Math 2374
Spring 2007
Midterm 2
March 28, 2007
Time Limit: 1 hour

> Name (Print):
> Student ID:
> Section Number:
> Teaching Assistant:
> Signature:
$\qquad$
$\longrightarrow$
$\qquad$
$\qquad$

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one-half of one (doubled-sided) 8.5 inch $\times 11$ inch sheet of notes into the exam.

Do not give numerical approximations to quantities such as $\sin 5$, $\pi$, or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{4}=\sqrt{2} / 2, e^{0}=1$, and so on.

The following rules apply:

- Show your work, in a reasonably neat and coherent way, in the space provided. All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- Mysterious or unsupported answers will not receive full credit. Your work should be mathematically correct and carefully and legibly written.
- A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

| 1 | 20 pts |  |
| :---: | :---: | :--- |
| 2 | 30 pts |  |
| 3 | 20 pts |  |
| 4 | 25 pts |  |
| 5 | 20 pts |  |
| 6 | 25 pts |  |
| TOTAL | 140 pts |  |

1. (20 points) Evaluate

$$
\int_{0}^{1} \int_{x^{2}}^{x}(2 x y+x) d y d x
$$

2. (30 points) Consider the wire parametrized by $\mathbf{r}(t)=(t \cos t, t \sin t, t)$ for $\sqrt{2} \leq t \leq \sqrt{7}$.
(i) (15 points) Set up, but do not evaluate, the integral for the length of the wire.
(ii) (15 points) Suppose the density of the wire at the point $(x, y, z)$ is $\delta(x, y, z)=z$. Find the mass of the wire. (This should be an easy integral to calculate.)
3. (20 points) Reverse the order of integration for this integral:

$$
\int_{0}^{1} \int_{0}^{3-3 y}(3 x-y+1) d x d y
$$

4. (25 points) Let $\mathbf{F}(x, y)=(2 y,-x)$. Compute $\int_{C} \mathbf{F} \cdot d \mathbf{s}$ where $C$ is the boundary of the shaded region below, oriented as pictured. Note that the outer circle has radius 4 and the two smaller circles have radius 1 .

5. (20 points) Find the value of the line integral $\int_{C} y d x+x^{2} d y$ along the parabola $C$ defined by $y=x^{2}$ from the point $(0,0)$ to the point $(1,1)$.
6. ( 25 points) Let $W$ be the region in $\mathbf{R}^{3}$ which is inside the cylinder $y^{2}+z^{2}=1$ and bounded by the $y z$-plane and the plane $z+x=1$.
(i) (15 points) Set up the integral $\iiint_{W} f(x, y, z) d V$ as an iterated integral with order $d x d z d y$.
(ii) (10 points) Set up the integral in (i) as an iterated integral with order $d y d z d x$.
