Financial Mathematics
Pricing/hedging in many subperiods
Part 2
\[ f(S) = (5000S - 5000) + \]

Goal: \(_{\text{approximately}}^\overset{\text{Compute}}{\text{the expected value of}} \] \[ f(e^{Hu+Td}). \]
Then multiply by \[ e^{-rN} = (e^r)^{-N}. \]

Coin-flipping game: Flip a fair coin \( N \) times.
If \( H \) heads and \( T \) tails, pay \( f(e^{Hu+Td}) \),
30 days from now.

\[ e^r = 1.000000001 \]
\[ N = 2,592,000 \]
\[ e^{rN}V = \text{expected payout} =: E = ??? \]
\[ V = e^{-rN}E \]
= discounted expected payout

**probability problems, then expected value problems**
\[ f(S) = (5000S - 5000) + \]

**Goal:** \( \underbrace{\text{approximately}}_{\text{Compute}} E \) the expected value of \( f(e^{Hu +Td}) \).

**Easier problem:** Compute the expected value of \( f(D_2) \).

\[ D_2 = H_2 - T_2 : \]

\[
\begin{array}{c|ccc}
 & 2 & f(2) \\
\hline
0.25 & 0.5 & 0 & f(0) \\
0.25 & -2 & f(-2) \\
\end{array}
\]

\[
[0.25][f(2)] + [0.5][f(0)] + [0.25][f(-2)] = 1,250
\]

works for any function \( f : \rightarrow g \)
Define:  \[ g(S) = 5e^S + S^2 \]

Easier problem:

Compute the expected value of \( g(D_2) \).

\[ D_2 = H_2 - T_2 \]

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>( g(2) )</th>
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<tbody>
<tr>
<td>0.25</td>
<td></td>
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<tr>
<td>0.5</td>
<td>0</td>
<td>( g(0) )</td>
</tr>
<tr>
<td>0.25</td>
<td>-2</td>
<td>( g(-2) )</td>
</tr>
</tbody>
</table>

\[
[0.25][g(2)] + [0.5][g(0)] + [0.25][g(-2)] = \text{Exercise 4}
\]
Recall: \( f(S) = (5000S - 5000)_+ \)

Goal: approximately \( E \)

Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem:

Compute the expected value of \( f(Z) \).

\( Z: \)

\[
\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx \mathrel{\left|\begin{array}{c}
 x \\
 f(x)
\end{array}\right.} \quad \text{Do this for all } x \in \mathbb{R}
\]

\[
\int_{-\infty}^{\infty} \left[ f(x) \right] \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[ f(x) \right] e^{-x^2/2} \, dx = \text{exercise}
\]
Recall: \( f(S) = (5000S - 5000)_+ \)

Goal: \( \underbrace{\text{approximately}}_{E} \text{Compute the expected value of } f(e^{Hu+Td}). \)

New easier problem:
\( \text{Compute the expected value of } f(Z). \)

\( Z: \)
\[
\frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx
\]

\( \quad x \quad f(x) \quad \text{Do this for all } x \in \mathbb{R} \)

\( E[f(Z)] \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx - \frac{x^2}{2} \, dx \)
Recall: \( f(S) = (5000S - 5000)_+ \)

Goal: \( \text{approximately} \quad \mathbb{E} f(e^{Hu+Td}) \)

Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem:
Compute the expected value of \( f(Z) \).

\[
Z: \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} \, dx \quad x \quad f(x)
\]

Do this for all \( x \in \mathbb{R} \)

works for any exp-bdd function \( f \mapsto g \)

\[
E[f(Z)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [f(x)] e^{-x^2/2} \, dx
\]
Recall: \( f(S) = (5000S - 5000)_+ \)

Goal: \( \text{approximately} \quad \mathbb{E} \quad \text{Compute the expected value of } f(e^{H_u + T_d}). \)

New easier problem:
Compute the expected value of \( g(Z) \).

\[
Z: \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} dx \quad \left| \begin{array}{cc} x & g(x) \end{array} \right|
\]

Do this for all \( x \in \mathbb{R} \)

works for any exp-bdd function \( g \)

temporary change of color...

\[
E[g(Z)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} dx
\]
Recall: \( f(S) = (5000S - 5000)_+ \)

Goal: \( \underbrace{\text{approximately}}_{E} \) Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem:
Compute the expected value of \( g(Z) \).

\[
Z: \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} dx \quad \begin{array}{c} x \quad g(x) \\ \text{Do this for} \\ \text{all } x \in \mathbb{R} \\ \text{works for} \\ \text{any exp-bdd} \\ \text{function } g \\
\end{array}
\]

Temporary change of color...

\[
E[g(Z)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} dx
\]
Recall: \[ f(S) = (5000S - 5000)_+ \]

Goal: \( \text{approximately} \quad E \]
Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem:
Compute the expected value of \( g(Z) \).

\[ Z: \quad \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx \]
\[ \bigg| \quad x \quad g(x) \quad \text{Do this for all } x \in \mathbb{R} \]

Change every \( Z \) to \( x \) and then integrate against \( h(x) \, dx \), from \(-\infty\) to \( \infty \).

\[ h(x) := \frac{e^{-x^2/2}}{\sqrt{2\pi}} \]

works for any exp-bdd function \( g \)

\[ E[g(Z)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx \]
Recall: \[ f(S) = (5000S - 5000)_+ \]

Goal: \( \underbrace{E}_{\text{approximately}} \) Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem:
Compute the expected value of \( g(Z) \).

\[ Z: \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} \, dx \begin{bmatrix} x \quad g(x) \end{bmatrix} \]

Do this for all \( x \in \mathbb{R} \)

works for any exp-bdd function \( g \)

\[ E[g(Z)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx \]
Recall: \( f(S) = (5000S - 5000)_+ \)

Goal: \( \text{approximately} \quad E \) Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem: \( \text{approximately} \quad \) Compute the expected value of \( g(X) \).

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]

\[
E[g(Z)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]

Do this for all \( x \in \mathbb{R} \)

works for any exp-bdd function \( g \)
Recall: \( f(S) = (5000S - 5000)_+ \)

Goal: \( \underbrace{E}_{\text{approximately}} f(\exp(Hu + Td)) \)

**Compute the expected value of** \( f(\exp(Hu + Td)) \).

**New easier problem:** Subgoal: Choose \( g \) s.t.: \( \| \)

**Compute the expected value of** \( g(X) \).

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \( f(S) = (5000S - 5000) + \)

Goal: \( \underbrace{\text{approximately}}_{E} \) Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem: Subgoal: Choose \( g \) s.t.: \( \|g\| \text{ exp-bdd?} \)
Compute \( \underbrace{\text{approximately}}_{\text{works for any exp-bdd function } g} \) the expected value of \( g(X) \).

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \( f(S) = (5000S - 5000) \)

Goal: \( \text{approximately } \mathbb{E} \text{Compute the expected value of } f(e^{H_u + T_d}). \)

New easier problem:
Compute \( \text{approximately } \mathbb{E} \text{the expected value of } g(X). \)

\[
X = \frac{(H - T)}{\sqrt{N}} 
\]

\[
H + T = N 
\]

\[
H - T = X \sqrt{N} 
\]

\[
2H = N + X \sqrt{N} 
\]

\[
2T = N - X \sqrt{N} 
\]

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx 
\]
Recall: \( f(S) = (5000S - 5000) + \)

Goal: \( \approx \)

Compute the expected value of \( f(e^{Hu + Td}) \).

New easier problem:

Compute \( \approx \) the expected value of \( g(X) \).

\[
H = \frac{[N + X\sqrt{N}]}{2} \\
T = \frac{[N - X\sqrt{N}]}{2}
\]

\( N := 30 \times 24 \times 60 \times 60 = 2,592,000 \)

\[
2H = N + X\sqrt{N} \\
2T = N - X\sqrt{N}
\]

\( E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx \)
Recall: \[ f(S) = (5000S - 5000) + \]

Goal: 

\[ \text{Compute approximately } E \]

Compute the expected value of \( f(e^{Hu +Td}) \).

New easier problem:

\[ \text{Compute approximately } \]

Compute the expected value of \( g(X) \).

\[
H = \frac{[N + X\sqrt{N}]/2}{xu}
\]

\[
T = \frac{[N - X\sqrt{N}]/2}{xd}
\]

\[
Hu = \frac{[Nu + X\sqrt{Nu}]/2}{ADD}
\]

\[
Td = \frac{[Nd - X\sqrt{Nd}]/2}{ADD}
\]

\[
Hu + Td = \frac{[N(u + d)/2] + [X\sqrt{N}(u - d)/2]}{ADD}
\]

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x) e^{-x^2/2} \, dx
\]
Recall: \( f(S) = (5000S - 5000) \)

Goal: approximately \( E \)

Compute the expected value of \( f(e^{Hu + Td}) \).

New easier problem: Compute approximately

Compute the expected value of \( g(X) \).

\[
H = \frac{[N + X\sqrt{N}]}{2} \quad \quad \quad T = \frac{[N - X\sqrt{N}]}{2}
\]

\[
e^{Hu + Td} = \left[e^{N(u+d)/2}\right] \left[e^{X\sqrt{N}(u-d)/2}\right]
\]

\[
= \left[e^{N(u+d)/2}\right] \left[e^{(\sqrt{N}(u-d)/2)X}\right]
\]

\[
Hu + Td = \frac{[N(u + d)/2]}{2} + \frac{[X\sqrt{N}(u − d)/2]}{2}
\]

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \[ f(S) = (5000S - 5000) + \]

Goal: \[ \text{Compute approximately } E \]

Compute the expected value of \( f(e^{Hu +Td}) \).

New easier problem:

\[ \text{Compute approximately } \]

Compute the expected value of \( g(X) \).

\[
H = \frac{[N + X\sqrt{N}]}{2} \quad T = \frac{[N - X\sqrt{N}]}{2}
\]

\[
e^{Hu +Td} = \left[ e^{N(u+d)/2} \right] \left[ e^{(\sqrt{N}(u-d)/2)X} \right] = Ce^{kX}
\]

\[
Hu +Td = \left[ N(u + d)/2 \right] + \left[ X\sqrt{N}(u - d)/2 \right]
\]

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \( f(S) = (5000S - 5000) \) +

Goal: \( \text{approximately } E \)

Compute the expected value of \( f(e^{Hu+Td}) \).

Restatement of goal:

Compute the expected value of \( g(X) \).

\[
H = \frac{[N + X\sqrt{N}]}{2} \quad T = \frac{[N - X\sqrt{N}]}{2}
\]

\[
e^{Hu+Td} = \left[e^{N(u+d)/2}\right]^{k} = C e^{kX}
\]

\[
g(x) := f(C e^{kx}) \quad g \text{ exp-bad?} \quad \text{YES}
\]

\[
f(e^{Hu+Td}) = f(C e^{kX}) = g(X)
\]

\[
E = E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]

write \( H, T \) as expr.s of \( X \)
Recall: \( f(S) = (5000S - 5000) \)

Goal: \( \text{approximately} \) \( E \) compute the expected value of \( f(e^{Hu+Td}) \).

Restatement of goal: \( \text{approximately} \) compute the expected value of \( g(X) \).

\[
N = 2,592,000 \\
u = 0.00003561536577 \\
d = -0.00003561463419 \\
k = 0.0573390439012 \\
C = 1.00094857729
\]

\[
e^{Hu+Td} = \left[ e^{N(u+d)/2} \right] \left[ e^{(\sqrt{N}(u-d)/2)X} \right] = Ce^{kX}
\]

\[
g(x) := f(Ce^{kx})
\]

\[
f(e^{Hu+Td}) = f(Ce^{kX}) = g(X)
\]

\[
E = E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \[ f(S) = (5000S - 5000)_+ \]
\[ = 5000(S - 1)_+ \]

\[ g(x) := f(Ce^{kx}) = 5000(Ce^{kx} - 1)_+ \]

\[ E \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 5000(Ce^{kx} - 1)_+ e^{-x^2/2} \, dx \]

\[ 1.00094857729 = C \]

\[ k = 0.0573390439012 \]

\[ E = E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx \]
\[ E \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 5000(Ce^{kx} - 1) + e^{-x^2/2} \, dx \]

\[ = \frac{5000}{\sqrt{2\pi}} \int_{-(\sqrt{2\pi})}^{\infty} (Ce^{kx} - 1) + e^{-x^2/2} \, dx + e^{-x^2/2} \, dx \]

\[ 1.00094857729 = C \]

\[ k = 0.0573390439012 \]

\[ 1.00094857729 = C \]
$$
E \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 5000(Ce^{kx} - 1) + e^{-x^2/2} \, dx
$$

$$
= \frac{5000}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Ce^{kx} - 1) e^{-x^2/2} \, dx
$$

$$
= \frac{5000}{\sqrt{2\pi}} \int_{a}^{\infty} (Ce^{kx} - 1) e^{-x^2/2} \, dx
$$

$$
Ce^{ka} - 1 = 0
$$

$$
Ce^{ka} = 1
$$

$$
e^{ka} = 1/C
$$

$$
ka = \ln(1/C) = -\ln C
$$

$$
a = -(\ln C)/k
$$

$$
C = 1.00094857729
$$

$$
k = 0.0573390439012
$$
\[ E \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 5000(Ke^{kx} - 1) + e^{-x^2/2} \, dx \]

\[ = \frac{5000}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Ke^{kx} - 1) + e^{-x^2/2} \, dx \]

\[ = \frac{5000}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Ke^{kx} - 1) \, e^{-x^2/2} \, dx \]

\[ k = 0.0573390439012 \quad a = -0.0165354585751 \]

\[ C = 1.00094857729 \]
\[ E \approx \frac{5000}{\sqrt{2\pi}} \left[ C \int_{a}^{\infty} e^{kx} e^{-x^2/2} \, dx \right. \]

\[ \left. - \int_{a}^{\infty} e^{-x^2/2} \, dx \right] + \sqrt{2\pi} \Phi(-a) \]

\[ \frac{5000}{\sqrt{2\pi}} \left[ C \int_{a}^{\infty} e^{kx} e^{-x^2/2} \, dx \right. \]

\[ \left. - \int_{a}^{\infty} e^{-x^2/2} \, dx \right] \]

\[ C = 1.00094857729 \]

\[ k = 0.0573390439012 \]

\[ a = -0.0165354585751 \]
\[ E \approx \frac{5000}{\sqrt{2\pi}} \left[ C \int_{a}^{\infty} e^{kx} e^{-x^2/2} \, dx - \int_{a}^{\infty} e^{-x^2/2} \, dx \right] \]

\[ \int_{a-k}^{\infty} e^{k(x+k)} e^{-(x+k)^2/2} \, dx \]

\[ e^{kx} e^{k^2/2} e^{-x^2/2} e^{-k^2/2} e^{-kx} \]

\[ \sqrt{2\pi} \Phi(k - a) \]

\[ C = 0.0573390439012 \quad a = -0.0165354585751 \]
\[ E \approx \frac{5000}{\sqrt{2\pi}} \left[ C \int_{a}^{\infty} e^{kx} e^{-x^2/2} \, dx - \int_{a}^{\infty} e^{-x^2/2} \, dx \right] \\
\int_{a-k}^{\infty} e^{k(x+k)} e^{-(x+k)^2/2} \, dx \\
e^{k^2/2} \int_{a-k}^{\infty} e^{-x^2/2} e^{-k^2/2} \\
\sqrt{2\pi} \Phi(k - a) \]

\[ C \approx 0.0573390439012 \]
\[ a = -0.0165354585751 \]
$$E \approx \frac{5000}{\sqrt{2\pi}} \left[ C \int_0^\infty e^{kx} e^{-x^2/2} dx - \int_a^\infty e^{-x^2/2} dx \right]$$

$$e^{k^2/2} \sqrt{2\pi} \Phi(k - a)$$

$$C \approx 1.00094857729 \quad k \approx 0.0573390439012 \quad a = -0.0165354585751$$
\[ E \approx \frac{5000}{\sqrt{2\pi}} \left[ C \int_a^\infty e^{kx} e^{-x^2/2} \, dx \underbrace{\int_a^\infty e^{-x^2/2} \, dx}_{\sqrt{2\pi} \Phi(-a)} \right] - \int_a^\infty e^{-x^2/2} \, dx \]

\[ = 5000 \left[ C e^{k^2/2} \Phi(k-a) \right] - \Phi(-a) \]

\[ = 121.07046876 \]

\[ C = 1.00094857729 \quad k = 0.0573390439012 \quad a = -0.0165354585751 \]
Coin-flipping game: Flip a fair coin $N$ times. If $H$ heads and $T$ tails, pay $f(u^H d^T)$, 30 days from now.

$$e^r = 1.000000001$$

$$N = 2,592,000$$

$$E \approx$$

$$e^{rN} V = \text{expected payout} =: E = ??? \text{ approx.}$$

$$V = e^{-rN} E$$

$$= \text{discounted expected payout}$$

$$V = e^{-rN} E \approx 120.757060394$$

$$1.000000001 = e^r$$

$$e^{-rN} = 0.997411356336$$

121.07046876

exact answer? soon...

$$E \approx 121.07046876$$

$$N := 30 \times 24 \times 60 \times 60 = 2,592,000$$
Scenarios with \( j \) upticks and \( N-j \) downticks:

There are \( \binom{N}{j} \) of them.

Each has (risk-neutral) probability: \( (0.5)^j (0.5)^{N-j} \)

\[
\text{prob.} \begin{pmatrix} j, N-j \\ j \end{pmatrix} = \binom{N}{j} (0.5)^j (0.5)^{N-j}
\]

\[
V = e^{-rN} E \approx 120.757060394
\]

\[
e^{-rN} = 0.997411356336
\]

\[
E \approx 121.07046876
\]

exact answer?

\[
N := 30 \times 24 \times 60 \times 60 = 2,592,000
\]
Scenarios with \( j \) upticks and \( N - j \) downticks:

\[
\ln(\text{stock price}) \text{ starts at } 0, \quad \text{ends at } ju + (N - j)d \\
f(S) = 5000(S - 1)^+ \\
\text{stock price ends at } e^{ju + (N-j)d} \\
\text{option pays } 5000(e^{ju + (N-j)d} - 1)^+ \\
\text{prob. } j, N-j = \binom{N}{j} (0.5)^j (0.5)^{N-j} \\
\text{ending value of hedge} \\
\]

\[
V = e^{-rN} E \approx 120.757060394 \\
1.0000000001 \\
\approx e^r \\
\]

**exact answer?**

\[
E \approx 121.07046876 \\
e^{-rN} = 0.997411356336 \\
\]

\[
N := 30 \times 24 \times 60 \times 60 = 2,592,000
\]
Scenarios with \( j \) upticks and \( N - j \) downticks:

\[
E := (\text{risk-neutral}) \text{ expected ending value of hedge}
\]

To compute it, multiply this by this, then add over \( j = 0, \ldots, N \).

\[
\text{option pays} \quad 5000(e^{ju} + (N-j)d - 1)
\]

\[
\text{ending value of hedge}
\]

\[
V = e^{-rN} E \approx 120.757060394
\]

\[
1.000000001
\]

\[
e^{-rN} = 0.997411356336
\]

**exact answer?**

\[
E \approx 121.07046876
\]

\[
N := 30 \times 24 \times 60 \times 60 = 2,592,000
\]
Scenarios with \( j \) upticks and \( N - j \) downticks:

\[
E := (\text{risk-neutral}) \text{ expected ending value of hedge}
\]

To compute it, **multiply** this by this, then **add** over \( j = 0, \ldots, N \).

Option pays 

\[
\text{prob. } j,N-j = \binom{N}{j} (0.5)^j (0.5)^{N-j}
\]

\[
5000(e^{ju}+(N-j)d - 1)^+
\]

ending value of hedge

\[
V = e^{-rN} E \approx 120.757060394
\]

Exact answer:

\[
E = \sum_{j=0}^{N} \binom{N}{j} [(0.5)^j (0.5)^{N-j}] 5000(e^{ju}+(N-j)d - 1)^+
\]

\[
E \approx 121.070464876
\]

**exact answer?**

\[
N := 30 \times 24 \times 60 \times 60 = 2,592,000
\]

35
\[ V = e^{-r^N} E \approx 120.757060394 \]

**Exact answer:**

\[ E = \sum_{j=0}^{N} \binom{N}{j} \left[ (0.5)^j (0.5)^{N-j} \right] 5000(e^{j\mu} + (N-j)d - 1) + e^{-r^N} = 0.997411356336 \]

\[ E \approx 121.07046876 \]

\[ N := 30 \times 24 \times 60 \times 60 = 2,592,000 \]
Another Central Limit Theorem application:

Recall: $S_0 = 1$ is the initial price of the stock.
Let $S_1$ denote the price after one year.

Exercise: Compute $E[S_1]$, approximately.

\[ V = e^{-rN}E \approx 120.757060394 \]

Exact answer:

\[
E = \sum_{j=0}^{N} \binom{N}{j} [(0.5)^j (0.5)^{N-j}] 5000(e^{ju+(N-j)d} - 1) + e^{-rN} = 0.997411356336
\]

\[ E \approx 121.07046876 \]

\[ V = e^{-rN}E = 120.7994402 \]

\[
\begin{align*}
  u &= 0.00003561536577 \\
  d &= -0.00003561463419
\end{align*}
\]

\[ (0.5)^N \]

\[ 5000(e^{ju+(N-j)d} - 1) \]
Market analyst: annual vol = 0.200002881086
annual drift = 0.03399864624

Recall: $S_0 = 1$ is the initial price of the stock.
Let $S_1$ denote the price after one year.

Exercise: Compute $\mathbb{E}[S_1]$, approximately.

\[
V = e^{-rN}E \approx 120.757060394
\]

**Exact answer:**

$$E = \sum_{j=0}^{N} \binom{N}{j} [ (0.5)^j (0.5)^{N-j} ] 5000(e^ju + (N-j)d - 1) + e^{-rN} = 0.997411356336$$

$$= 121.1129585417487$$

$$\approx 121.07046876$$

\[
V = e^{-rN}E = 120.7994402
\]
Market analyst: annual vol = 0.200002881086
annual drift = 0.03399864624

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Solution: By the Central Limit Theorem, $\ln S_1$, being a large sum of iid PCRVs, is approximately normal.

Then exponentiation nearly almost commutes with expectation.

That is, $\mathbb{E}[e^{\ln S_1}] \approx e^{\mathbb{E}[\ln S_1]}$. That is, $\mathbb{E}[S_1] \approx e^{\mathbb{E}[\ln S_1] + \frac{1}{2} \text{Var}[\ln S_1]}$. 
Market analyst: annual vol = 0.200002881086
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Solution:

$$\mathbb{E}[S_1] \approx e^{\mathbb{E}[\ln S_1] + \frac{1}{2} \text{Var}[\ln S_1]}$$

$$= e^{(0.03399864624) + \frac{1}{2}(0.200002881086)^2}$$

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Solution:

\[
E[S_1] \approx e^{E[\ln S_1] + \frac{1}{2} \text{Var}[\ln S_1]}
\]
\[
= e^{(0.03399864624) + \frac{1}{2}(0.200002881086)^2}
\]
\[
= 1.05548378145
\]

Expected annual return is about 5.5%.
Annual drift is 0.03399864624.
Annual augmented drift is, by definition,
\[
(0.03399864624) + \frac{1}{2}(0.200002881086)^2
\]

annual risk-free factor = \( e^{\text{annual augmented drift}} \)