Financial Mathematics
Conditional probability, independence and the Central Limit Theorem
0047-1. Suppose $\Pr[A|B] = 0.6$, 
$\Pr[A] = 0.3$ and $\Pr[B] = 0.5$.

a. Find $\Pr[A \text{ and } B]$.

b. Find $\Pr[B|A]$.

0047-2. a. Find two PCRVs $X$ and $Y$ s.t.
$\Pr[(X = 1)|(Y = 2)] = 0.6$, 
$\Pr[X = 1] = 0.3$ and $\Pr[Y = 2] = 0.5$.

b. Compute $\Pr[(Y = 2)|(X = 1)]$. 
Let $C_1, C_2, C_3, \ldots$ be our standard sequence of coin-flipping PCRVs. For all integers $n \geq 1$, let $D_n := (C_1 + \cdots + C_n)/\sqrt{n}$.

a. Compute $\lim_{n \to \infty} E[D_n^6]$.

b. Compute $\lim_{n \to \infty} E[80(e^{4D_n-3} - e)_+]$. 


0047-4. Let $X$ and $Y$ be PCRVs
s.t. $\Pr[X = 4] = 0.8$,
and s.t. $\Pr[(X = 4) \& (Y = 9)] = 0.35$
and s.t. $\Pr[(X = 4) \& (Y = 2)] = 0.45$.

a. Find $\Pr[(Y = 2)|(X = 4)]$.
b. Find $\mathbb{E}[Y|(X = 4)]$.

0047-5. Find two PCRVs
which are uncorrelated,
but not independent.
Sketch the graphs
of the two PCRVs.

WARNING: Neither can be deterministic.
WARNING: At least one must have
three values of positive probability.
0047-6. Let $X_1, X_2, X_3, \ldots, X_{40}$ be iid.

Suppose $E[X_1 + X_2 + X_3 + \cdots + X_{40}] = 60$ and $SD[X_1 + X_2 + X_3 + \cdots + X_{40}] = 10$.

Let $\mu := E[X_1] = E[X_2] = \cdots = E[X_{40}]$ and $\sigma := SD[X_1] = SD[X_2] = \cdots = SD[X_{40}]$.

Compute $\mu$ and $\sigma$. 