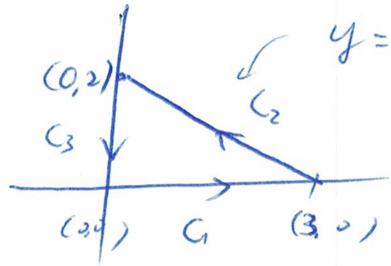
S2008 #6



$y = -\frac{2}{3}x + 2 \Rightarrow x = 3 - \frac{3}{2}y$

$C_1(t) = (t, 0), \quad 0 \le t \le 3$

$C_2(t) = (3 - \frac{3}{2}t, t), \quad 0 \le t \le 2$

$C_3(t) = (0, 2-t), \quad 0 \le t \le 2.$

$$\int_{C^+} F \cdot ds = \int_{C_1} F \cdot ds + \int_{C_2} F \cdot ds + \int_{C_3} F \cdot ds$$

① $$\int_{C_1} F \cdot ds = \int_0^3 (t^2, 0) \cdot (1, 0)\, dt = \frac{1}{3}t^3 \Big|_0^3 = 9$$

② $$\int_{C_2} F \cdot ds = \int_0^2 \langle (3 - \tfrac{3}{2}t)^2, (3 - \tfrac{3}{2}t)t \rangle \cdot \langle -\tfrac{3}{2}, 1 \rangle\, dt$$

$$= \int_0^2 -\frac{3}{2}(3 - \tfrac{3}{2}t)^2 + 3t - \frac{3}{2}t^2\, dt$$

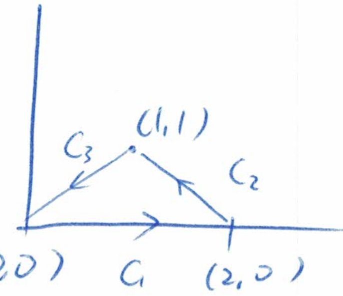
$$= \int_0^2 -\frac{3}{2}(9 - 9t + \tfrac{9}{4}t^2) + 3t - \frac{3}{2}t^2\, dt$$

$$= -7$$

③ $$\int_{C_3} F \cdot ds = \int_0^2 \langle 0, 0 \rangle \cdot \langle 0, -1 \rangle\, dt = 0$$

① + ② + ③ $= 9 - 7 + 0 = \underline{2}$ #

S2010 #6



$$\int_C xy\,dx + x^2\,dy = \int_{C_1} xy\,dx + x^2\,dy + \int_{C_2} \sim + \int_{C_3} \sim$$

$C_1(t) = (t, 0)$, $0 \le t \le 2$

$C_2(t) = (2-t, t)$, $0 \le t \le 1$

$C_3(t) = (1-t, 1-t)$, $0 \le t \le 1$

① $$\int_{C_1} xy\,dx + x^2\,dy = \int_0^2 \left[t \cdot 0 \frac{dx}{dt} + t^2 \frac{dy}{dt}\right] dt = 0$$

② $$\int_{C_2} xy\,dx + x^2\,dy = \int_0^1 \left[(2-t)t \overset{-1}{\overbrace{\frac{dx}{dt}}} + (2-t)^2 \overset{1}{\overbrace{\frac{dy}{dt}}}\right] dt$$

$$= \int_0^1 2t^2 - 6t + 4\,dt$$

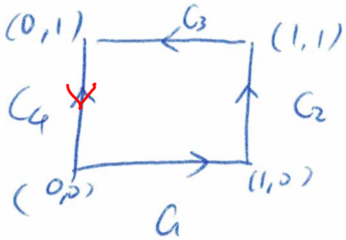
$$= \frac{2}{3}t^3 - 3t^2 + 4t \Big|_0^1 = \frac{5}{3}.$$

③ $$\int_{C_3} xy\,dx + x^2\,dy = \int_0^1 (1-t)^2(-1) + (1-t)^2(-1)\,dt$$

$$= -\frac{2}{3}$$

① + ② + ③ $= 1.$

Sp 2011 #5.



$C_1(t) = (t, 0), \quad 0 \le t \le 1$

$C_2(t) = (1, t), \quad 0 \le t \le 1$

$C_3(t) = (1-t, 1), \quad 0 \le t \le 1$

$C_4(t) = (0, 1-t), \quad 0 \le t \le 1$

$$\int (-xy)dx + (xy)dy = \int_{C_1} (-xy)dx + (xy)dy + \cdots + \int_{C_4} (-xy)dx + (xy)dy$$

① $\displaystyle\int_{C_1} (-t \cdot 0)\,dx + (t \cdot 0)\,dy = 0$

② $\displaystyle\int_0^1 \left[(-t)\overbrace{\frac{dx}{dt}}^{0} + t \cdot \overbrace{\frac{dx}{dt}}^{1}\right] dt = \frac{t^2}{2}\Big|_0^1 = \frac{1}{2}$

③ $\displaystyle\int_0^1 \left[-(1-t)\frac{dx}{dt} + (1-t)\frac{dy}{dt}\right] dt = \int_0^1 +(1-t) + (1-t)\cdot 0 \; dt = t - \frac{t^2}{2}\Big|_0^1 = \frac{1}{2}.$

④ $\displaystyle\int_{C_4} \sim \; = 0$

Sum ① – ④, we have $\displaystyle\int (-xy)\,dx + (xy)\,dy = 1.$