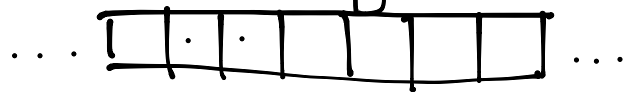
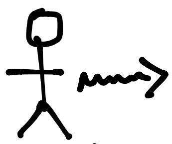
REU 2017 Day 3 S. Chepuri

Box-ball systems

We have ∞ many boxes, finitely many balls

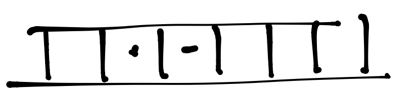


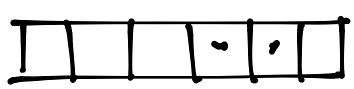


carrier

1) If the carrier passes a box with a ball in it, they pick it up.
2) If the carrier passes an empty box while carrying a ball, the carrier puts a ball in the box.

EXAMPLE

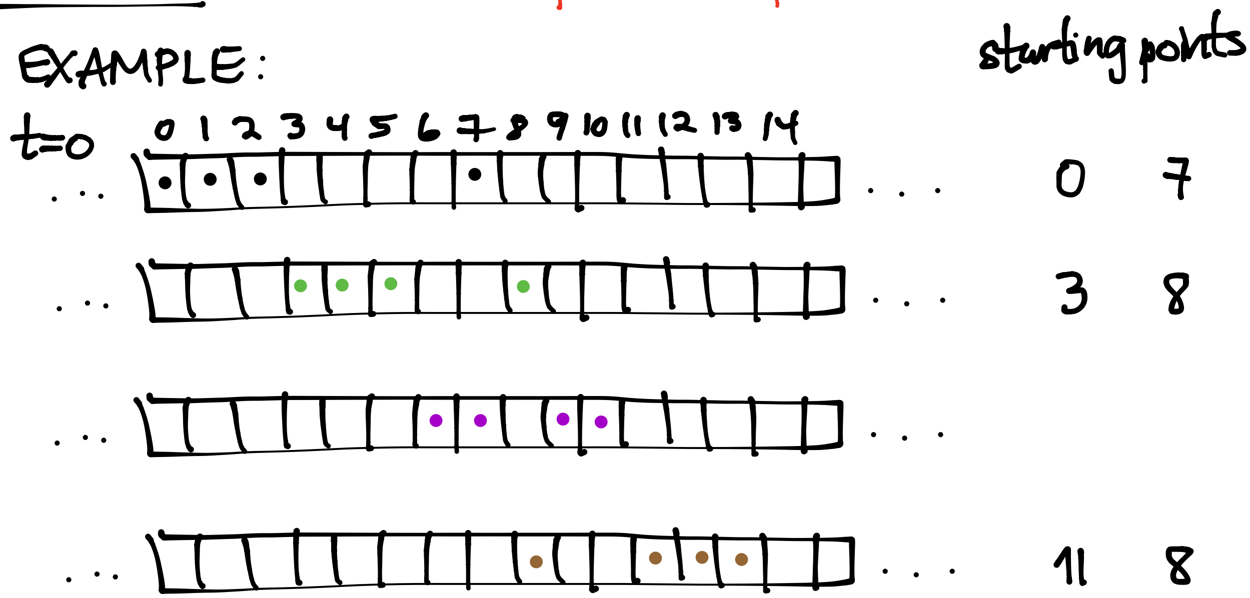
$t=0$ 

$t=1$ 

DEF'N: A series of balls with no empty boxes is called a **soliton**. Its **amplitude** is the number of balls it has.

REMARK: If there is no interference, a solution moves at **speed = amplitude**

EXAMPLE:



REU EXERCISE 5: When a solution of of amplitude m passes one of amplitude m, we get ones of amplitude n & m.

REV EXERCISE 6

After a collision of solitons, the bigger is pushed right, and the smaller pushed left.

We would like a notion of speed for the whole system so that the system moves with constant speed.

IDEA: Take the average position of the balls.

EXAMPLE

$$\frac{3+4+5+8}{4} - \frac{0+1+2+7}{4} = \frac{5}{2}$$

$$\frac{6+7+9+10}{4} - \frac{3+4+5+8}{4} = 3$$

not equal!

IDEA 2: Average position of solitons' centers

EXAMPLE:

$$\frac{4+8}{2} - \frac{1+7}{2} = 2$$

$$\frac{6.5+9.5}{2} - \frac{4+8}{2} = 2$$

$$\frac{8+12}{2} - \frac{6.5+9.5}{2} = 2$$

REU EXERCISE 7

Prove this idea of speed gives a constant for any box-ball system.

Generalized box-ball systems

We can generalize to allow multiple kinds of balls.
For notation, consider an empty box to be a ball of type 1.
Let the carrier have a bag with space for k = total # of balls of type >1 in the system

1) The carrier starts with all balls of type 1.

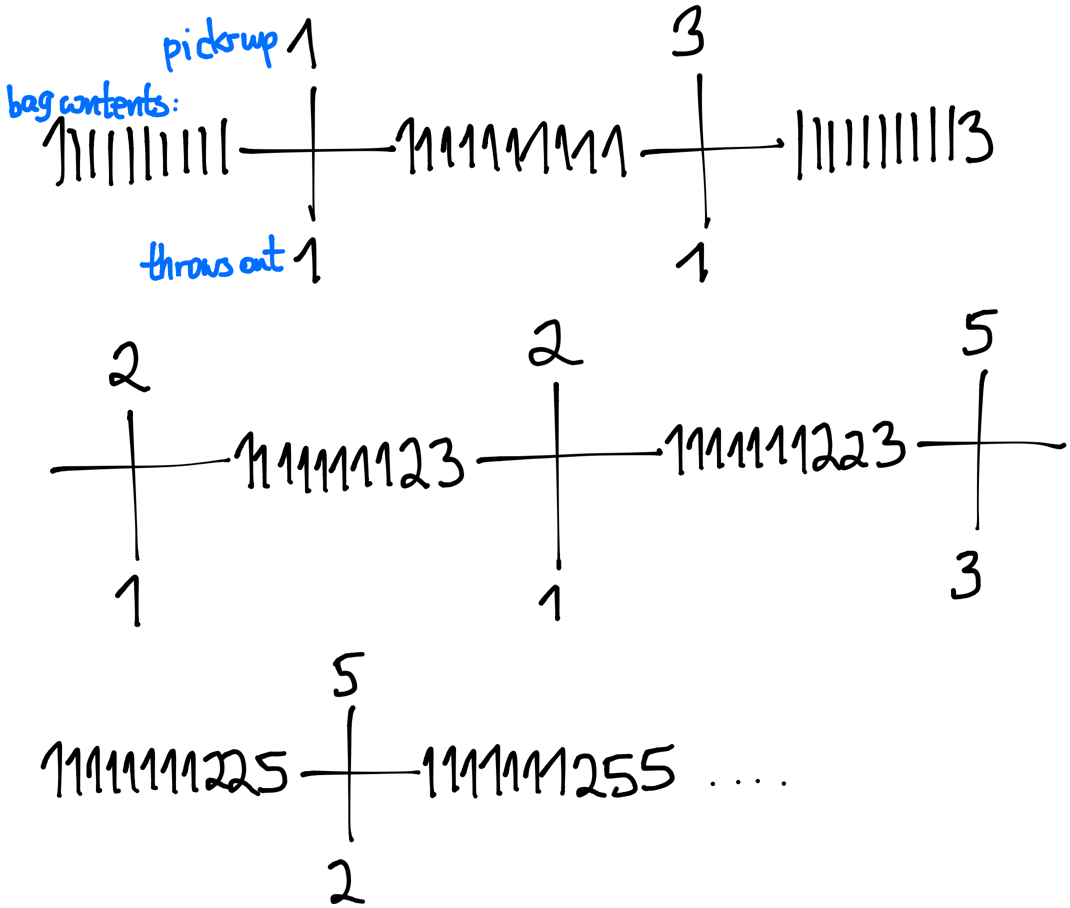
2) As the carrier passes a ball of type x, they pick it up and replace it with the largest (type) ball of type $< x$ in their bag.
If they have no balls of type $< x$ in their bag, they replace it with the ball of largest type in their bag.

EXAMPLE

$t=0$

. . . 132225544331611111 . . .

$\Longrightarrow k=10$



$t=1$

. . . 1111322111556443111 . . .

Another way to think about this...
Assuming there are n ball types $1, 2, \ldots, n$,

1) Move the leftmost n to the nearest right empty space.

2) Repeat with next leftmost n, and continue until all n's have been moved once.

3) Repeat steps (1), (2) for $n-1$,
for $n-2$,
$\vdots$
for 3,
for 2.

EXAMPLE

$t=0$

$\ldots 13225544331611111 \ldots$ ↓ move 6

$\ldots 13225544331161111 \ldots$ ↓ move 5's

$\ldots 13221144335561111 \ldots$ ↓ move 4's

$\ldots 11223111335564411 \ldots$ ↓ move 3's

$\ldots 11223111115564433 \ldots$ ↓ move 2's

$\ldots 11113221115564433 \ldots$

DEF'N: A **soliton** is a (maximal) sequence of balls in decreasing order with no 1's.

CONJECTURE: The average center position of the solitons gives a constant speed for generalized box-ball systems.

$\boxed{\text{REU Problem 3: Prove or disprove it.}}$