

Math 2374  
Spring 2009  
Midterm 2  
April 1, 2009  
Time Limit: 1 hour

Name (Print): \_\_\_\_\_  
Student ID: \_\_\_\_\_  
Section Number: \_\_\_\_\_  
Teaching Assistant: \_\_\_\_\_  
Signature: \_\_\_\_\_

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This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one-half of one (doubled-sided) 8.5 inch  $\times$  11 inch sheet of notes into the exam.

Do not give numerical approximations to quantities such as  $\sin 5$ ,  $\pi$ , or  $\sqrt{2}$ . However, you should simplify  $\cos \frac{\pi}{4} = \sqrt{2}/2$ ,  $e^0 = 1$ , and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit.** Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit;** an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

1	25 pts	
2	25 pts	
3	20 pts	
4	25 pts	
5	20 pts	
6	25 pts	
TOTAL	140 pts	

1. (25 points) Consider the region enclosed by the cylinder  $x^2 + z^2 = 9$ , the plane  $x - y + z = 0$  and the plane  $x - y + z + 4 = 0$ . Compute the volume of this region by setting up the triple integral as  $dy dx dz$ .

2. (25 points) Let  $\mathbf{c}$  be the straight line path from  $(1, 1, 3)$  to  $(2, 1, 5)$  and let  $\mathbf{F}(x, y, z) = (xz, e^x \cos(y + z), z - 2x - y)$ . Compute

$$\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}.$$

3. (20 points) Consider the triangle with vertices  $(1, 1)$ ,  $(4, 1)$  and  $(4, 2)$  and let  $\mathbf{c}$  be the *negatively oriented* closed path along the boundary of the triangle. Consider the vector field  $\mathbf{F}(x, y) = (\cos x - xy^2, e^y + 2x^2y)$ . Using Green's Theorem, compute

$$\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}.$$

This problem specifies to use Green's Theorem which is not in the range for Midterm 2. Notice that for this problem, if you try to compute it directly without using Green's Theorem, you might find it is very complicated.

4. (25 points) Consider the path  $\mathbf{c}(t) = (3t \cos t, 3t \sin t, \sqrt{8}t^{3/2})$  for  $0 \leq t \leq 2$ .

(a) Show that the speed is increasing with time.

(b) Compute the length of the path.

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5. (20 points) Show that the path  $\mathbf{c}(t) = (\cos t, \sin t, -\cos 2t)$  is a flow line of the vector field  $\mathbf{F}(x, y, z) = (-y, x, 4xy)$ .

6. (25 points) Change the order of integration in the integral

$$\int_0^{1/\sqrt{2}} \int_{\arccos x}^{\pi/2} dy dx$$

and then compute the resulting integral.