FM 5001 Fall 2011, Final Exam
Ending time for in-person students: 8:00 pm on Wednesday 14 December 2011
Time for exam: 1.5 HOURS (ONE AND ONE HALF HOURS)

For PROCTORS of online students:
Email scan to: adams@math.umn.edu
Preferred FAX: 612-624-6702    Alternate FAX: 612-626-2017
Exam must be received by 24 hours after the ending time for in-person students. Thank you.

STUDENT, PLEASE PRINT NAME:

Remember to read to the bottom and to SIGN YOUR NAME BELOW!

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.
Show work; a correct answer, by itself, may be insufficient for credit.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

STUDENT, PLEASE REMEMBER TO SIGN YOUR NAME:
I. Definitions: Complete the following sentences.

a. (Topic 0031(15), 3 pts.) A matrix $R \in \mathbb{R}^{n \times n}$ is a \textbf{rotation matrix} if . .

b. (Topic 0022(16), 3 pts.) Let $V$ and $W$ be two subspaces and let $T: V \to W$ be a linear transformation. The \textbf{kernel} of $T$ is $\text{ker}(T) = \cdots$

c. (Topic 0016(9), 3 pts.) Let $f: \mathbb{R} \to \mathbb{R}$ be smooth. The \textit{kth order Maclaurin approximation} to $f$ is the polynomial $P: \mathbb{R} \to \mathbb{R}$ such that . .

d. (Topic 0029(36), 3 pts.) Let $Q: \mathbb{R}^{n} \to \mathbb{R}$ be a quadratic form. The \textbf{polarization} of $Q$ is the bilinear form $B: \mathbb{R}^{n} \times \mathbb{R}^{n} \to \mathbb{R}$ such that . .
e. (Topic 0034(12), 3 pts.) A matrix \( M \in \mathbb{R}^{n \times n} \) is **rotationally diagonalizable** if . . .

f. (Topic 0023(46), 3 pts.) Two matrices \( A, B \in \mathbb{R}^{n \times n} \) are **conjugate** if . . .

g. (Topic 0032(53), 3 pts.) Let \( M \in \mathbb{R}^{n \times n} \) and let \( a \) be an eigenvalue of \( M \). Then the **\( a \)-eigenspace** of \( M \) is . . .

h. (Topic 0024(12), 3 pts.) Let \( M \in \mathbb{R}^{n \times n} \). Then the **exponential** of \( M \) is the matrix defined by \( e^M = \cdots \).
II. True or False. (No partial credit.)

a. (Topic 0002(11), 2 pts.) Any compact subset of \( \mathbb{R}^n \) is bounded.

b. (Topic 034(17), 2 pts.) Any symmetric real matrix is rotationally diagonalizable.

c. (Topic 0027(19,24), 2 pts.) For any \( A, B \in \mathbb{R}^{n \times n} \), if \( A \) and \( B \) are conjugate, then \( \det(A) = \det(B) \).

d. (Topic 0033(10), 2 pts.) Every eigenvalue of an antisymmetric real matrix is a real number.

e. (Topic 0017(26), 2 pts.) If a series converges, then any rearrangement of it converges as well.

f. (Topic 0033(20), 2 pts.) Any \( 2 \times 2 \) Jordan block is diagonalizable.

g. (Topic 0036(2), 2 pts.) For any matrix \( M \), there is a nonzero polynomial \( f \) such that \( F(M) = 0 \), where \( F \) is the matrix extension of \( f \).

h. (Topic 0024(6), 2 pts.) Every nilpotent matrix is invertible.
I.a-d.

I.e-h.

II.a-d.

II.e-h.

III(1).

III(2,3).

III(4).

III(5).

III(6).

III(7).

III(8).
III. Computations. Some of your answers may involve $\Phi$, the cumulative distribution function of the standard normal distribution. (Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.)

1. In this problem, all answers can be expressed using trigonometric functions. You don’t need to calculate, e.g., $\sin 3$.

a. (Topic 0019(27), 5 pts.) Compute real numbers $a, b, c, d$ such that $e^{3i} = a + bi$ and $e^{4i} = c + di$.

b. (Topic 0019(15), 5 pts.) Using $a, b, c, d$ from Part a, expand $(a + bi)(c + di)$, and compute its real part.

c. (Topic 0019(27), 5 pts.) Compute the real part of $e^{7i}$. 
2. (Topic 0008(10-16), 20 pts.) How many monomials are there of degree = 7 in 15 variables? Write your answer as a product of integers.

3. Let
\[
M := \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \quad \text{and} \quad N := \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}.
\]

a. (Topic 0023(19), 5 pts.) Compute \( M \oplus N \).

b. (Topic 0023(20), 10 pts.) Compute \( M \otimes N \).
4. (Topic 0026(41), 20 pts.) Let \( M := \begin{bmatrix} 1 & 6 & 8 \\ 1 & 7 & 6 \\ 0 & 1 & -3 \end{bmatrix} \). Find \( M^{-1} \).
5. (Topic 0026(26), 20 pts.) Find the dimensions of the image and kernel of

\[
\begin{bmatrix}
1 & 2 & 4 & 2 & 0 \\
1 & 1 & 2 & 2 & 0 \\
2 & 3 & 6 & 4 & 0 \\
3 & 4 & 8 & 6 & 1
\end{bmatrix}.
\]
6. (Topic 0027(19) and 0027(23) and 0028(42) and 0028(43), 20 pts.) Compute the determinant of

\[ A := \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & -1 & 2 & 0 \\ 2 & 3 & 6 & 4 \\ 3 & 4 & 9 & 6 \end{bmatrix}. \]
7. (Topic 0034(22-36), 25 pts.) Define $Q : \mathbb{R}^2 \to \mathbb{R}$ by $Q(x, y) = 9x^2 + 4xy + 6y^2$. Find a $2 \times 2$ rotation matrix $R$ such that $Q \circ L_R$ is a diagonal quadratic form.
8. (Topic 0024(23), 0032(27), 25 pts.) Let \( S = \begin{bmatrix} 73 & 36 \\ 36 & 52 \end{bmatrix} \). Find a symmetric matrix \( T \in \mathbb{R}^{2 \times 2} \) such that \( T^2 = S \). \textbf{Hint:} Let \( R = \frac{1}{5} \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix} \). Then \( R^t S R = \begin{bmatrix} 25 & 0 \\ 0 & 100 \end{bmatrix} \).