$L_{K(2)}\, tmf = EO_2$ at prime 2, 3

Mike

at 2 : to do $K(2)$ local. of this
reduce mod a power of p,
— n ———— v_1
invert v_2

$v_1 = 0$: supersingular ell. curves
v_2^{-1} : non-singular i.e. really an elliptic curve

$\Rightarrow$ $K(2)$-localization is the same as completion at supersingular curve

at $p=2$ there is only one supersing

$$y^2 + y = x^3$$

Aut of that (over $\mathbb{F}_4$) is 24
= binary tetrahedral group.

Silverman

The stack reduced mod p & v_1 is equivalent to the stack of

$$\bullet \circlearrowleft G_{24}$$

Toy example : $\mathbb{Z}[b,c]$ $\quad x^2 + bx + c$

$$\mathbb{Z}[b,c] \rightrightarrows \mathbb{Z}[b,c][r] \quad [(b^2-4c)^{-1}]$$

maybe ?

$x \mapsto x + r \rightsquigarrow$

$$x^2 + (b+2r)x + (r^2 + br + c)$$

$b \mapsto b \qquad b \mapsto b + 2r$

$c \mapsto c \qquad c \mapsto c + br + r^2$

this defines a stack $\mathcal{M}$

covering

$$\begin{tikzcd} & \mathbb{A}^2_{b,c} \arrow[d] \\ \operatorname{Spec}\mathbb{Z}[b] \arrow[ur, "c=0"] \arrow[r] & \mathcal{M} \end{tikzcd}$$

~~define~~

$$x^2+bx$$

$$x^2+(b+2x)+(\underbrace{r^2+br}_{0})$$

$$\begin{tikzcd} & & \mathbb{A}^2_{b,c} \arrow[d] \\ \operatorname{Spec}\mathbb{Z}[b][r]/_{r^2+br} \arrow[r, shift left] \arrow[r, shift right] & \operatorname{Spec}\mathbb{Z}[b] \arrow[ur] \arrow[r] & \mathcal{M} \end{tikzcd}$$

$$\simeq \quad \mathbb{Z}[b^{\pm 1}] \circlearrowleft \mathbb{Z}/2$$

Why is $L_{K(2)}$ a completion?

What $L_{K(2)}$ tries to be is

$$L_{K(2)}(X) \quad \text{— " —} \quad \varprojlim_{n_1,n_2} v_2^{-1} X \wedge M(p^{n_1}, v^{n_2})$$

(telescope conjecture) true for they