STUDENT, PLEASE PRINT NAME:

Remember to read to the bottom and to SIGN YOUR NAME BELOW!

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.
Show work; a correct answer, by itself, may be insufficient for credit.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

STUDENT, PLEASE REMEMBER TO SIGN YOUR NAME:
I. Definitions: Complete the following sentences.

a. (Topic 0022(11), 3 pts.) Let $V$ and $W$ be subspaces of Euclidean spaces. A map $L : V \rightarrow W$ is **linear** if . . .

b. (Topic 0015(13), 3 pts.) $s_1 + s_2 + s_3 + \cdots = s$ means . . .

c. (Topic 0002(29), 3 pts.) Let $S \subseteq \mathbb{R}$, $b \in \mathbb{R}$. We say $b$ is a **lower bound** of $S$, written $b \leq S$, if . . .

d. (Topic 0002(29), 3 pts.) Let $S \subseteq \mathbb{R}$, $b \in \mathbb{R}$. We say $b$ is the **infimum** or **glb** of $S$, written $b = \inf S$, if . . .

e. (Topic 0015(4), 3 pts.) Let $a_1, a_2, a_3, \ldots$ be a sequence of real numbers. Then the **liminf** of $a_j$ is . . .
II. True or False. (No partial credit.)

a. (Topic 0017(32), 3 pts.) If a series has only finitely many nonpositive terms, then all of its rearrangements have the same sum.

b. (Topic 0020(15), 3 pts.) Every subset of $\mathbb{R}$ is open or closed (or both).

c. (Topic 0022(18), 3 pts.) A linear transformation is one-to-one iff its kernel is $\{0\}$.

d. (Topic 0016(8), 3 pts.) For any smooth function $f : \mathbb{R} \to \mathbb{R}$, there is a polynomial $p : \mathbb{R} \to \mathbb{R}$ of degree $\leq 3$ such that $J_3^0 p = J_3^0 f$.

e. (Topic 0022(14), 3 pts.) Let $A, B : \mathbb{R}^n \to \mathbb{R}^n$ be linear transformations. Assume, for all $v, w \in \mathbb{R}^n$, that $(A(v)) \cdot w = (B(v)) \cdot w$. Then $A = B$. 

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PLEASE DO NOT WRITE BELOW THE LINE
III. Computations. Some of your answers may involve $\Phi$, the cumulative distribution function of the standard normal distribution. (Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.)

1. (Topic 0008(16), 5 pts.) How many monomials are there of degree exactly 4 in 9 variables? (Express your answer as a product of integers.)

2. (Topic 0009(23-25), 5 pts.) Compute $\int x^2 e^{-x^2/2} dx$. 
3. Let \( A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 2 & 0 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \).

a. (Topic 0023(11), 5 pts.) Find a matrix \( C \) such that \( L_C = L_A \circ L_B \).

b. (Topic 0023(19), 5 pts.) Compute \( A \oplus B \). (This is a matrix of scalars, not a matrix of matrices.)

c. (Topic 0023(20), 5 pts.) Compute \( A \otimes B \). (This is a matrix of matrices.)
4. (Topic 0015(13), 5 pts.) Let \( s := \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \frac{5}{2^5} + \cdots \). Write \( s - \frac{s}{2} \) as a sum of a geometric series, and use this to compute \( s \).
5. (Topic 0016(22), 10 pts.) Assume that \( f'''(x) \leq 6 \), for all \( x \in \mathbb{R} \). Assume that \( f(0) = f'(0) = f''(0) = 2 \). Among all functions \( f \) satisfying those two conditions, find the maximum possible value of \( f(3) \).
6. (Topic 0018(15), 10 pts.) Compute \( \lim_{n \to \infty} (e^{1/n} + \sin(2/n))^n \).
7. (Topic 0019(29), 10 pts.) Let \( i := \sqrt{-1} \) and let \( f(x, y) = |x + iy|^2 + e^{(x+iy)^2} \). (Here \( x \) and \( y \) are real variables.) Let \( U \) and \( V \) be, respectively, the real and imaginary parts of \( f(x, y) \). Compute \( U \) and \( V \) as expressions of \( x \) and \( y \).
8. Let \( f(x) = (\cos x) + (\sin^2(x/2)) \).

   a. (Topic 0016(6), 5 pts.) Find the second order Maclaurin approximation of \( f \).

   b. (Topic 0018(35), 5 pts.) Compute \( \lim_{n \to \infty} [f(5/\sqrt{n})]^n \).