Math 2374
Spring 2008
Exam 2
April 2, 2008
Time Limit: 1 hour

Name (Print): ______________________
Student ID: ______________________
Section Number: ______
Teaching Assistant: ______________________
Signature: ______________________

This exam contains 6 numbered problems on four sheets of paper. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to use one-half of one (doubled-sided) 8.5 inch $\times$ 11 inch sheet of notes.

Give **exact answers**, not numerical approximations, to the questions in problems 2, 4, 5, and 6.

Show your work, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

1	30 pts	
2	20 pts	
3	20 pts	
4	30 pts	
5	20 pts	
6	20 pts	
TOTAL	140 pts	

1. (30 points) (a) (3 points each) Let $\mathbf{F}$ be a vector field in $\mathbf{R}^3$, and let $f(x, y, z)$ be a real-valued function of three variables. For each expression below, circle YES if the expression is meaningful or circle NO if it is not. No justification necessary. No need to evaluate any of the expressions. The $\times$ represents cross product, the $\cdot$ represents dot product, and ∇ is the del (gradient) operator.

$\mathrm{curl}(\nabla f)$	YES	NO
$\mathrm{curl}(\mathrm{div}\,\mathbf{F})$	YES	NO
$\mathrm{div}(\mathrm{curl}\,\mathbf{F})$	YES	NO
$\nabla \times (\nabla \times \mathbf{F})$	YES	NO
$\nabla \times (\nabla \cdot \mathbf{F})$	YES	NO

(b) (15) Suppose that $\mathbf{c}(t)$ is a path in $\mathbf{R}^n$ with the property that $||\mathbf{c}(t)|| = 1$ for all t. Show that $\mathbf{c}'(t)$ is perpendicular to $\mathbf{c}(t)$ for all t.

(Hint: remember that $||\mathbf{c}(t)||^2 = \mathbf{c}(t) \cdot \mathbf{c}(t)$.)

2. (20 points) Evaluate

$$\int_0^4 \int_{\sqrt{x}}^2 e^{(y^3)}\, dy\, dx.$$

3. (20 points) Is $\mathbf{F} = (yz^2, xy^2, zx^2)$ the gradient of a function $f(x, y, z)$? If so, write the function $f(x, y, z)$, and if not, show why not.

4. (30 points) Let $\mathbf{c}(t) = (\cos t, \sin t, 2t)$ for $2\pi \le t \le 4\pi$.

(a) (5) What is the distance in $\mathbf{R}^3$ between the endpoints of the path?

(b) (10) Find the length of the path.

(c) (15) Suppose that the path represents a wire whose density is given by the function $\delta(x, y, z) = \frac{1}{z}$, in grams per unit length. Find the mass of the wire.

5. (20 points) Let $\mathbf{F} = (yz^2, xz^2, 2xyz)$, and let $\mathbf{c}$ be a straight-line path from $(1, -2, -3)$ to $(3, 1, 2)$. Find

$$\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}.$$

6. (20 points) Let $\mathbf{F} = (x^2, xy)$. Let C^+ be the positively oriented perimeter of the triangle that has vertices $(0,0)$, $(3,0)$, and $(0,2)$. Find

$$\int_{C^+} \mathbf{F} \cdot d\mathbf{s}.$$