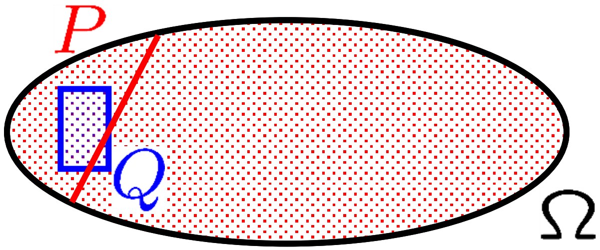
Financial Mathematics

Bayes' Law

Definition: The **conditional probability** of P given Q is

$$\boxed{\text{“}\frac{P \,\&\, Q}{Q}\text{”}}$$



$$\boxed{\Pr[P \mid Q]} = \frac{\Pr[P \,\&\, Q]}{\Pr[Q]}$$

Warning: Only defined when $\Pr[Q] \neq 0$.

$$\Pr[C \mid B] \cdot \Pr[B \mid A] \overset{??}{\neq} \Pr[C \mid A]$$

$$\cancel{\frac{C}{B} \qquad \frac{B}{A} \qquad \frac{C}{A}}$$

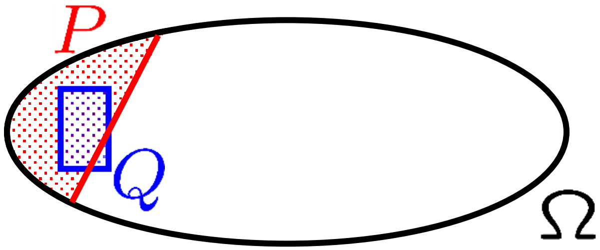
$$A \,\&\, \frac{B \,\&\, C}{\boxed{\cancel{A \,\&\, B}}} \qquad \frac{\cancel{A \,\&\, B}}{A} = \frac{\overbrace{A \,\&\, C}^{\&\, B}}{A}$$

Fact: $\Pr[C \mid A \,\&\, B] \cdot \Pr[B \mid A] = \Pr[B \,\&\, C \mid A]$

Definition: The **conditional probability** of P given Q is

$$\boxed{\frac{\text{“}P \,\&\, Q\text{”}}{Q}}$$

short form



$$\boxed{\Pr[P \mid Q]} = \frac{\Pr[P \,\&\, Q]}{\Pr[Q]}$$

$$\| $$

$$\Pr[P \& Q | Q]$$

long form

$$\Pr[C \mid B] \cdot \Pr[B \mid A] \overset{??}{\cancel{=}} \Pr[C \mid A]$$

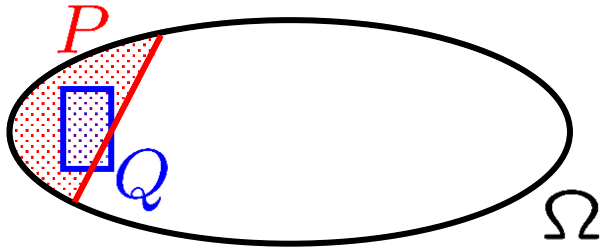
$$\cancel{\frac{C}{B} \qquad \frac{B}{A} \qquad \frac{C}{A}}$$

$$\frac{A \,\&\, B \,\&\, C}{A \,\&\, B} \qquad \frac{A \,\&\, B}{A} = \frac{\overbrace{A \,\&\, C}^{\&\, B}}{A}$$

Fact: $\Pr[C \mid A \,\&\, B] \cdot \Pr[B \mid A] = \Pr[B \,\& C \mid A]$

Definition: The **conditional probability** of P given Q is

$$\boxed{\frac{\text{“}P \,\&\, Q\text{”}}{Q}}$$



short form

$$\boxed{\Pr[P \mid Q]} = \frac{\Pr[P \,\&\, Q]}{\Pr[Q]}$$

$$\boxed{\Pr[P \mid Q]} = \Pr[P\&Q|Q]$$

long form

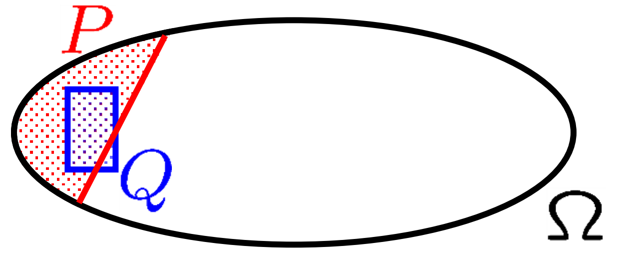
Suggestion: Move to the long form before doing any cancellations.
THEN move to the short form.

$$\frac{A\&B\&C}{\cancel{A\&B}} \quad \frac{\cancel{A\&B}}{A} = \frac{A\&B\&C}{A}$$

$$\Pr[\boxed{\cancel{A\&B}}\&C|\boxed{\cancel{A\&B}}]\cdot\Pr[\cancel{A\&B}|A] = \Pr[\cancel{A}\&B\&C|A]$$

Fact: $\boxed{\Pr[C \mid A \,\&\, B]} \cdot \boxed{\Pr[B \mid A]} = \boxed{\Pr[B \,\&C \mid A]}$

Definition: The **conditional probability** of P given Q is $\boxed{\dfrac{\text{“}P \,\&\, Q\text{”}}{Q}}$



short form

$$\boxed{\Pr[P \mid Q]} = \frac{\Pr[P \,\&\, Q]}{\Pr[Q]}$$

$$\| $$

$$\Pr[P\&Q|Q]$$

long form

Suggestion: Move to the long form before doing any cancellations.
THEN move to the short form.

$$\Pr[A\&B|\cancel{A}] \cdot \Pr[\cancel{A}] = \Pr[A\&B]$$

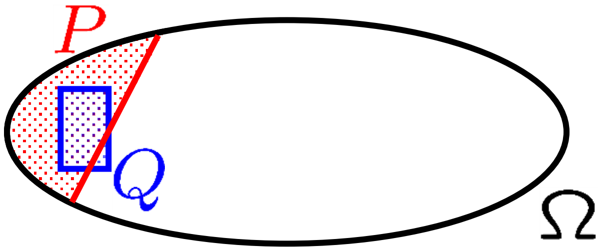
$$\xcancel{\frac{B}{A} \quad A \quad B}$$

$$\boxed{\Pr[B \mid A]} \cdot \boxed{\Pr[A]} \overset{??}{\neq} \Pr[B]$$

Fact: $\Pr[C \mid A \,\&\, B] \cdot \Pr[B \mid A] = \Pr[B \,\&C \mid A]$

Definition: The **conditional probability** of P given Q is

$$\boxed{\text{“}\frac{P \ \& \ Q}{Q}\text{”}}$$



$$\overset{\text{short form}}{\boxed{\Pr[P \mid Q]}} = \frac{\Pr[P \ \& \ Q]}{\Pr[Q]}$$

$$\| $$

$$\Pr[P\&Q|Q] \leftarrow \boxed{\text{long form}}$$

Suggestion: Move to the long form before doing any cancellations. *THEN* move to the short form.

$$\Pr[A\&B|A] \cdot \Pr[A] = \Pr[A\&B]$$

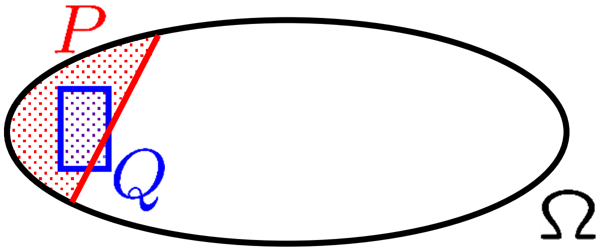
$$\frac{A\&B}{\cancel{A}} \qquad \cancel{A} \qquad A\&B$$

$$\Pr[B \mid A] \cdot \Pr[A] \overset{??}{\cancel{=}} \boxed{\Pr[B]}$$

Fact: $\Pr[C \mid A \,\&\, B] \cdot \Pr[B \mid A] = \Pr[B \,\&C \mid A]$

Definition: The **conditional probability** of P given Q is

$$\boxed{\text{“}\frac{P \,\&\, Q}{Q}\text{”}}$$



short form

$$\boxed{\Pr[P\,|\,Q]} = \frac{\Pr[P \,\&\, Q]}{\Pr[Q]}$$

$$\|$$

$$\Pr[P\&Q|Q]$$

long form

Suggestion: Move to the long form before doing any cancellations. *THEN* move to the short form.

$$\boxed{\Pr[\cancel{A}\&B|A]}\cdot\Pr[A] = \Pr[A\&B]$$

Fact: $$\Pr[B|A]\cdot\Pr[A] = \Pr[A\&B]$$

$$\Pr[B\underbrace{\cancel{|A}}_{\&A}]\cdot\Pr[\cancel{A}] = \Pr[A\&B]$$

$$\Pr[B\,|\,A]\cdot\Pr[A] \overset{??}{\neq} \boxed{\Pr[B]}$$

Fact: $$\Pr[C\,|\,A\,\&\,B]\cdot\Pr[B\,|\,A] = \Pr[B\,\&C\,|\,A]$$

Definition:

For any expression E involving an event A,

let $\boxed{\{E\}^{\times}_{A}}$ denote the same expression,

but with every "A" replaced by "not A"

and let $\boxed{\{E\}^{\circ}_{A}} := \dfrac{E}{\{E\}^{\times}_{A}}$.

e.g.: $\{\mathrm{Pr}[Q|\underline{P}]\}^{\times}_{P} = \mathrm{Pr}[Q|(\underline{\text{not } P})]$

$$\{\mathrm{Pr}[Q|\underline{P}]\}^{\circ}_{P} = \frac{\mathrm{Pr}[Q|\underline{P}]}{\mathrm{Pr}[Q|(\underline{\text{not } P})]}$$

$$\{\mathrm{Pr}[X|(\underline{Y}\&Z)]\}^{\circ}_{Y} = \frac{\mathrm{Pr}[X|(\underline{Y}\&Z)]}{\mathrm{Pr}[X|((\underline{\text{not } Y})\&Z)]}$$

Definition:

For any expression E involving an event A,

let $\boxed{\{E\}_A^{\times}}$ denote the same expression,

but with every "A" replaced by "not A"

and let $\{E\}_A^{\circ} := \dfrac{E}{\{E\}_A^{\times}}$.

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e.g.: $\{\text{Pr}[(\underline{S}\&T)|(R\&U)] \cdot \boxed{\cancel{\text{Pr}[T|(R\&U)]}}\}_S^{\circ}$

$$= \frac{\text{Pr}[(\underline{S}\&T)|(R\&U)] \cdot \cancel{\text{Pr}[T|(R\&U)]}}{\text{Pr}[(\underline{(\text{not } S})\&T)|(R\&U)] \cdot \cancel{\text{Pr}[T|(R\&U)]}}$$

$$\{\text{Pr}[X|(Y\&Z)]\}_Y^{\circ} = \frac{\text{Pr}[X|(Y\&Z)]}{\text{Pr}[X|((\text{not } Y)\&Z)]}$$

Definition: For any event A,

$$\boxed{\text{Odds}[A]} := \{\text{Pr}[A]\}_A^{\circ} = \frac{\text{Pr}[A]}{\text{Pr}[\text{not } A]}$$

Definition:
For any expression E involving an event A,
let $\boxed{\{E\}_A^{\times}}$ denote the same expression,
but with every "A" replaced by "not A"
and let $\{E\}_A^{\circ} := \dfrac{E}{\{E\}_A^{\times}}$.

e.g.: If the horse wins 3/4 of the time,
written $\Pr[W] = 3/4$,
then its odds of winning are
3 to 1, or, equivalently 6 to 2,
written $\text{Odds}[W] = \frac{3}{1} = \frac{6}{2} = 3$

$$\boxed{3 = \frac{3/4}{1/4} = \frac{\Pr[W]}{\Pr[\text{not } W]}}$$

Definition: For any event A,

$$\boxed{\text{Odds}[A]} := \{\Pr[A]\}_A^{\circ} = \frac{\Pr[A]}{\Pr[\text{not } A]}$$

Definition: For any events A, B,

$$\boxed{\text{Odds}[A|B]} := \{\Pr[A|B]\}_A^{O} = \frac{\Pr[A|B]}{\Pr[(\text{not } A)|B]}$$

e.g.: If the horse wins $3/4$ of the time,

written $\Pr[W] = 3/4$,

then its odds of winning are

3 to 1, or, equivalently 6 to 2,

written $\text{Odds}[W] = \frac{3}{1} = \frac{6}{2} = 3$

$$\boxed{3 = \frac{3/4}{1/4} = \frac{\Pr[W]}{\Pr[\text{not } W]}}$$

Definition: For any event A,

$$\boxed{\text{Odds}[A]} := \{\Pr[A]\}_A^{O} = \frac{\Pr[A]}{\Pr[\text{not } A]}$$

Bayes' Law

You're worried you may have a certain disease.
You're given a medical test and told
that the test comes up positive
for 99% of those with the disease.

$$\Pr[P|S] = 0.99$$

Question:
If test comes out positive,
what are the chances you're sick?

$$\Pr[S|P] = ???$$

Say: $\Pr[S] = 0.2\%$ and $\Pr[P] = 0.8\%$

$\Pr[S|P] = ???$ $\Pr[P|S] = 0.99$

Events: P = test comes out positive
S = you're sick (have the disease)

Bayes' Law

Fact: $$\Pr[A|B] \quad = \quad \Pr[B|A] \cdot \frac{\Pr[A]}{\Pr[B]}$$

Pf:

$$\frac{A\&B}{B} \quad = \quad \frac{B\&A}{\cancel{A}} \qquad \frac{\cancel{A}}{B}$$

QED

$A :\rightarrow S$
$B :\rightarrow P$

Say: $\Pr[S] = 0.2\%$ and $\Pr[P] = 0.8\%$

$\Pr[S|P] = ???$ $\qquad$ $\Pr[P|S] = 0.99$

Events: P = test comes out positive
S = you're sick (have the disease)

Bayes' Law

Fact: $$\Pr[S|P] = \underbrace{\Pr[P|S]}_{0.99} \cdot \underbrace{\frac{\Pr[S]}{\Pr[P]}}_{\frac{0.2\%}{0.8\%}}$$

$$\underbrace{0.99 \quad \frac{0.2\%}{0.8\%}}_{0.2475} = 24.75\%$$

Say: $\Pr[S] = 0.2\%$ and $\Pr[P] = 0.8\%$

$\Pr[S|P] =$???

$\Pr[P|S] = 0.99$

Events: P = test comes out positive

S = you're sick (have the disease)

Bayes' Law

Fact: $$\Pr[S|P] = \underbrace{\underbrace{\Pr[P|S]}_{0.99} \cdot \underbrace{\frac{\Pr[S]}{\Pr[P]}}_{\frac{0.2\%}{0.8\%}}}_{0.2475 \,=\, 24.75\%}$$

Say: $\Pr[S] = 0.2\%$ and $\Pr[P] = 0.8\%$

$\Pr[S|P] = 0.2475 < 25\%$ $\Pr[P|S] = 0.99$

Events: P = test comes out positive

S = you're sick (have the disease)

Bayes' Law

$$\begin{aligned}\Pr[P\&S] &= \Pr[P|S]\cdot\Pr[S]\\ &= 0.99\cdot 0.2\%\\ &= 0.198\%\end{aligned}$$

$$\boxed{\Pr[A\,\&\,B] \;=\; \Pr[\underbrace{A|\cancel{B}}_{\&B}]\cdot\Pr[\cancel{B}]}$$

$$\text{“}\;\frac{A\&B}{\cancel{B}}\cdot\cancel{B}\;\text{”}$$

Let's find: $\Pr[P|(\text{not } S)]$

Say: $\Pr[S] = 0.2\%$ and $\Pr[P] = 0.8\%$

$\Pr[P|S] = 0.99$

Events: P = test comes out positive

S = you're sick (have the disease)

Bayes' Law

$$\begin{aligned}\Pr[P\&S] &= \Pr[P|S]\cdot\Pr[S]\\ &= 0.99\cdot 0.2\%\\ &= 0.198\%\end{aligned}$$

$$\underbrace{\Pr[P\&S]}_{0.198\%} + \Pr[P\&(\text{not } S)] = \underbrace{\Pr[P]}_{0.8\%}$$

$$\Pr[P\&(\text{not } S)] = 0.602\%$$

Let's find: $\Pr[P|(\text{not } S)]$

Say: $\Pr[S] = 0.2\%$ and $\Pr[P] = 0.8\%$

$$\Pr[P|S] = 0.99$$

Events: P = test comes out positive
S = you're sick (have the disease)

Bayes' Law

$$0.60320641\% \;\Vert$$

$$\Pr[P|(\text{not } S)] = \frac{\Pr[P\&(\text{not } S)]}{\Pr[\text{not } S]} = \frac{0.602\%}{99.8\%}$$

$$\Pr[P\&(\text{not } S)] = 0.602\%$$

Let's find: $\Pr[P|(\text{not } S)] = 0.60320641\%$

Say: $\Pr[S] = 0.2\%$ and $\Pr[P] = 0.8\%$

$\Pr[\text{not } S] = 99.8\%$

$\Pr[P|S] = 0.99$

Events: P = test comes out positive

S = you're sick (have the disease)

Bayes' Law

Fact: $$\text{Odds}[A|B] = \frac{\Pr[B|A]}{\Pr[B|(\text{not } A)]} \cdot \text{Odds}[A]$$

$$\{\boxed{\Pr[A|B]}\}_A^{\text{O}} = \left\{ \boxed{\Pr[B|A]} \cdot \frac{\boxed{\Pr[A]}}{\boxed{\cancel{\Pr[B]}}} \right\}_A^{\text{O}}$$

$$\Pr[P|(\text{not } S)] = 0.60320641\%$$

Say: $\Pr[S] = 0.2\%$ and $\Pr[P] = 0.8\%$

$$\Pr[P|S] = 0.99$$

Events: P = test comes out positive

S = you're sick (have the disease)

Bayes' Law

Fact: $$\text{Odds}[A|B] = \text{Odds}[A] \cdot \frac{\Pr[B|A]}{\Pr[B|(\text{not } A)]}$$

Fact: $$\text{Odds}[A|B] = \frac{\Pr[B|A]}{\Pr[B|(\text{not } A)]} \cdot \text{Odds}[A]$$

$$\{\Pr[A|B]\}_A^{\text{O}} = \left\{ \Pr[B|A] \cdot \frac{\Pr[A]}{\boxed{\cancel{\Pr[B]}}} \right\}_A^{\text{O}}$$

$$\Pr[P|(\text{not } S)] = 0.60320641\%$$

Say: $\Pr[S] = 0.2\%$ and $\Pr[P] = 0.8\%$

$$\Pr[P|S] = 0.99$$

Events: P = test comes out positive

S = you're sick (have the disease)

Bayes' Law

Fact: $\underbrace{\text{Odds}[A|B]}_{\substack{\| \\ \frac{\Pr[A|B]}{\Pr[(\text{not A})|B]}}} = \underbrace{\text{Odds}[A]}_{\substack{\| \\ \frac{\Pr[A]}{\Pr[\text{not } A]}}} \cdot \underbrace{\frac{\Pr[B|A]}{\Pr[B|(\text{not } A)]}}_{\substack{\| \\ \boxed{\boxed{\text{LQ}}^A[B]}}}$

To update the odds based on new info, multiply by a carefully defined "likelihood quotient".

$A :\to S$
$B :\to P$

Likelihood Quotient

$$\Pr[P|(\text{not } S)] = 0.60320641\%$$

Say: $\Pr[S] = 0.2\%$ and $\Pr[P] = 0.8\%$

$$\Pr[P|S] = 0.99$$

Events: P = test comes out positive
S = you're sick (have the disease)

Bayes' Law

Fact: $$\underbrace{\text{Odds}[S|P]}_{\substack{=:\ \frac{\Pr[S|P]}{\Pr[(\text{not S})|P]}\ =\ \frac{24.75\%}{75.25\%}}} = \underbrace{\text{Odds}[S]}_{\substack{=:\ \frac{\Pr[S]}{\Pr[\text{not } S]}\ =\ \frac{0.2\%}{\boxed{99.8\%}}}} \cdot \underbrace{\frac{\Pr[P|S]}{\Pr[P|(\text{not } S)]}}_{\substack{=:\ \text{LQ}^S[P]\ =\ \frac{99\%}{0.60320641\%}}}$$

24.75% $=$ $\boxed{\Pr[S|P]}$

These add to 100%

$\Pr[P|(\text{not } S)] = 0.60320641\%$

Say: $\Pr[S] = 0.2\%$ and $\Pr[P] = 0.8\%$

$\text{Odds}[S|P] = ???$ $\qquad$ $\Pr[P|S] = 0.99$

Events: P $=$ test comes out positive

S $=$ you're sick (have the disease)

Bayes' Law

Fact: $$\text{Odds}[S|P] = \text{Odds}[S] \cdot \frac{\Pr[P|S]}{\Pr[P|(\text{not } S)]}$$

$$\text{Odds}[S|(P\&R)] = \text{Odds}[S|P] \cdot \;???$$

$$\{\Pr[S|(P\&R)]\}_S^{\text{o}} = \{\Pr[S|P] \cdot \;???\}_S^{\text{o}}$$

$$\frac{\boxed{R\&P\&S}}{\boxed{R\&P}} = \frac{\cancel{P\&S}}{\cancel{P}} \; \frac{\cancel{P}}{\boxed{R\&P}} \; \frac{\boxed{R\&P\&S}}{\cancel{P\&S}}$$

$$\{???\}_S^{\text{o}} = \left\{\frac{\Pr[P]}{\Pr[R\&P]} \; \frac{\Pr[R\&P\&S]}{\Pr[P\&S]}\right\}_S^{\text{o}}$$

Events: R = you're in a high-risk group
P = test comes out positive
S = you're sick (have the disease)

Bayes' Law

Fact: $$\text{Odds}[S|P] = \text{Odds}[S] \cdot \frac{\Pr[P|S]}{\Pr[P|(\text{not } S)]}$$

$$\text{Odds}[S|(P\&R)] = \text{Odds}[S|P] \cdot ???$$

$$\{\Pr[S|(P\&R)]\}^{\text{O}}_{S} = \{\Pr[S|P] \cdot ???\}^{\text{O}}_{S}$$

$$??? = \{\Pr[R|(P\&S)]\}^{\text{O}}_{S}$$

$$||$$

$$\{???\}^{\text{O}}_{S} = \left\{ \boxed{\cancel{\frac{\Pr[P]}{\Pr[R\&P]}}} \overbrace{\frac{\Pr[R\&P\&S]}{\Pr[P\&S]}}^{\Pr[R|(P\&S)]} \right\}^{\text{O}}_{S}$$

Events:
- R = you're in a high-risk group
- P = test comes out positive
- S = you're sick (have the disease)

Bayes' Law

Fact: $$\text{Odds}[S|P] = \text{Odds}[S] \cdot \frac{\Pr[P|S]}{\Pr[P|(\text{not } S)]}$$

$$\text{Odds}[S|(P\&R)] = \text{Odds}[S|P] \cdot \boxed{???}$$

$$\{\Pr[S|(P\&R)]\}^{\text{O}}_{S} = \{\Pr[S|P] \cdot ???\}^{\text{O}}_{S}$$

$$??? = \underline{\{\Pr[R|(P\&S)]\}^{\text{O}}_{S}}$$
$$= \text{LQ}^{S}[R|P]$$

Def'n: $$\boxed{\text{LQ}^{A}[C|B]} := \{\Pr[C|(B\&A)]\}^{\text{O}}_{A}$$

Events: $\begin{cases} \text{R} = \text{you're in a high-risk group} \\ \text{P} = \text{test comes out positive} \\ \text{S} = \text{you're sick (have the disease)} \end{cases}$

Bayes' Law

Fact: $$\text{Odds}[S|P] = \text{Odds}[S] \cdot \frac{\Pr[P|S]}{\Pr[P|(\text{not } S)]}$$

$$\text{Odds}[S|(P\&R)] = \text{Odds}[S|P] \cdot \text{LQ}^S[R|P]$$

$$\{\Pr[S|(P\&R)]\}^{\text{O}}_S = \{\Pr[S|P] \cdot ???\}^{\text{O}}_S$$

$$??? = \{\Pr[R|(P\&S)]\}^{\text{O}}_S$$
$$= \text{LQ}^S[R|P]$$

Def'n: $\boxed{\text{LQ}^A[C|B]} := \{\Pr[C|(B\&A)]\}^{\text{O}}_A$

Events: $\begin{cases} \text{R} = \text{you're in a high-risk group} \\ \text{P} = \text{test comes out positive} \\ \text{S} = \text{you're sick (have the disease)} \end{cases}$

Bayes' Law

Fact: $$\text{Odds}[S|P] = \text{Odds}[S] \cdot \frac{\Pr[P|S]}{\Pr[P|(\text{not } S)]}$$

$$\underbrace{\text{Odds}[S|(P\&R)]}_{65.78\%} = \underbrace{\text{Odds}[S|P]}_{\frac{24.75\%}{75.25\%}} \cdot \underbrace{\text{LQ}^S[R|P]}_{2}$$

Fact:

$$\text{Odds}[A|(B\&C)] = \text{Odds}[A|B] \cdot \text{LQ}^A[C|B]$$

Def'n: $$\boxed{\text{LQ}^A[C|B]} := \{\Pr[C|(B\&A)]\}^{\text{O}}_A$$

Events:
- R = you're in a high-risk group
- P = test comes out positive
- S = you're sick (have the disease)

Bayes' Law

Fact:

$$\text{Odds}[A|(B\&C)] \quad = \quad \text{Odds}[A|B] \ \cdot \ \text{LQ}^A[C|B]$$

Def'n:

$$\boxed{\text{LQ}^A[C|B]} ::= \{\Pr[C|(B\&A)]\}^{\text{O}}_{A}$$

$$= \frac{\Pr[C|(B\&A)]}{\Pr[C|(B\&(\text{not } A))]}$$

Bayes' Law

Fact:

$$\mathrm{Odds}[A|(B\&C)] = \mathrm{Odds}[A|B] \cdot \mathrm{LQ}^A[C|B]$$

Def'n: $\boxed{\mathrm{LQ}^A[C|B]} := \{\Pr[C|(B\&A)]\}^{\circ}_{A}$

$$= \frac{\Pr[C|(B\&A)]}{\Pr[C|(B\&(\text{not } A))]}$$

Fact:

$$\mathrm{Odds}[A|(B\&C)] = \mathrm{Odds}[A|B] \cdot \mathrm{LQ}^A[C|B]$$

Def'n: $\boxed{\mathrm{LQ}^A[C|B]} := \{\Pr[C|(B\&A)]\}^{\circ}_{A}$

To update the odds based on new info, multiply by a carefully defined "likelihood quotient".

$$= \frac{\Pr[C|(B\&A)]}{\Pr[C|(B\&(\text{not } A))]}$$

Bayes' Law

Fact:

$$\text{Odds}[A|(B\&C)] = \text{Odds}[A|B] \cdot \text{LQ}^A[C|B]$$

Def'n: $\boxed{\text{LQ}^A[C|B]} ::= \{\Pr[C|(B\&A)]\}^{\circ}_{A}$

$$= \frac{\Pr[C|(B\&A)]}{\Pr[C|(B\&(\text{not } A))]}$$

Fact:

$$\text{Odds}[A|(B\&C\&D)] = \text{Odds}[A|(B\&C)] \cdot \text{LQ}^A[D|(B\&C)]$$

Def'n: $\boxed{\text{LQ}^A[D|(B\&C)]} ::= \{\Pr[D|(B\&C\&A)]\}^{\circ}_{A}$

$$= \frac{\Pr[D|(C\&B\&A)]}{\Pr[D|(C\&B\&(\text{not } A))]}$$

To update the odds based on new info, multiply by a carefully defined "likelihood quotient".

Next: Transitivity of LQ

Bayes' Law

$$\frac{D\&C\&B\&A}{\cancel{C\&B\&A}} \quad \frac{\cancel{C\&B\&A}}{B\&A} = \frac{D\&C\&B\&A}{B\&A}$$

$$\{\Pr[D|(C\&B\&A)]\cdot\Pr[C|(B\&A)]\}^{\circ}_{A} = \{\Pr[(D\&C)|(B\&A)]\}^{\circ}_{A}$$

$$\mathrm{LQ}^{A}[D|(\underline{C\&B})] \cdot \mathrm{LQ}^{A}[C|B] = \mathrm{LQ}^{A}[(D\&C)|B]$$

Fact:

$$\mathrm{Odds}[A|(B\&C\&D)] = \mathrm{Odds}[A|(B\&C)] \cdot \mathrm{LQ}^{A}[D|(B\&C)]$$

Def'n: $\boxed{\mathrm{LQ}^{A}[D|(B\&C)]} := \{\Pr[D|(B\&C\&A)]\}^{\circ}_{A}$

To update the odds based on new info, multiply by a carefully defined "likelihood quotient".

$$= \frac{\Pr[D|(C\&B\&A)]}{\Pr[D|(C\&B\&(\text{not } A))]}$$

Bayes' Law

$$\frac{D\&C\&B\&A}{C\&B\&A} \quad \frac{C\&B\&A}{B\&A} = \frac{D\&C\&B\&A}{B\&A}$$

$$\{\mathsf{Pr}[D|(C\&B\&A)]\cdot\mathsf{Pr}[C|(B\&A)]\}^{\circ}_{A} = \{\mathsf{Pr}[(D\&C)|(B\&A)]\}^{\circ}_{A}$$

$$\mathsf{LQ}^{A}[D|(\underline{C\&B})] \cdot \mathsf{LQ}^{A}[C|B] = \mathsf{LQ}^{A}[(D\&C)|B]$$

$$\mathsf{LQ}^{A}[C|B] \cdot \mathsf{LQ}^{A}[B] = \mathsf{LQ}^{A}[C\&B]$$

$$\mathsf{LQ}^{A}[(D\&\underline{C\&B})|(\underline{\cancel{C\&B}})]$$

$$\cdot \ \mathsf{LQ}^{A}[(\cancel{C\&B})|B]$$

$$= \mathsf{LQ}^{A}[(D\&C\&\boxed{\cancel{B}})|\boxed{B}]$$

Bayes' Law

$$\frac{D\&C\&B\&A}{C\&B\&A} \quad \frac{C\&B\&A}{B\&A} = \frac{D\&C\&B\&A}{B\&A}$$

$$\{\mathrm{Pr}[D|(C\&B\&A)]\cdot\mathrm{Pr}[C|(B\&A)]\}^{\circ}_{A} = \{\mathrm{Pr}[(D\&C)|(B\&A)]\}^{\circ}_{A}$$

$$\mathrm{LQ}^{A}[D|(C\&B)] \cdot \mathrm{LQ}^{A}[C|B] = \mathrm{LQ}^{A}[(D\&C)|B]$$

$$\mathrm{LQ}^{A}[C|B] \cdot \mathrm{LQ}^{A}[B] = \mathrm{LQ}^{A}[C\&B]$$

$$\mathrm{LQ}^{A}[\underbrace{C|\cancel{B}}_{\&B}] \cdot \mathrm{LQ}^{A}[\cancel{B}] = \mathrm{LQ}^{A}[C\&B]$$

Next: from transitivity to Bayes' Laws

Bayes' Law

$$\frac{D\&C\&B\&A}{C\&B\&A} \quad \frac{C\&B\&A}{B\&A} = \frac{D\&C\&B\&A}{B\&A}$$

$$\{\Pr[D|(C\&B\&A)]\cdot\Pr[C|(B\&A)]\}^{\circ}_{A} = \{\Pr[(D\&C)|(B\&A)]\}^{\circ}_{A}$$

$$\mathrm{LQ}^A[D|(C\&B)] \cdot \mathrm{LQ}^A[C|B] = \mathrm{LQ}^A[(D\&C)|B]$$

$$\boxed{\mathrm{LQ}^A[C|B]} \cdot \boxed{\mathrm{LQ}^A[B]} = \boxed{\mathrm{LQ}^A[C\&B]}$$

$$\mathrm{Odds}[A|\underline{B}] = \boxed{\mathrm{Odds}[A] \cdot \mathrm{LQ}^A[\underline{B}]}$$

$$\mathrm{Odds}[A|(\underline{B\&C})] = \boxed{\mathrm{Odds}[A]}\cdot\underbrace{\boxed{\mathrm{LQ}^A[\underline{B\&C}]}}_{\boxed{\mathrm{LQ}^A[B]}\cdot\mathrm{LQ}^A[C|B]}$$

To update the odds based on new info, multiply by a carefully defined "likelihood quotient".

$$\mathrm{Odds}[A|(B\&C)] = \mathrm{Odds}[A|B] \cdot \mathrm{LQ}^A[C|B]$$

Bayes' Law

$$\frac{D\&C\&B\&A}{C\&B\&A} \quad \frac{C\&B\&A}{B\&A} = \frac{D\&C\&B\&A}{B\&A}$$

$$\{\mathsf{Pr}[D|(C\&B\&A)]\cdot\mathsf{Pr}[C|(B\&A)]\}^{\circ}_{A}=\{\mathsf{Pr}[(D\&C)|(B\&A)]\}^{\circ}_{A}$$

$$\mathsf{LQ}^{A}[D|(C\&B)] \cdot \mathsf{LQ}^{A}[C|B] = \mathsf{LQ}^{A}[(D\&C)|B]$$

$$\mathsf{LQ}^{A}[C|B] \cdot \mathsf{LQ}^{A}[B] = \mathsf{LQ}^{A}[C\&B]$$

$$\mathsf{Odds}[A|(B\&C\&D)]=\mathsf{Odds}[A|(B\&C)]\cdot$$

$$\mathsf{LQ}^{A}[D|(C\&B)]$$

$$\boxed{\mathsf{LQ}^{A}[D|(C\&B)]}$$

$$\cdot\boxed{\mathsf{LQ}^{A}[C|B]}$$

$$\boxed{\mathsf{Odds}[A|(B\&\underline{C\&D})]}=\boxed{\mathsf{Odds}[A|B]}\cdot\overbrace{\mathsf{LQ}^{A}[(\underline{C\&D})|B]}$$

$$\mathsf{Odds}[A|(B\&\underline{C})]=\boxed{\mathsf{Odds}[A|B]\cdot\mathsf{LQ}^{A}[\underline{C}|B]}$$

Bayes' Law

$$\frac{D\&C\&B\&A}{C\&B\&A} \quad \frac{C\&B\&A}{B\&A} = \frac{D\&C\&B\&A}{B\&A}$$

$$\{\Pr[D|(C\&B\&A)]\cdot\Pr[C|(B\&A)]\}^{\circ}_{A} = \{\Pr[(D\&C)|(B\&A)]\}^{\circ}_{A}$$

$$\mathrm{LQ}^{A}[D|(C\&B)] \cdot \mathrm{LQ}^{A}[C|B] = \mathrm{LQ}^{A}[(D\&C)|B]$$

$$\mathrm{LQ}^{A}[C|B] \cdot \mathrm{LQ}^{A}[B] = \mathrm{LQ}^{A}[C\&B]$$

$$\boxed{\mathrm{Odds}[A|(B\&C\&D)] = \mathrm{Odds}[A|(B\&C)] \cdot \mathrm{LQ}^{A}[D|(C\&B)]}$$

etc., etc., etc., . . .

To update the odds based on new info, multiply by a carefully defined "likelihood quotient".

STOP

$$\boxed{\mathrm{Odds}[A|B] = \mathrm{Odds}[A] \cdot \mathrm{LQ}^{A}[B]}$$

$$\boxed{\mathrm{Odds}[A|(B\&C)] = \mathrm{Odds}[A|B] \cdot \mathrm{LQ}^{A}[C|B]}$$