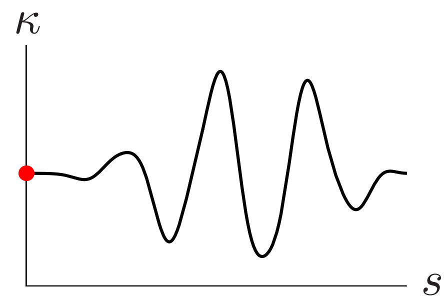
Theorem (Élie Cartan 1908)

Shapes are related if and only if they have the same relationships among their **differential invariants**.

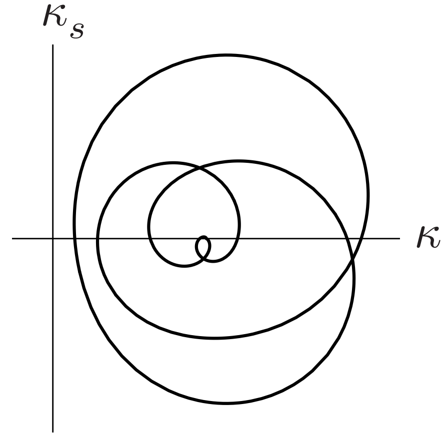
Signatures



Original curve

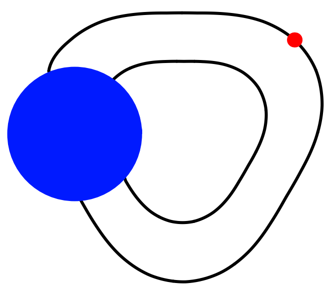


Classical Signature



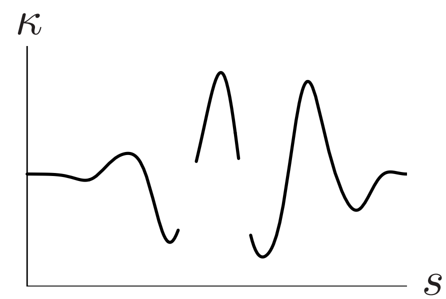
Differential invariant signature

Occlusions

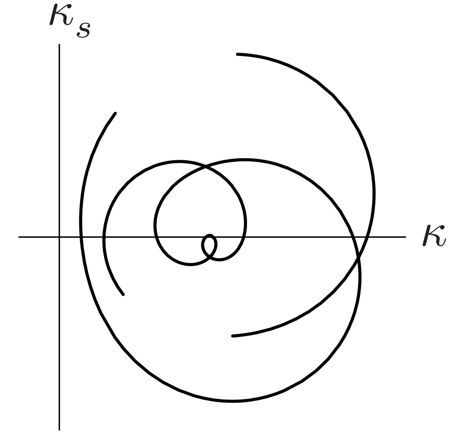


Original curve

$\longrightarrow$



Classical Signature



Differential invariant signature

3D Differential Invariant Signatures

Euclidean space curves: $C \subset \mathbb{R}^3$

$$\Sigma = \{ \, (\, \kappa \, , \, \kappa_s \, , \, \tau \,) \, \} \ \subset \ \mathbb{R}^3$$

- κ — curvature, τ — torsion

Euclidean surfaces: $S \subset \mathbb{R}^3$ (generic)

$$\Sigma = \left\{ \, \left(\, H \, , \, K \, , \, H_{,1} \, , \, H_{,2} \, , \, K_{,1} \, , \, K_{,2} \right) \, \right\} \ \subset \ \mathbb{R}^6$$

or $$\widehat{\Sigma} = \left\{ \, \left(\, H \, , \, H_{,1} \, , \, H_{,2} \, , \, H_{,11} \right) \, \right\} \ \subset \ \mathbb{R}^4$$

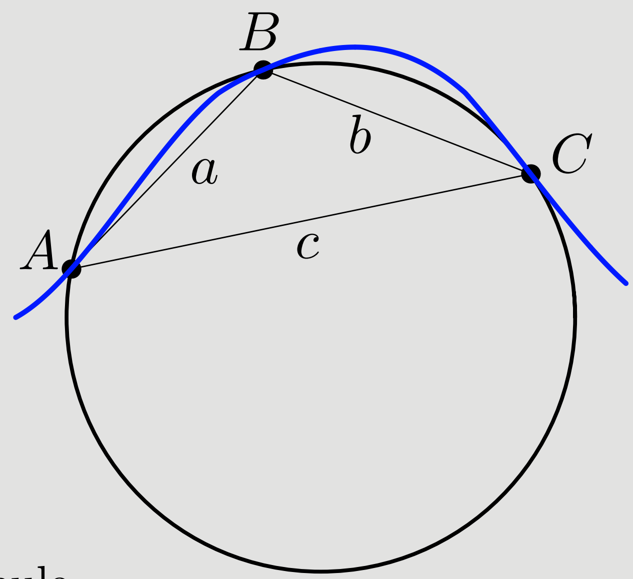
- H — mean curvature, K — Gauss curvature

Symmetry–Preserving Numerical Methods

- Invariant numerical approximations to differential invariants.
- Invariantization of numerical integration methods.

$\Longrightarrow$ Structure-preserving algorithms

Numerical approximation to curvature



Heron's formula

$$\tilde{\kappa}(A,B,C) = 4\,\frac{\Delta}{abc} = 4\,\frac{\sqrt{s(s-a)(s-b)(s-c)}}{abc}$$

$$s = \frac{a+b+c}{2} \quad — \quad \text{semi-perimeter}$$

Higher order invariants

$$\kappa_s = \frac{d\kappa}{ds}$$

Invariant finite difference approximation:

$$\widetilde{\kappa}_s(P_{i-2}, P_{i-1}, P_i, P_{i+1}) = \frac{\widetilde{\kappa}(P_{i-1}, P_i, P_{i+1}) - \widetilde{\kappa}(P_{i-2}, P_{i-1}, P_i)}{\mathbf{d}(P_i, P_{i-1})}$$

Unbiased centered difference:

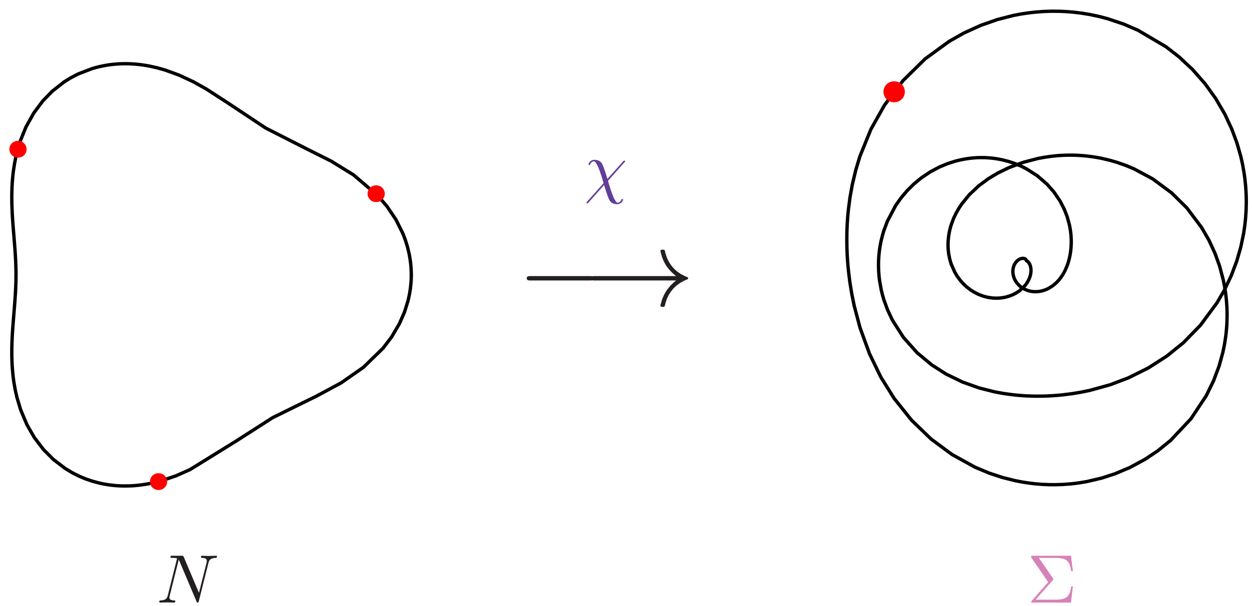
$$\widetilde{\kappa}_s(P_{i-2}, P_{i-1}, P_i, P_{i+1}, P_{i+2}) = \frac{\widetilde{\kappa}(P_i, P_{i+1}, P_{i+2}) - \widetilde{\kappa}(P_{i-2}, P_{i-1}, P_i)}{\mathbf{d}(P_{i+1}, P_{i-1})}$$

Better approximation (M. Boutin):

$$\widetilde{\kappa}_s(P_{i-2}, P_{i-1}, P_i, P_{i+1}) = 3\ \frac{\widetilde{\kappa}(P_{i-1}, P_i, P_{i+1}) - \widetilde{\kappa}(P_{i-2}, P_{i-1}, P_i)}{\mathbf{d}_{i-2} + 2\,\mathbf{d}_{i-1} + 2\,\mathbf{d}_i + \mathbf{d}_{i+1}}$$

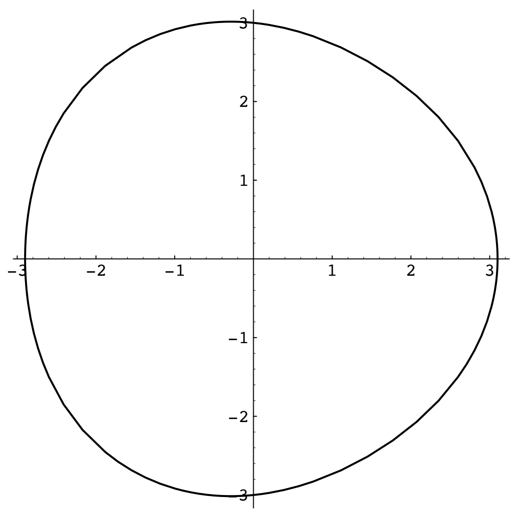
$$\mathbf{d}_j = \mathbf{d}(P_j, P_{j+1})$$

The Index

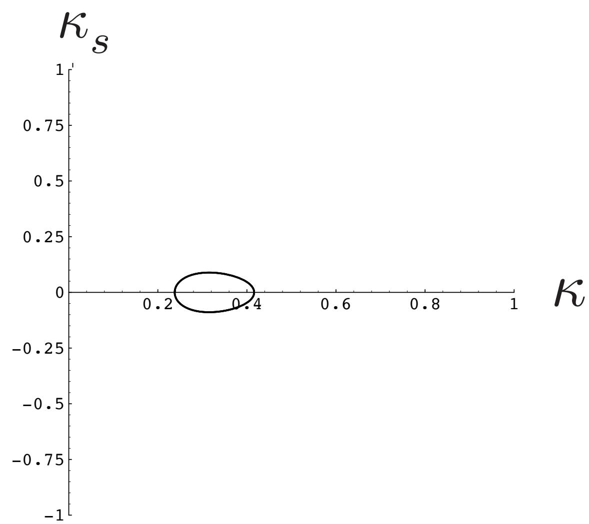


$$\text{index} = 3 = \#\ \text{symmetries}$$

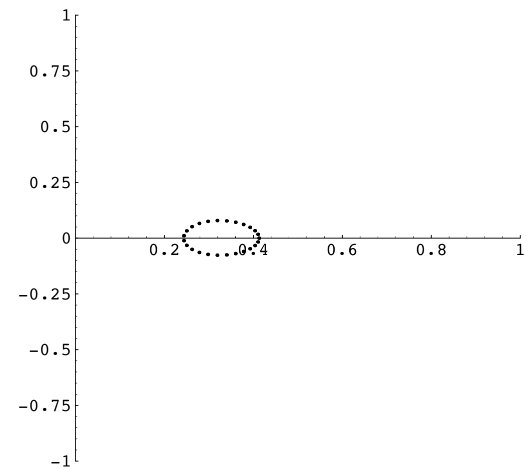
The polar curve $r = 3 + \frac{1}{10}\cos 3\theta$



The Original Curve

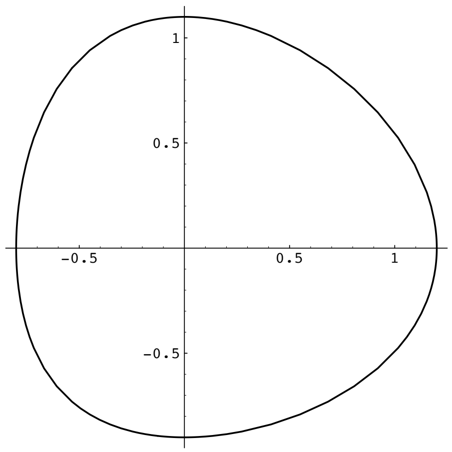


Euclidean Signature

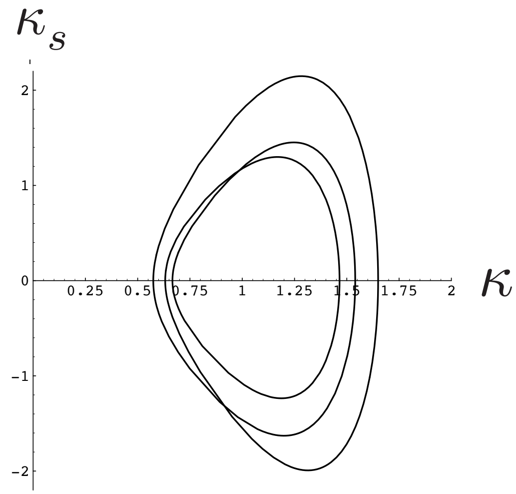


Numerical Signature

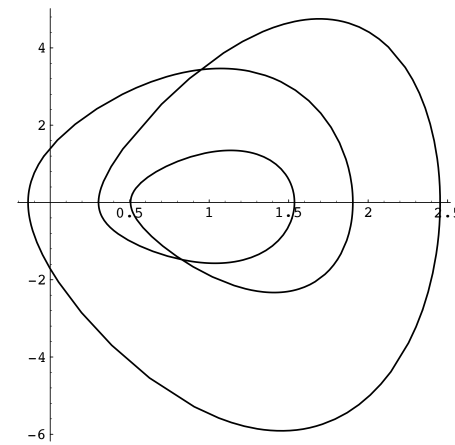
The Curve $x = \cos t + \frac{1}{5}\cos^2 t, \quad y = \sin t + \frac{1}{10}\sin^2 t$



The Original Curve

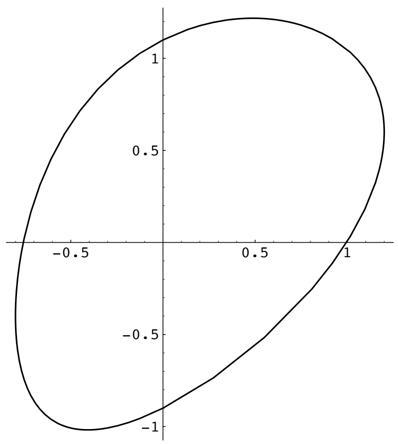


Euclidean Signature

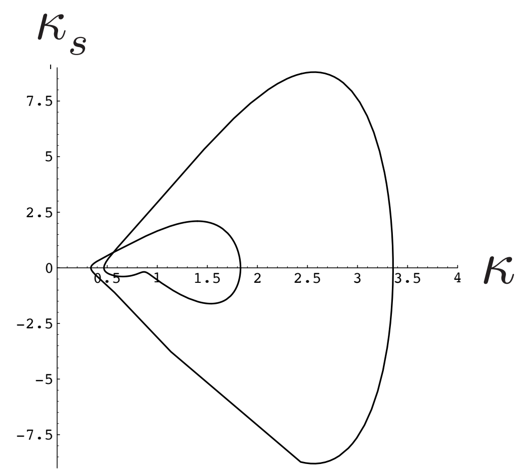


Equi-affine Signature

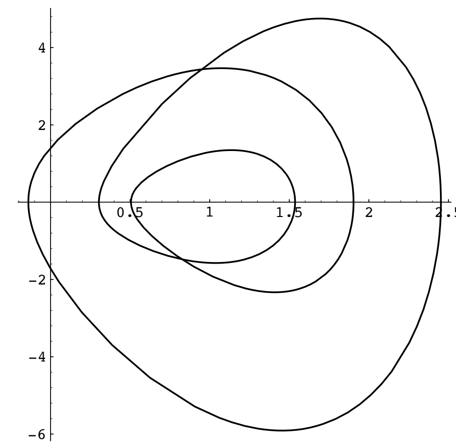
The Curve $x = \cos t + \frac{1}{5}\cos^2 t, \quad y = \frac{1}{2}x + \sin t + \frac{1}{10}\sin^2 t$



The Original Curve

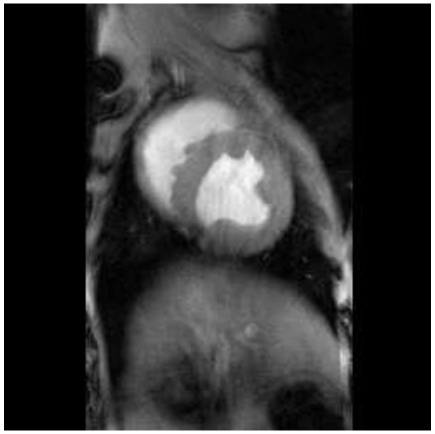


Euclidean Signature

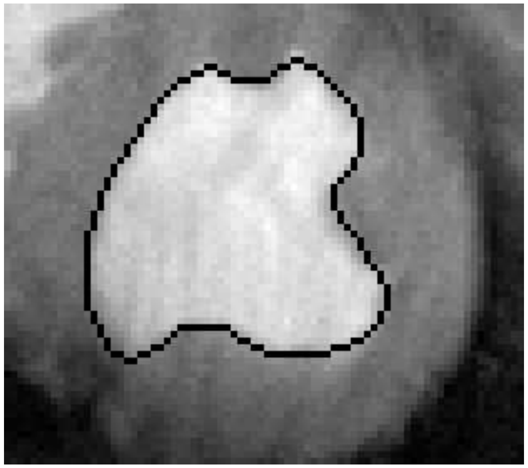


Equi-affine Signature

Canine Left Ventricle Signature

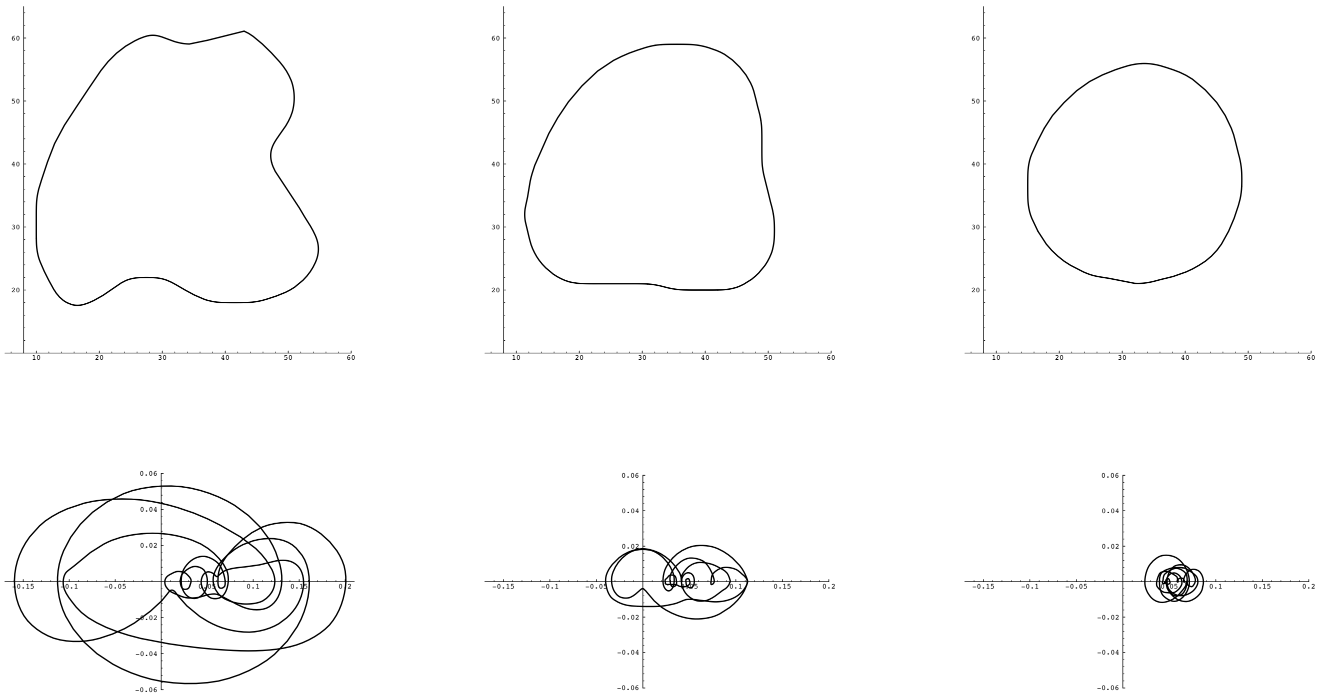


Original Canine Heart
MRI Image

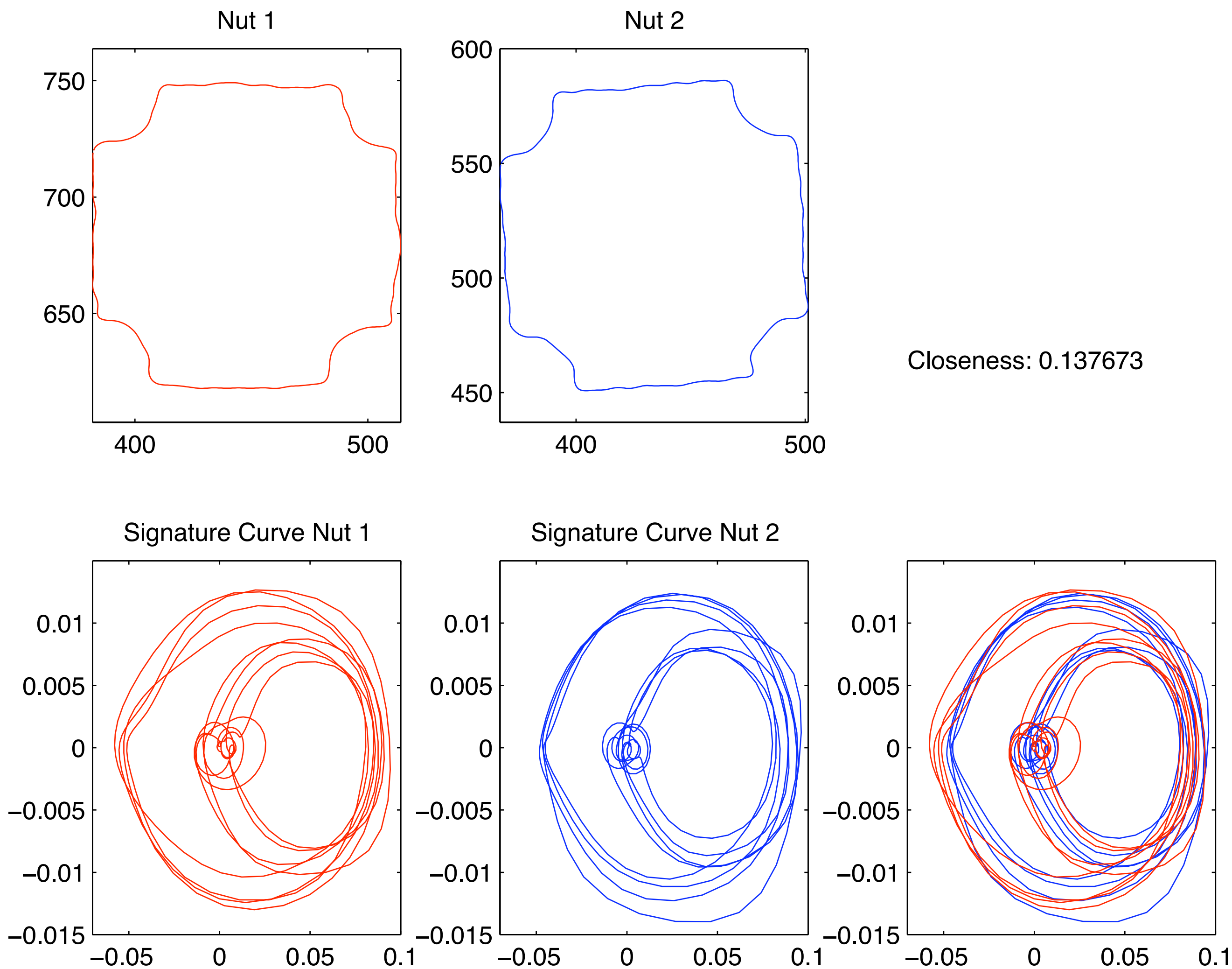


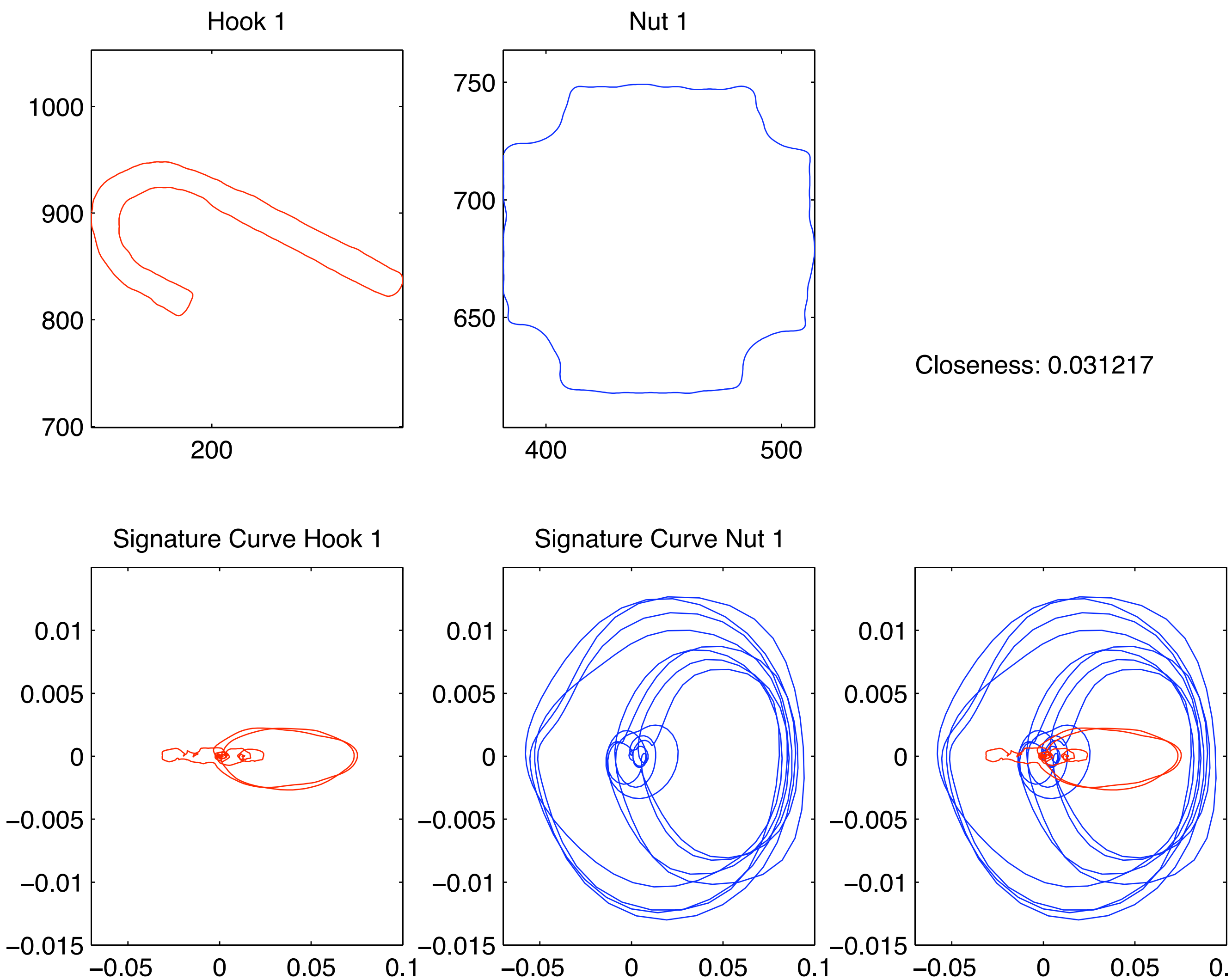
Boundary of Left Ventricle

Smoothed Ventricle Signature



Object Recognition


Nut 1
Nut 2
Closeness: 0.137673
Signature Curve Nut 1
Signature Curve Nut 2


Hook 1
Nut 1
1000
900
800
700
200
750
700
650
400
500
Closeness: 0.031217
Signature Curve Hook 1
Signature Curve Nut 1
0.01
0.005
0
−0.005
−0.01
−0.015
−0.05
0
0.05
0.1

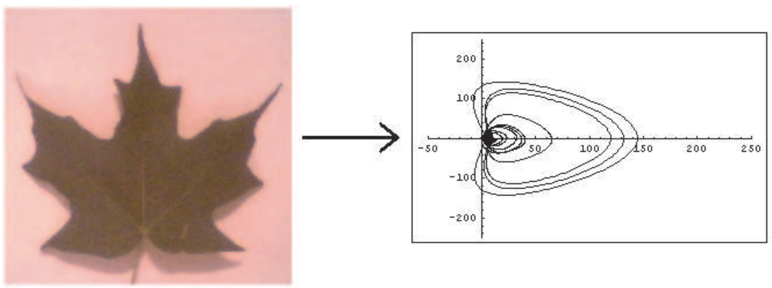


Figure 8: The Maple Leaf and its Signature Curve

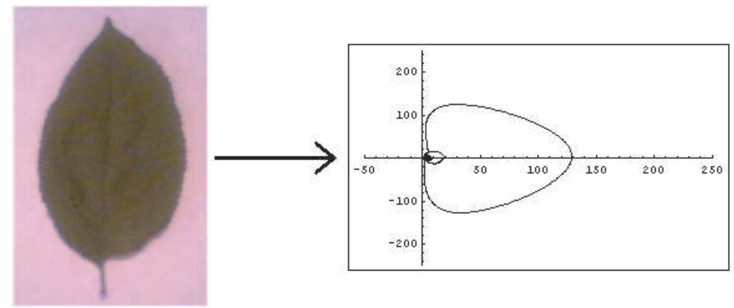


Figure 9: The Buckthorn Leaf and its Signature Curve

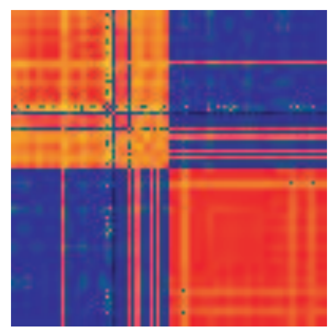
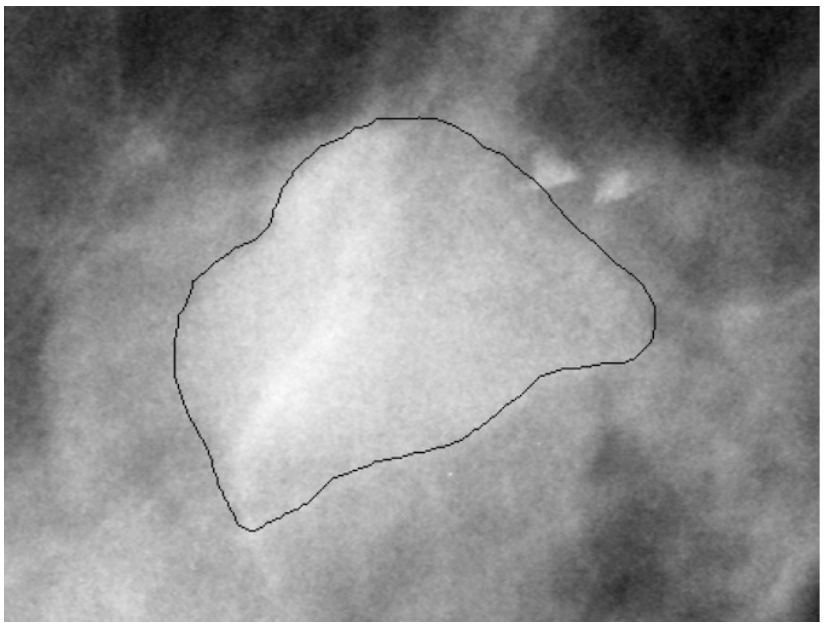


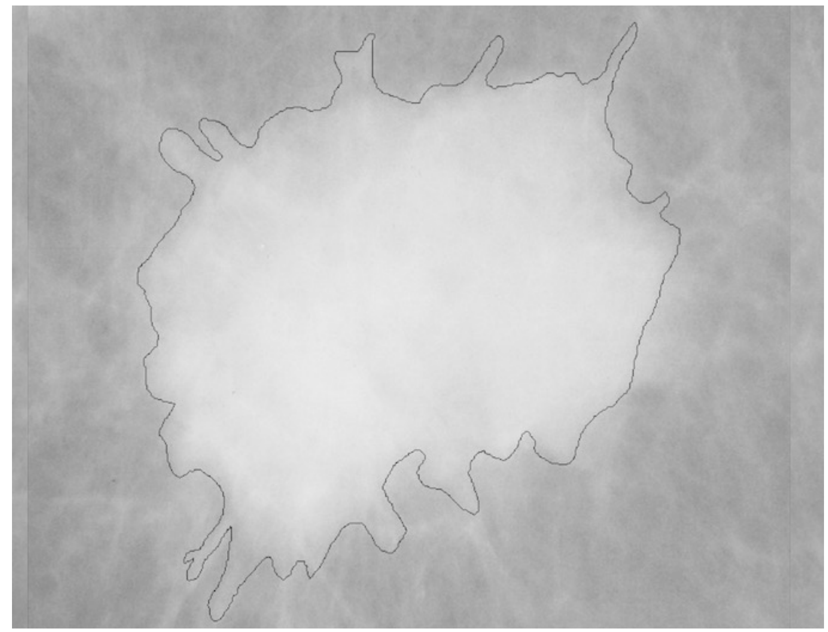
Figure 10: Correlation Matrix for Maple versus Buckthorn

Ryan Lloyd, Cheri Shakiban (2004)

Diagnosing breast tumors



Benign — cyst

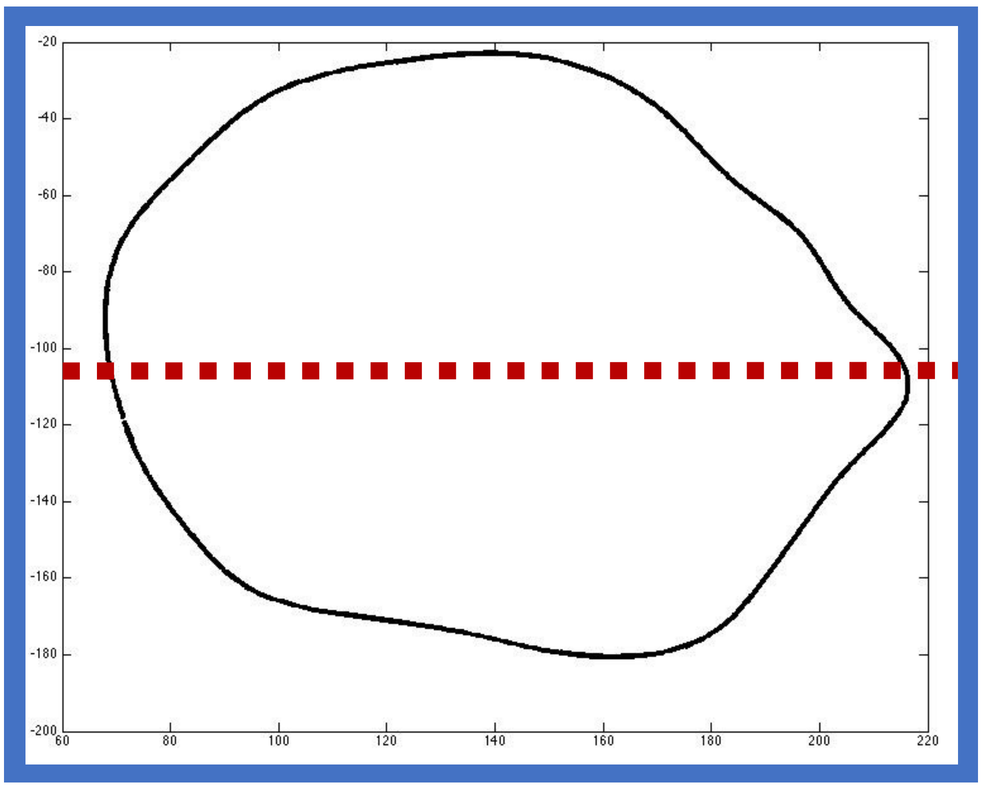


Malignant — cancerous

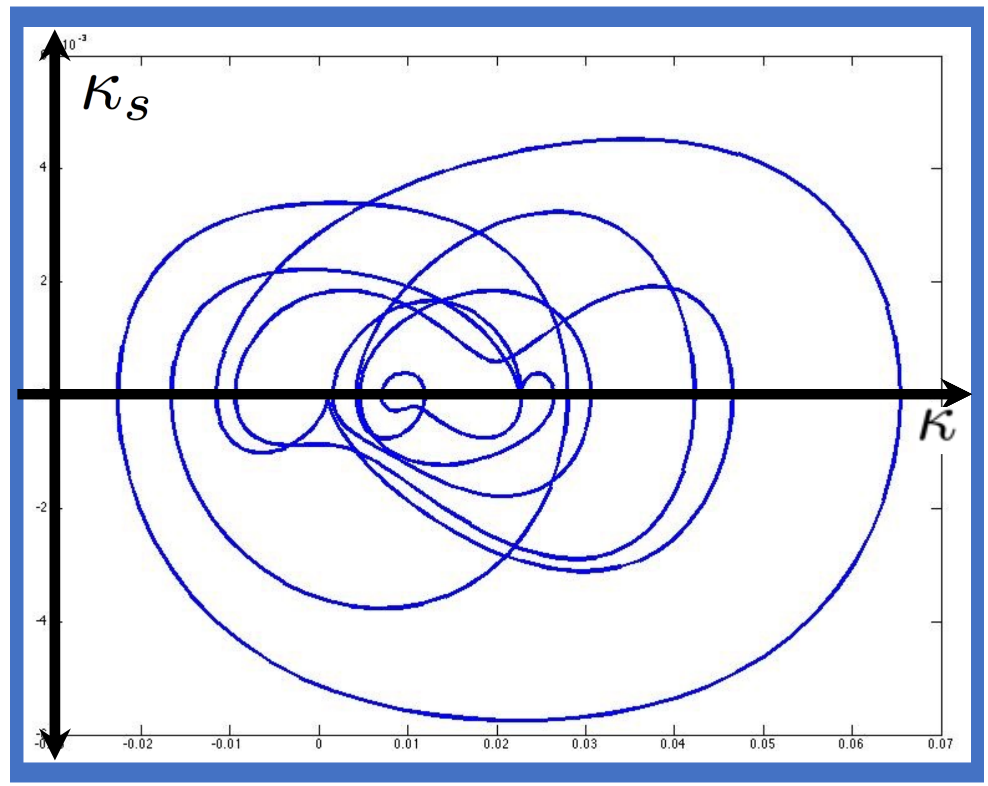
Anna Grim, Cheri Shakiban (2017)

A BENIGN TUMOR

Contour

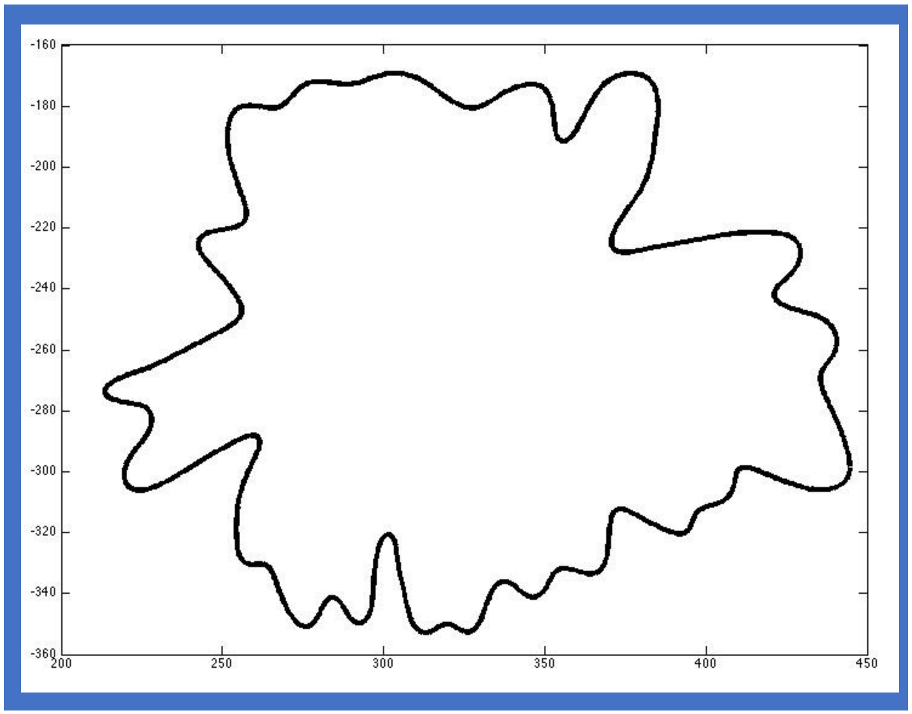


Signature Curve

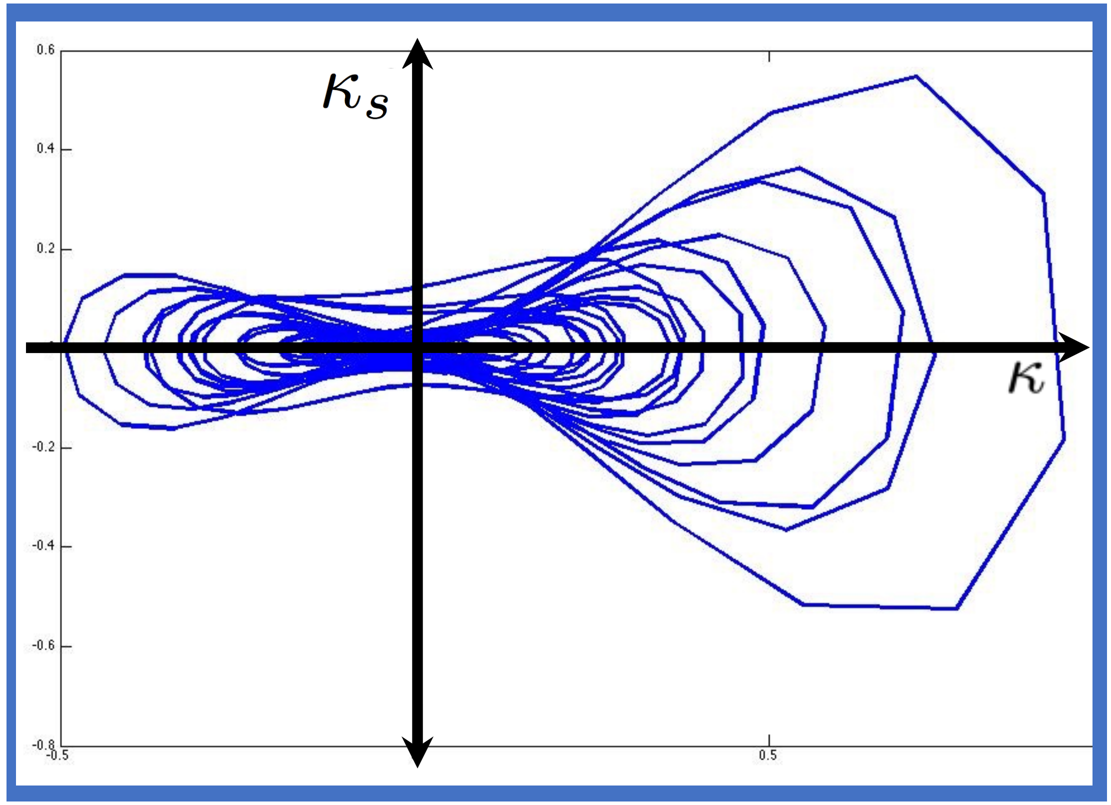


A MALIGNANT TUMOR

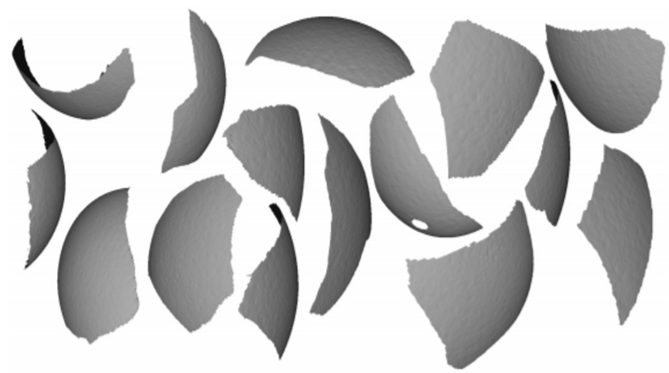
Contour



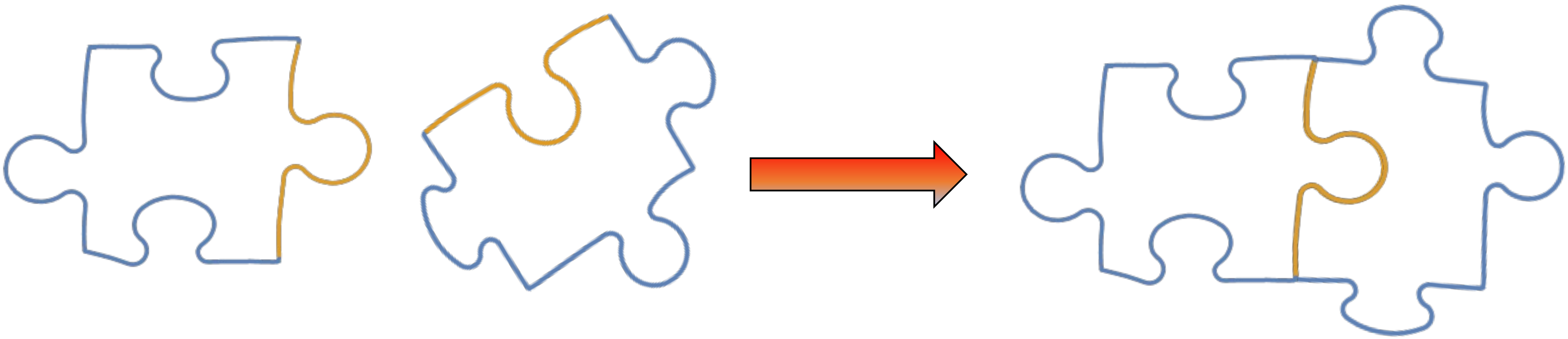
Signature Curve



Reassembly of Broken Objects



Apictorial jigsaw puzzle reassembly



Step 0. Digitally photograph and smooth the puzzle pieces.

Step 1. Numerically compute invariant signatures of (parts of) pieces.

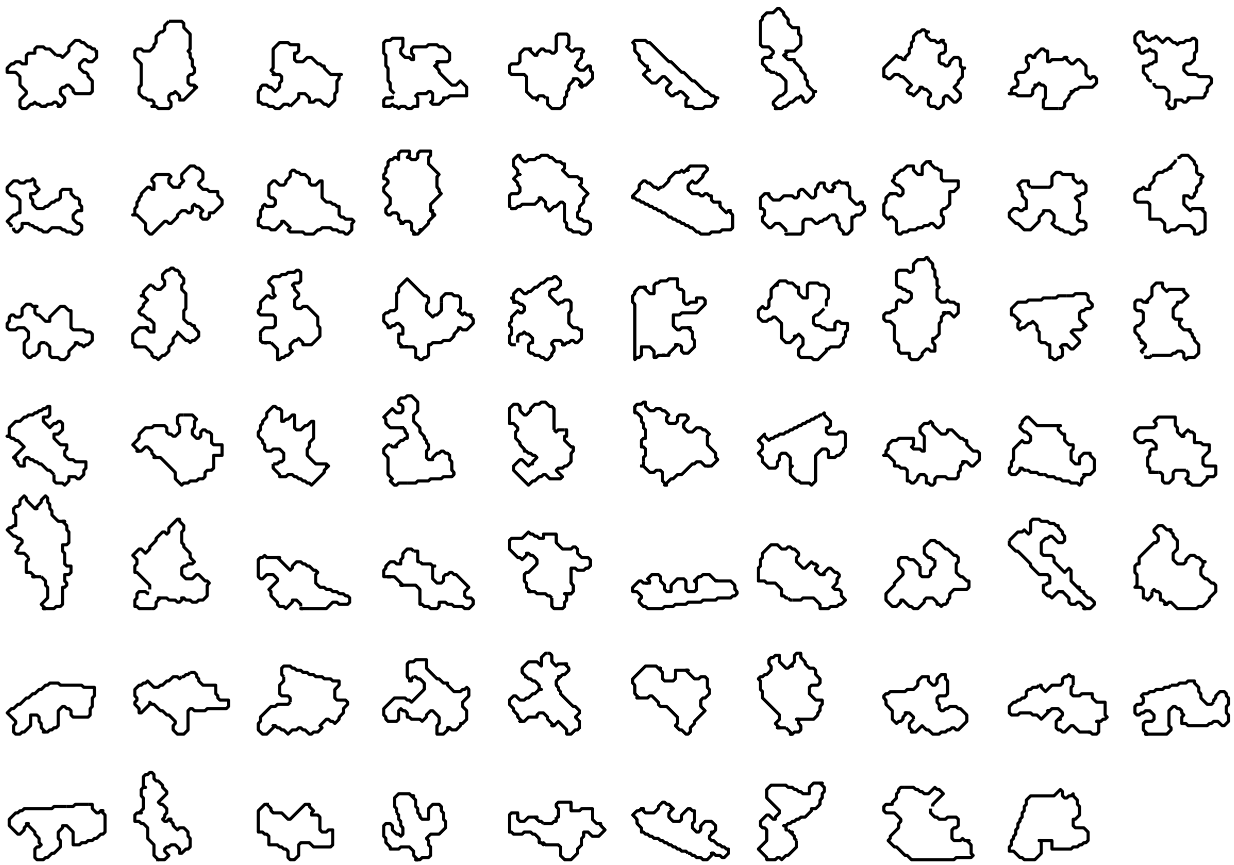
Step 2. Compare signatures to find potential fits.

Step 3. Put them together, if they fit, as closely as possible.

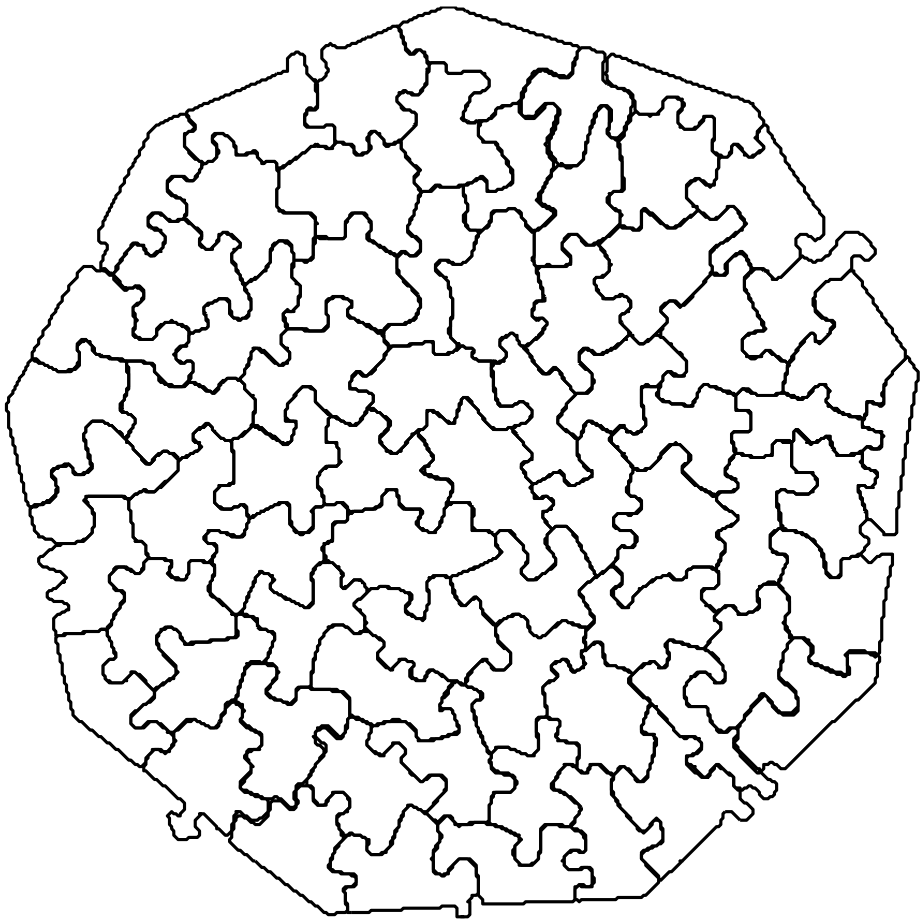
Repeat steps 1–3 until puzzle is assembled….


the most unique puzzle ever
the BAFFLER™
by CHRIS Yates
The Nonagon
67 pieces

The Baffler Nonagon



The Baffler Nonagon — Solved

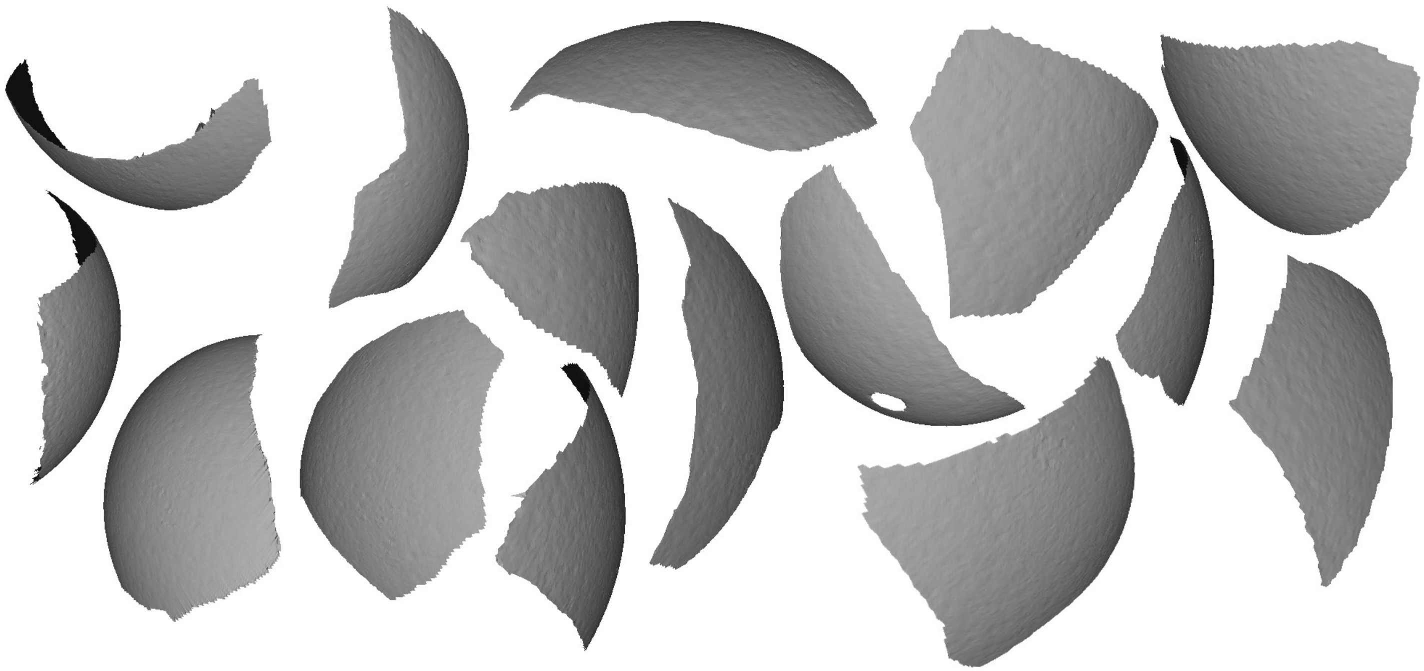


$\Longrightarrow$ D. Hoff & PJO, Automatic solution of jigsaw puzzles,
J. Math. Imaging Vision **49** (2014) 234–250.

Putting Humpty Dumpty Together Again

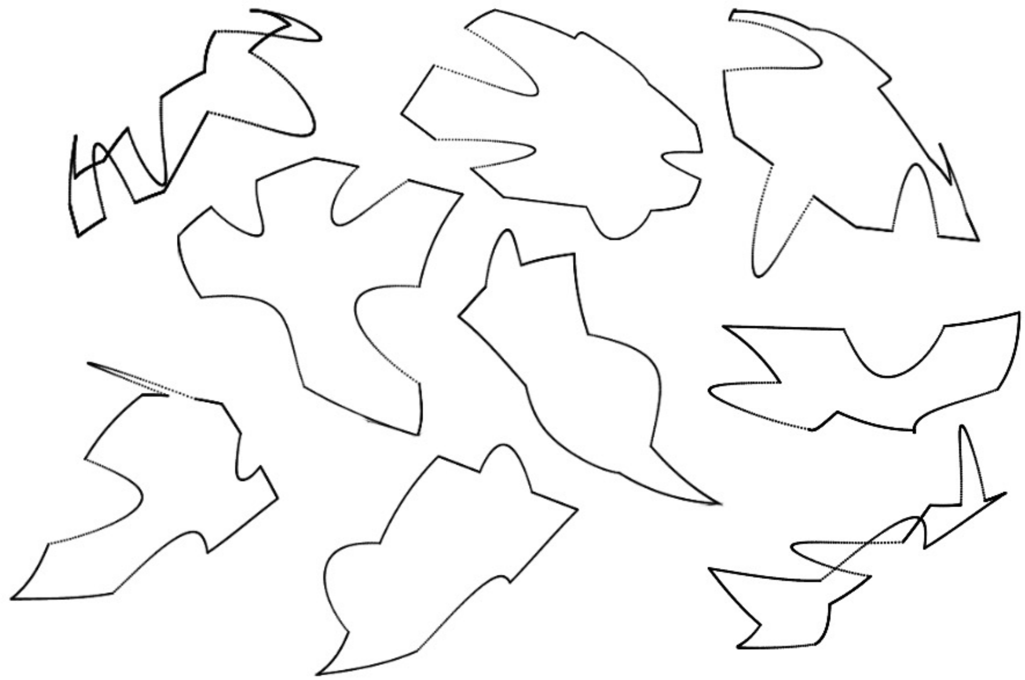
Anna Grim, Ryan Slechta, Tim O'Connor, Rob Thompson, Cheri Shakiban, PJO

A broken ostrich egg

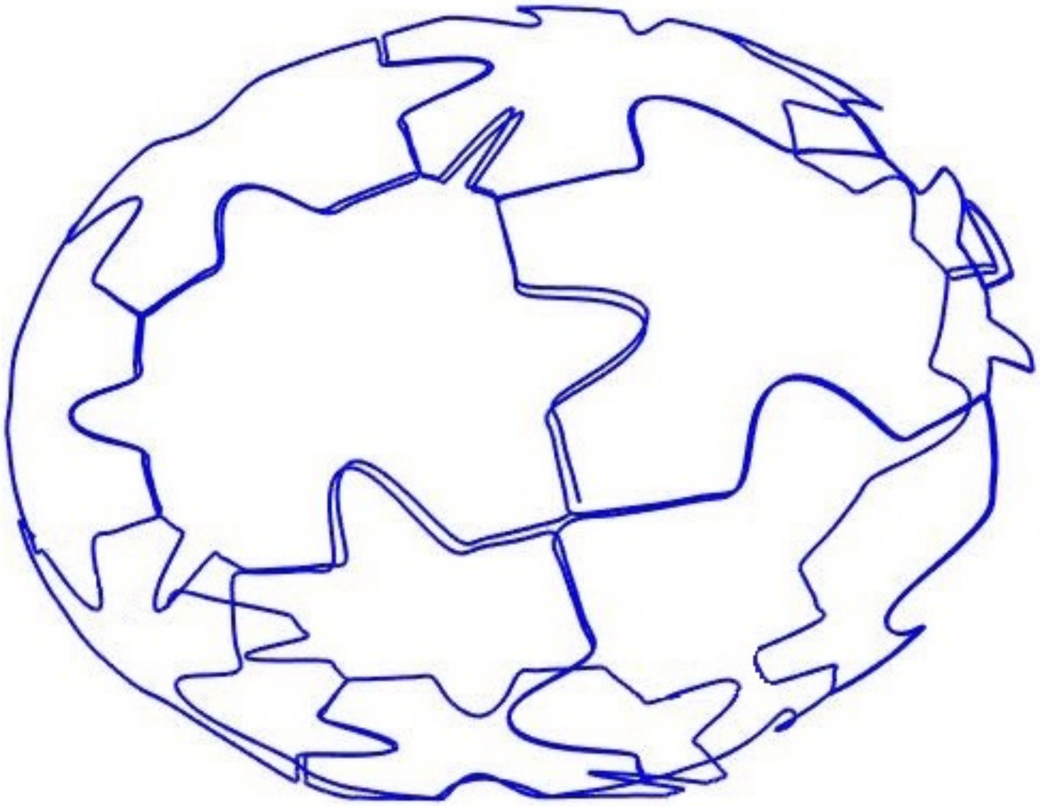


(Scanned by M. Bern, Xerox PARC)

A synthetic 3d jigsaw puzzle

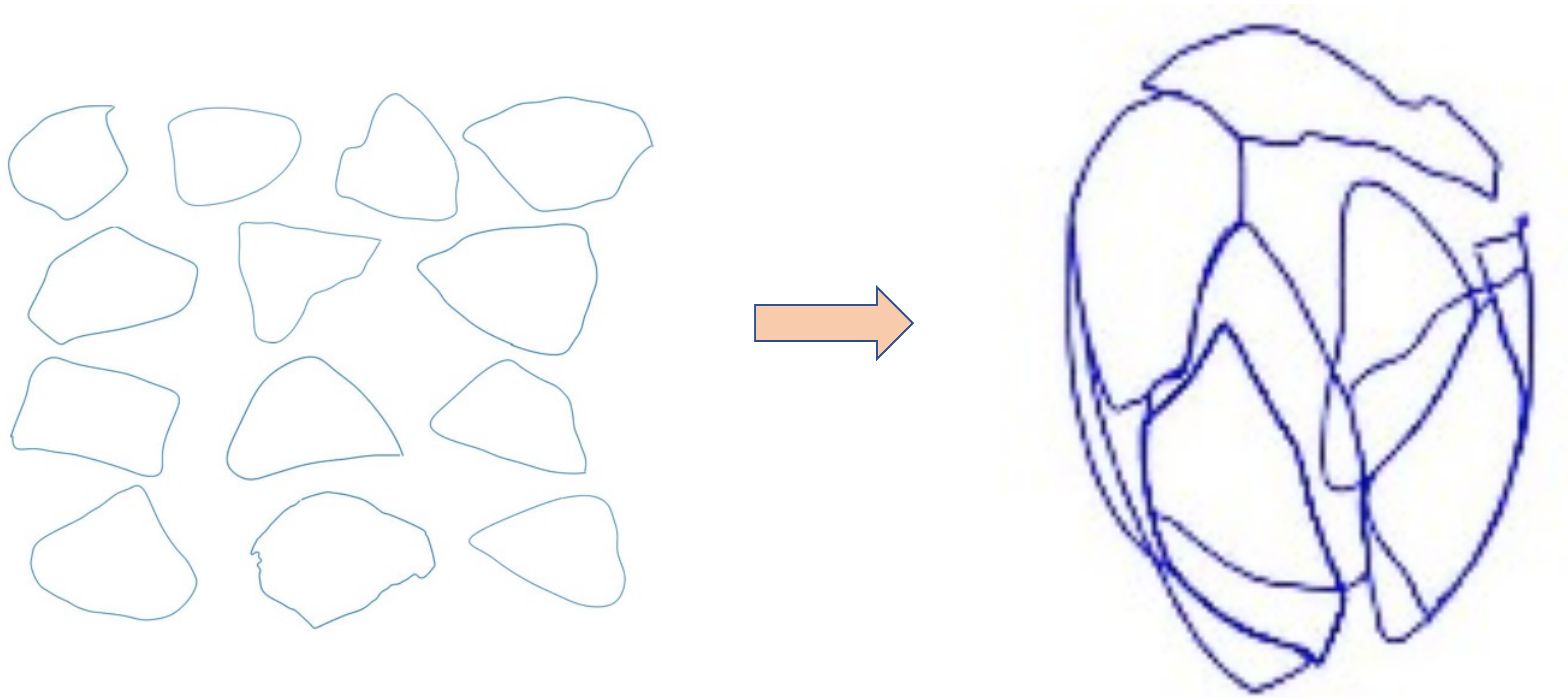


Assembly of Synthetic Ellipsoidal Puzzle



- Uses curvature and torsion invariants

All the king's horses and men



Broken Bones

Katrina Yezzi-Woodley, Martha Tappen, Reed Coil, Gilbert Tostevin, Annie Melton,

Jeff Calder, Peter Olver, Cheri Shakiban, Riley O’Neill

and many undergrad researchers…..

Breaking Bones

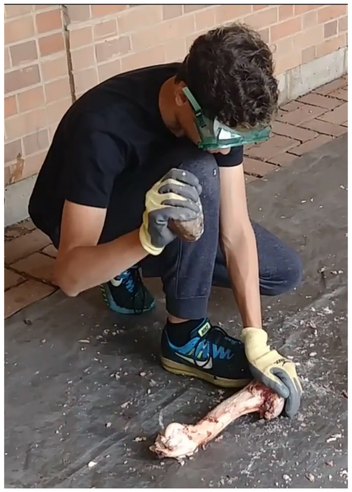
Carnivore

Crocuta crocuta = *hyena*

Hominin

Batting

Hammerstone and anvil



Hammerstone only

Geological

Rock fall

Working Hypothesis

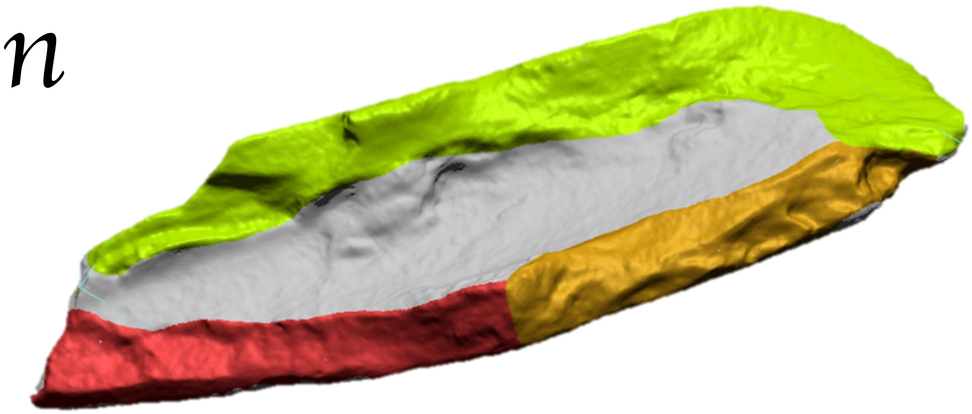
The geometry of the bone fragments,
their identity (taxon and element),
and how they are reassembled
will tell us the actor of breakage

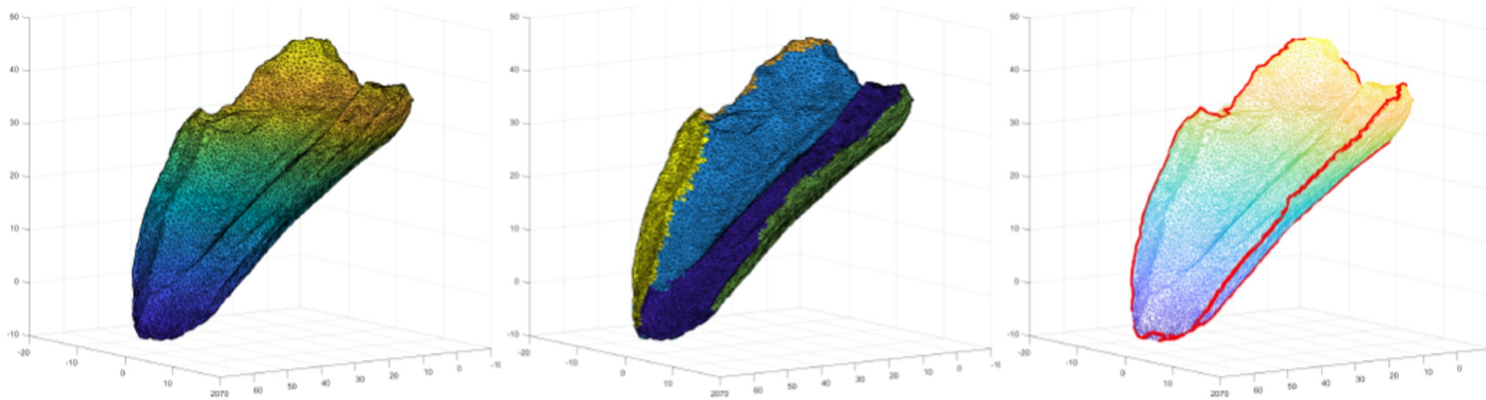
Anthropological Implications

- Meat eater vs. vegetarian
- Brain development
- Scavenging vs. hunting
- Food sharing
- Social structures
- Cooperative behavior
- Home bases/central places
- Carcass transport
- Butchering behavior

OR ?

Segmentation

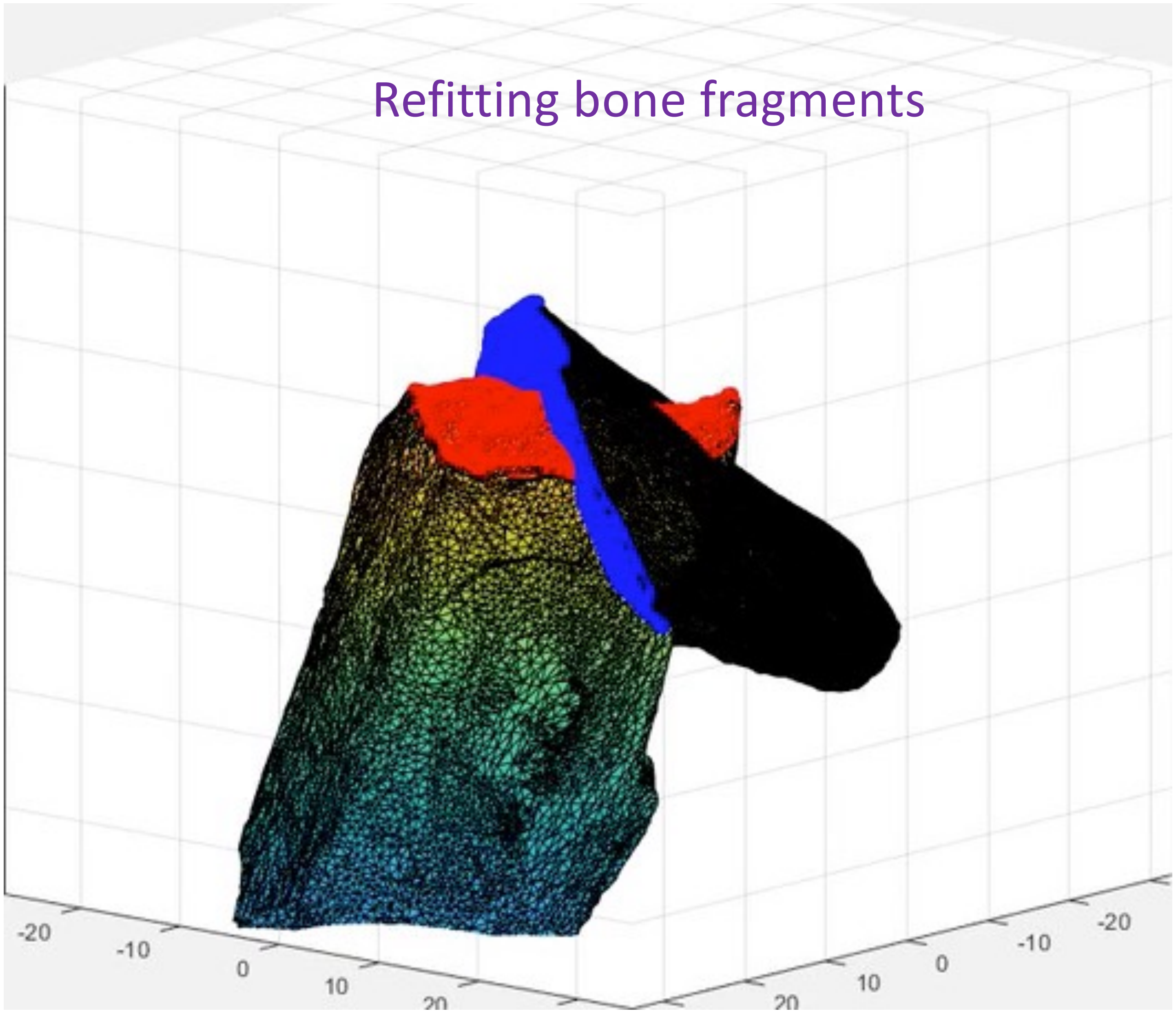




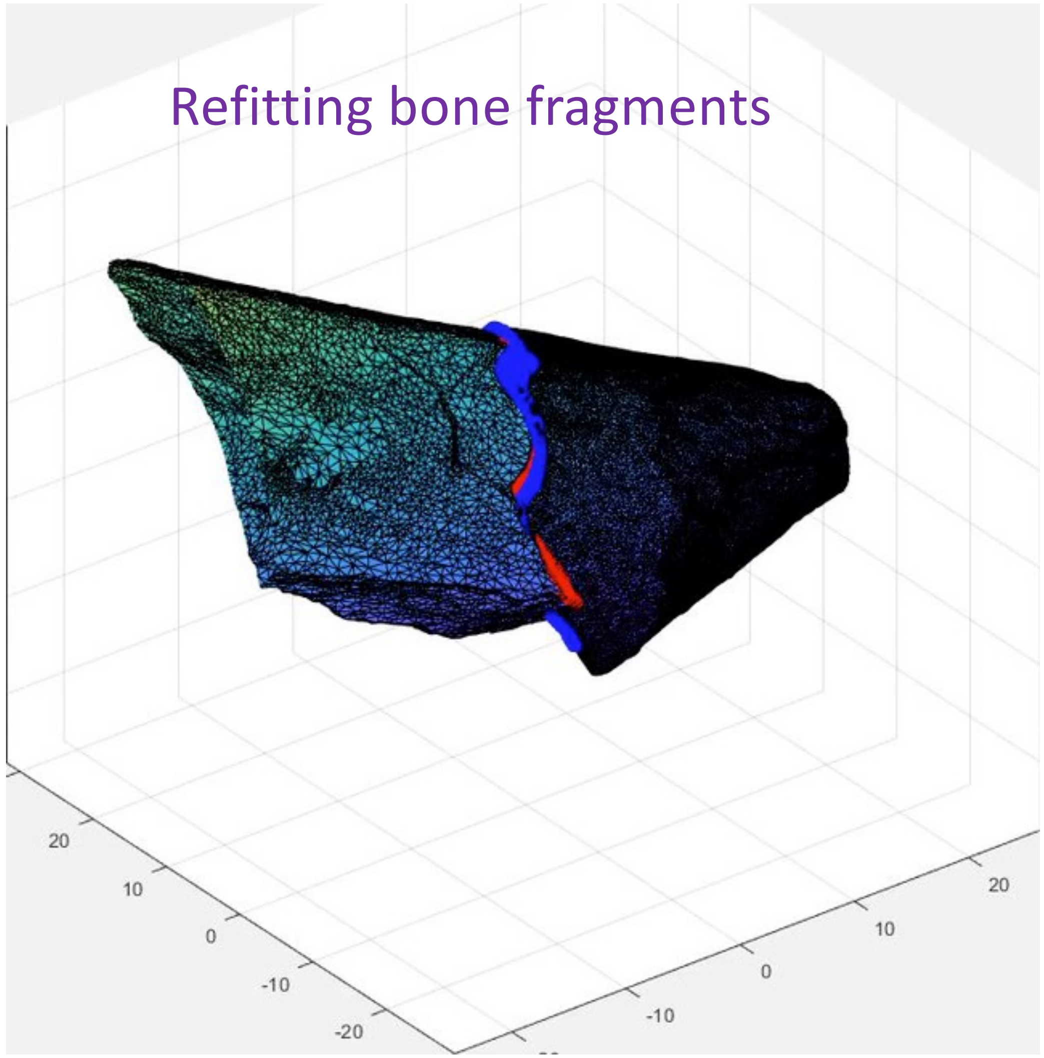
(a) Bone fragment (b) Face segmentation (c) Edge tracing

FIGURE 1: Results of preliminary experiments with face segmentation and edge tracing.

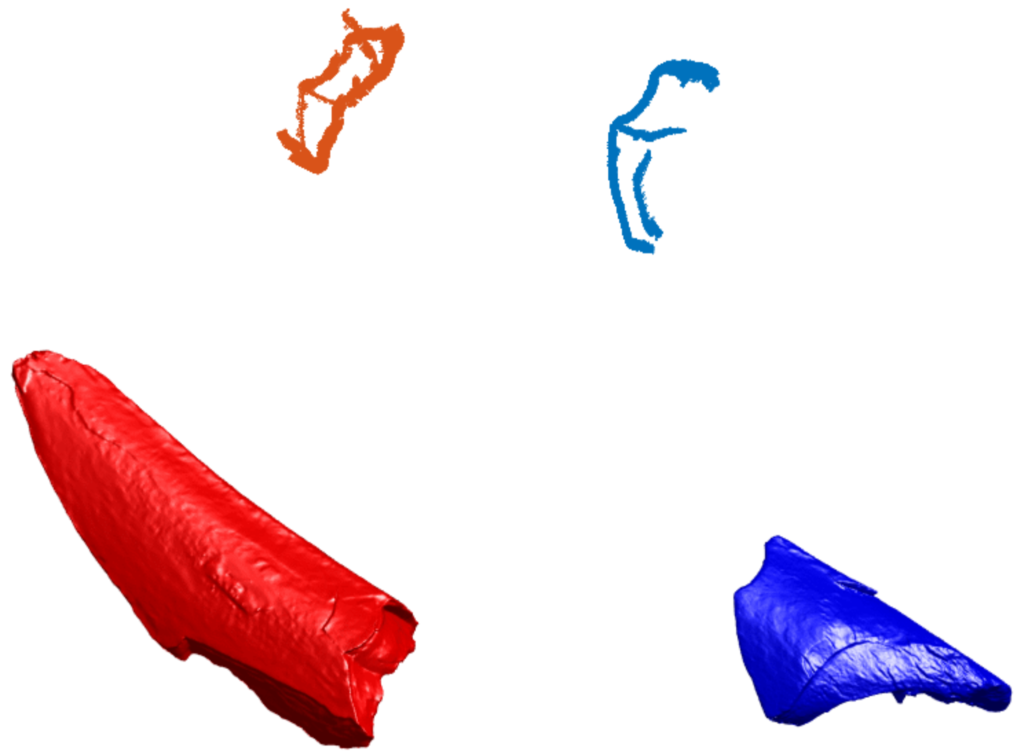
Refitting bone fragments



Refitting bone fragments



Refitting bone fragments:
Gradient descent on SE(3)
using an objective function based on
segmented break edges and surface normals



Riley O'Neill

Thank you for your attention!