Financial Mathematics
Clicker review session, Midterm 02
(a) \((0.1)(e^4) + (0.9)(e^6)\)

(b) \((4)(e^{0.1}) + (6)(e^{0.9})\)

(c) \((0.1)(4) + (0.9)(6)\)

(d) none of the above

\[\Pr[X = 4] = 0.1\]
\[\Pr[X = 6] = 0.9\]

Compute \(E[e^X]\).
(a) \((0.4)(2) + (0.6)(3)\)

(b) \((0.4)(2^4) + (0.6)(3^4)\)

(c) \((2)(0.4)^4 + (3)(0.6)^4\)

(d) none of the above

Pr\([X = 2]\) = 0.4
Pr\([X = 3]\) = 0.6

Compute \(E[X^4]\).
(a) \( \delta_{0.4} + \delta_{3.6} \)

(b) \((4)(\delta_{0.1}) + (6)(\delta_{0.9})\)

(c) \((0.1)(\delta_{4}) + (0.9)(\delta_{6})\)

(d) none of the above

\[
\begin{align*}
\Pr[X = 4] &= 0.1 \\
\Pr[X = 6] &= 0.9
\end{align*}
\]

Compute \( \delta[X] \).
(a) \((0.4)(\delta_2) + (0.6)(\delta_3)\)

(b) \((0.4)(\delta_{24}) + (0.6)(\delta_{34})\)

(c) \((2^4)(\delta_{0.4}) + (3^4)(\delta_{0.6})\)

(d) none of the above

Pr\([X = 2]\) = 0.4
Pr\([X = 3]\) = 0.6

Compute \(\delta[X]\).
(a) \((0.1)(\delta_{e^4}) + (0.9)(\delta_{e^6})\)

(b) \((e^4)(\delta_{0.1}) + (e^6)(\delta_{0.9})\)

(c) \((0.1)(\delta_4) + (0.9)(\delta_6)\)

(d) none of the above

Pr\([X = 4]\) = 0.1
Pr\([X = 6]\) = 0.9

Compute \(\delta[e^X]\).
(a) \((0.4)(\delta_2) + (0.6)(\delta_3)\)

(b) \((0.4)(\delta_{24}) + (0.6)(\delta_{34})\)

(c) \((2^4)(\delta_{0.4}) + (3^4)(\delta_{0.6})\)

(d) none of the above

\[ \Pr[X = 2] = 0.4 \]
\[ \Pr[X = 3] = 0.6 \]

Compute \(\delta[X^4]\).
(a) \((0.1)(\delta_4) + (0.9)(\delta_6)\)

(b) \((0.1)(\delta_{0.1}) + (0.9)(\delta_{0.9})\)

(c) \((0.1)(\delta_{0.1}) + (0.9)(\delta_1)\)

(d) none of the above

\[\text{Pr}[X = 4] = 0.1 \quad \text{Pr}[X = 6] = 0.9\]

\[
F(x) := \text{Pr}[X \leq x]
\]

Compute \(\delta[F(X)]\).
(a) \((0.4)(\delta_{0.4}) + (0.6)(\delta_{1})\)

(b) \((0.4)(\delta_{0.4}) + (0.6)(\delta_{0.6})\)

(c) \((0.4)(\delta_{2}) + (0.6)(\delta_{3})\)

(d) none of the above

\[\begin{align*}
\Pr[X = 2] &= 0.4 \\
\Pr[X = 3] &= 0.6 \\
F(x) &:= \Pr[X \leq x] \\
\text{Compute } \delta[F(X)].
\end{align*}\]
(a) $\sqrt{2\pi} \Phi(8)$

(b) $\sqrt{2\pi} \Phi(-8)$

(c) 0

(d) none of the above

\[ \int_{-\infty}^{\infty} e^{-x^2/2} \, dx \]
(a) \[\sqrt{2\pi} \Phi(8)\]

(b) \[\sqrt{2\pi} \Phi(-8)\]

(c) 0

(d) none of the above
Compute \( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (4x^6 + 2x^5)e^{-x^2/2} \, dx \)

(a) \( 4(6!) + 2(5!) \)

(b) 0

(c) \( 4(1)(3)(5) \)

(d) none of the above
Compute \[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (x^9 + 2x^7 - 4x^4)e^{-x^2/2} \, dx \]

(a) \( 9 + 2(7) - 4(4) \)

(b) \((9!) + 2(7!) - 4(4!))\)

(c) 0

(d) none of the above

\(-4(1)(3)\)
Compute \[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (x^9 + 2x^7 - 4x^3)e^{-x^2/2} \, dx \]

(a) \( (9!) + 2(7!) - 4(3!) \)

(b) 0

(c) \( 9 + 2(7) - 4(3) \)

(d) none of the above
Compute \( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (x^8 + 2x^6 - 4x^2) e^{-x^2/2} \, dx \)

(a) \((3)(5)(7) + 2(3)(5) - 4\)

(b) \((3)(5)(7) + 2(3)(5)\)

(c) \(2(1)(3)(5) - 4(1)\)

(d) none of the above
(a) \( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} \, dx \)

(b) \( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{3x} + 7 \, dx \)

(c) \( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{3x} + 7 e^{-x^2/2} \, dx \)

(d) none of the above

\[
Z := \Phi^{-1} : (0, 1) \to \mathbb{R}
\]

Compute \( \int_{0}^{1} e^{3Z(\omega) + 7} \, d\omega \).
(a) 0

(b) 1

(c) $e^{(3^2/2)} + 7$

(d) none of the above

$Z := \Phi^{-1} : (0, 1) \to \mathbb{R}$

Compute $\int_0^1 e^{3Z(\omega)} + 7 \, d\omega$. 

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(a) 1

(b) 2

(c) 3

(d) none of the above

\[ Z := \Phi^{-1} : (0, 1) \to \mathbb{R} \]

Compute \( \int_0^1 (Z(\omega))^3 \, d\omega \).
(a) 8!

(b) (2)(4)(6)(8)

(c) (1)(3)(5)(7)

(d) none of the above

\[ Z := \Phi^{-1} : (0, 1) \rightarrow \mathbb{R} \]

Compute \( \int_0^1 (Z(\omega))^8 \, d\omega \).
(a) \[ \frac{1}{\sqrt{2\pi}} \int_{-7}^{\infty} (e^{x} - 7)e^{-x^2/2} \, dx \]

(b) \[ \frac{1}{\sqrt{2\pi}} \int_{\ln 7}^{\infty} (e^{x} - 7)e^{-x^2/2} \, dx \]

(c) \[ \frac{1}{\sqrt{2\pi}} \int_{7}^{\infty} (e^{x} - 7)e^{-x^2/2} \, dx \]

(d) none of the above

\[ Z := \Phi^{-1} : (0, 1) \to \mathbb{R} \]

Compute
\[ \int_{0}^{1} (e^{Z(\omega)} - 7) + \, d\omega. \]
(a) \[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (e^{-x} - e^{-7})e^{-x^2/2} \, dx \]

(b) \[ \frac{1}{\sqrt{2\pi}} \int_{7}^{\infty} (e^{-x} - e^{-7})e^{-x^2/2} \, dx \]

(c) \[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{7} (e^{-x} - e^{-7})e^{-x^2/2} \, dx \]

(d) none of the above

\[ Z := \Phi^{-1} : (0, 1) \to \mathbb{R} \]

Compute
\[ \int_{0}^{1} (e^{-Z(\omega)} - e^{-7}) + d\omega. \]
(a) \( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} \, dx \)

(b) \( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{3x+7} \, dx \)

(c) \( \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{3x+7} e^{-x^2/2} \, dx \)

(d) none of the above

\[ Y_1, Y_2, Y_3, \ldots \text{ iid, std} \]

\[ Z_n = \frac{(Y_1 + \cdots + Y_n)}{\sqrt{n}} \]

Compute \( \lim_{n \to \infty} \mathbb{E}[e^{3Z_n+7}] \).
\( Y_1, Y_2, Y_3, \ldots \) iid, std
\[
Z_n = \frac{Y_1 + \cdots + Y_n}{\sqrt{n}}
\]

Compute \( \lim_{n \to \infty} E[e^{3Z_n + 7}] \).

(a) 0

(b) 1

(c) \( e^{(3^2/2) + 7} \)

(d) none of the above
(a) 1

(b) 2

(c) 3

(d) none of the above

\[ Y_1, Y_2, Y_3, \ldots \text{ iid, std} \]

\[ Z_n = \frac{Y_1 + \cdots + Y_n}{\sqrt{n}} \]

Compute \( \lim_{n \to \infty} E[Z_n^3] \).
(a) $8!$

(b) $(2)(4)(6)(8)$

(c) $(1)(3)(5)(7)$

(d) none of the above

$Y_1, Y_2, Y_3, \ldots$ iid, std

$Z_n = (Y_1 + \cdots + Y_n)/\sqrt{n}$

Compute $\lim_{n \to \infty} E[Z_n^8]$. 

0 of 5
(a) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[ e^{3x+7} \right] \left[ e^{-x^2/2} \right] \, dx$

(b) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[ e^{-(3x+7)^2/2} \right] \, dx$

(c) $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[ 3e^{-x^2/2} + 7 \right] \, dx$

(d) none of the above

$Z$ standard normal

$E[e^{3Z+7}] = ??$
\( Z \text{ standard normal} \)

\[
E[Z^4 + \sin Z] = ??
\]

(a) \[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x^4 + \sin x)^2/2} \, dx
\]

(b) \[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (e^{-x^2/2})^4 + \sin(e^{-x^2/2}) \, dx
\]

(c) \[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [x^4 + \sin x] \left[ e^{-x^2/2} \right] \, dx
\]

(d) none of the above
(a) $\Phi(x)$

(b) $\Phi(x^3)$

(c) $\Phi(\sqrt[3]{x})$

(d) none of the above
(a) $\Phi(x)$

(b) $\Phi(x^3)$

(c) $\Phi(\sqrt[3]{x})$

(d) none of the above
(a) $\delta[Z]$

(b) $\delta[Z^3]$

(c) $\delta[Z^{1/3}]$

(d) none of the above

Correct answer: $\Phi(Z)$

$\text{Gr}[Z]$
(a) $\phi \left( Z^3 \right)$

(b) $\phi \left( \frac{1}{3} Z^3 \right)$

(c) $\phi \left( \frac{1}{3} \sqrt{Z} \right)$

(d) none of the above
(a) $7!$

(b) $7$

(c) $2^7$

(d) none of the above

A $\sigma$-alg. with 7 atoms has how many sets?
(a) \((12)!\)

(b) \(2^{12}\)

(c) 12

(d) none of the above

A \(\sigma\)-alg. with 12 atoms has how many sets?
(a) $\text{Var} \left[ \sum_{1}^{100} \mathcal{B}_{0.6, 4}^{0.4, 10} \right]$

(b) $(10)(0.4)(0.6)(10 - 4)^2$

(c) $(0.4)(0.6)(10 - 4)^2$

(d) none of the above
(a) \((400)(0.4)(0.6)(3 - 1)^2\)

(b) \((20)(0.4)(0.6)(3 - 1)^2\)

(c) \(\sqrt{20}(0.4)(0.6)(3 - 1)^2\)

(d) none of the above
(a) \( \sqrt{44}(0.7)(0.3)(9 - 2)^2 \)

(b) \( (44^2)(0.7)(0.3)(9 - 2)^2 \)

(c) \( (44)(0.7)(0.3)(9 - 2)^2 \)

(d) none of the above

\[ \text{Var}\left[\sum_{i=1}^{44} B_{0.3,2}^{0.7,9}\right] \]
(a) \((36)\sqrt{(0.4)(0.6)(10 - 4)}\)

(b) \(6\sqrt{(0.4)(0.6)(10 - 4)}\)

(c) \(\sqrt{(0.4)(0.6)(10 - 4)}\)

(d) none of the above
(a) \((400)\sqrt{(0.4)(0.6)(3 - 1)}\)

(b) \((20)\sqrt{(0.4)(0.6)(3 - 1)}\)

(c) \(\sqrt{20}\sqrt{(0.4)(0.6)(3 - 1)}\)

(d) none of the above
(a) $\sqrt{15} \sqrt{(0.7)(0.3)(9 - 2)}$

(b) $(15^2) \sqrt{(0.7)(0.3)(9 - 2)}$

(c) $(15) \sqrt{(0.7)(0.3)(9 - 2)}$

(d) none of the above
(a) 0
(b) 1
(c) 2
(d) none of the above

\[ I := [4, 7) \]
\[ \delta_7(I) = ? \]
$I := [4, 7]$

$\delta_7(I) = ?$

(a) 0

(b) 1

(c) 2

(d) none of the above
\[ I := (2, 4) \]
\[ \delta_5(I) = ? \]

(a) 0
(b) 1
(c) 2
(d) none of the above
(a) 0

(b) 1

(c) 2

(d) none of the above
\[ \lambda := \text{Lebesgue measure} \]

\[ \int_{0}^{1} 2x \, d\lambda(x) \]

(a) 0

(b) 1

(c) 2

(d) none of the above
\[ \int_0^3 2x \, d\delta_1(x) \]

(a) 0
(b) 1
(c) 2
(d) none of the above
\[ \lambda := \text{Lebesgue measure} \]

\[ \int_0^2 2x \ d\lambda(x) \]

(a) 0
(b) 1
(c) 2
(d) none of the above
(a) 0
(b) 1
(c) 2
(d) none of the above

\[ \int_{-3}^{3} 2x \, d\delta_0(x) \]
Fourier transform of $\delta_8$

(a) 0

(b) $e^{8it}$

(c) $e^{-8it}$

(d) none of the above
Fourier transform of $\delta_4$

(a) $e^{-4it}$

(b) $e^{4it}$

(c) 0

(d) none of the above
\[ h(x) = e^{-x^2/2} / \sqrt{2\pi} \]
\[ \lambda := \text{Lebesgue measure} \]

Fourier transform of \( h\lambda \)

(a) \( \int_{-\infty}^{\infty} h(x) \, dx \)
(b) \( \int_{-\infty}^{\infty} [h(x)][e^{itx}] \, dx \)
(c) 0
(d) none of the above

\[ \int_{-\infty}^{\infty} [h(x)][e^{-itx}] \, dx = e^{-t^2/2} \]
\[ f(x) = \frac{1}{x^2 + 1} \]
\[ \lambda := \text{Lebesgue measure} \]
\[ \text{Fourier transform of } f \lambda \]

(a) \[ \int_{-\infty}^{\infty} [f(x)][e^{-itx}] \, dx \]

(b) 0

(c) \[ \int_{-\infty}^{\infty} [f(x)][e^{itx}] \, dx \]

(d) none of the above
\( \mu \coloneqq \text{std normal distr. on } \mathbb{R} \)

\[
\int_{2}^{3} x^2 \, d\mu(x) = ??
\]

(a) \([1 \pm 1 = 0] \) of 5

(b) \[\int_{2}^{3} x^2 e^{-x^2/2} \, dx\]

(c) 0

(d) none of the above
\[ \mu := \text{std normal distr. on } \mathbb{R} \]
\[ \int_{-\infty}^{\infty} (\cos x) \, d\mu(x) = ?? \]

(a) \[ \int_{-\infty}^{\infty} (\cos x) e^{-x^2/2} \, dx \]

(b) 0

(c) \[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\cos x) \, dx \]

(d) none of the above

\[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\cos x) e^{-x^2/2} \, dx \]
\[ \mu := \text{std normal distr. on } \mathbb{R} \]

\[ \int_{-\infty}^{\infty} (\sin x) \, d\mu(x) = ?? \]

(a) \[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sin(e^{-x^2/2})) \, dx \]

(b) 0

(c) \[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sin x) \, dx \]

(d) none of the above
CDF = \Phi(2x - 4)

PDF?

(a) \ e^{-(2x-4)^2/2}
(b) \ [e^{-(2x-4)^2/2}]^2
(c) \ [e^{-(2x-4)^2/2}/\sqrt{2\pi}]^2
(d) none of the above
CDF = \Phi(x^3 + x) \\
PDF? \\

(a) \ e^{-(x^3+x)^2/2} \\
(b) \ [e^{-(x^3+x)^2/2}[3x^2 + 1] \\
(c) \ [e^{-(x^3+x)^2/2}/\sqrt{2\pi}][3x^2 + 1] \\
(d) \ none \ of \ the \ above
(a) \[ \frac{1}{1 + x^2} \]

\[ \lambda := \text{Leb msr on } \mathbb{R} \]

\[ f(x) = \frac{1}{1 + x^2} \]

\[ \text{CDF}_{\lambda}(x) = ? \]

(b) \[ \int_{-\infty}^{x} \frac{dt}{1 + t^2} \]

(c) \( \Phi(x) \)

(d) none of the above
(a) \( \frac{1}{1 + x^2} \)

(b) \( \int_{-\infty}^{x} \frac{dt}{1 + t^2} \)

(c) \( \Phi(x) \)

(d) none of the above

\[ \lambda := \text{Leb mssr on } \mathbb{R} \]
\[ h(x) = e^{-x^2/2}/\sqrt{2\pi} \]
\[ \text{CDF}_{h\lambda}(x) = ? \]
(a) \( e^{1/2} \)

(b) \( \int_{-\infty}^{x} e^t \, dt \)

(c) \( \int_{-\infty}^{x} e^t e^{-t^2} \, dt \)

(d) none of the above

\[ \lambda := \text{Leb msr on } \mathbb{R} \]

\[ f(x) = e^x e^{-x^2} \]

\[ \text{CDF}_{f \lambda}(x) = ? \]
\( Z \) standard normal
\[ \mu := \delta_{4Z+5} \]
\[ \text{CDF}_{\mu}(x) = ? \]

(a) \( \Phi(4x + 5) \)

(b) \( \Phi((x - 5)/4) \)

(c) \( \Phi((x/4) - 5) \)

(d) none of the above
\( Z \) standard normal
\[ \mu := \delta_{Z^3 - 4} \]
\[ \text{CDF}_{\mu}(x) = ? \]

(a) \( \Phi(x^3 - 4) \)
(b) \( \Phi(x^{1/3} + 4) \)
(c) \( \Phi((x + 4)^{1/3}) \)
(d) none of the above
\( Z \text{ standard normal} \)
\[ \mu := \delta_{Z^2 - 4} \]
\[ \text{CDF}_{\mu}(x) = ? \]

(a) \( \Phi(x^2 - 4) \)

(b) \( \Phi(x^{1/2} + 4) \)

(c) \( \Phi((x + 4)^{1/2}) \)

(d) none of the above

\[ [\Phi((x + 4)^{1/2})] - [\Phi(-(x + 4)^{1/2})] \]
$g(x) = e^{3x}$

\[ \int_{2}^{9} (\cos x) \, dg(x) \]

(a) $\int_{2}^{9} \cos e^{3x} \, dx$

(b) $\int_{2}^{9} (\cos x)(e^{3x})(3) \, dx$

(c) $\int_{2}^{9} e^{3} \cos(x) \, dx$

(d) none of the above
\[ g(x) = \begin{cases} 
0, & \text{if } x < 4 \\
3, & \text{if } x \geq 4 
\end{cases} \]

\[ \int_{2}^{9} (\cos x) \, dg(x) \]

(a) \( 3 \cos(4) \)

(b) \(-3 \cos(4) \)

(c) \(-4 \cos(3) \)

(d) none of the above
\[ g(x) = \begin{cases} 
3, & \text{if } x < 4 \\
0, & \text{if } x \geq 4 
\end{cases} \]

\[ \int_2^9 (\cos x) \, dg(x) \]

(a) \(3 \cos(4)\)

(b) \(-3 \cos(4)\)

(c) \(-4 \cos(3)\)

(d) none of the above
\[ g(x) = \begin{cases} 
  x^2 + 3, & \text{if } x < 4 \\
  0, & \text{if } x \geq 4 
\end{cases} \]

\[ \int_{2}^{9} (\cos x) \, dg(x) \]

(a) \[ \int_{2}^{9} (\cos x)(2x) \, dx \]

(b) \[ \int_{2}^{4} (\cos x)(2x) \, dx \]

(c) \[ \left[ \int_{2}^{4} (\cos x)(2x) \, dx \right] - [3 \cos(4)] \]

(d) none of the above

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Topic 2800
$X$ and $Y$ independent, standard

$\text{Cov}[2X + 4Y, 3X] = ??$

(a) 18

(b) 12

(c) 6

(d) none of the above
Given that $X$ and $Y$ are independent, standard normal variables, we know that $\text{Cov}(X, Y) = 0$.

For the given problem, we need to find $\text{Cov}[2X + 4Y, 3Y]$.

Using the properties of covariance,

$$\text{Cov}[2X + 4Y, 3Y] = 2 \cdot 3 \cdot \text{Cov}(X, Y) + 4 \cdot 3 \cdot \text{Cov}(Y, Y)$$

Since $\text{Cov}(X, Y) = 0$ and $\text{Var}(Y) = 1$,

$$\text{Cov}[2X + 4Y, 3Y] = 2 \cdot 3 \cdot 0 + 4 \cdot 3 \cdot 1 = 12$$

Thus, the answer is (b) $12$. 

(d) none of the above
$X$ and $Y$ independent, standard

$\text{Var}[2X + 4Y] = ??$

(a) 18
(b) 12
(c) 6
(d) none of the above

Correct answer: 20
Financial Mathematics
Regular review session, Midterm 02
Discussion:
How do you multiply a measure by a function? What is “functoriality” of push-forward?

Discussion:
\[ f \text{ continuous on } [a, b], \]
\[ g \text{ piecewise differentiable on } [a, b] \]
How to compute \[ \int_{a}^{b} f \, dg \]?

Discussion:
Supports of prob msrs \( \mu_n \) decrease to \( \{x\} \).

How to compute \( \lim_{n \to \infty} \int_{a}^{b} f \, d\mu_n \), for \( f \in C_B \)?

How to compute \( \lim_{n \to \infty} \mu_n \)?
Discussion:

Borel sets in $\mathbb{R}$.
Measurable sets in $\mathbb{R}$.
Borel sets in $\mathbb{R}^n$.
Measurable sets in $\mathbb{R}^n$.
Def’n of Borel space.
Def’n of measure space.
Borel sets in a Borel space.
Measurable sets in a Borel space.
Borel sets in a measure space.
Measurable sets in a measure space.
Borel functions.
Measurable functions.
Def’n of isomorphism of Borel spaces.
Def’n of isomorphism of measure spaces.
Discussion:
completion of a $\sigma$-algebra w.r.t. a measure.
completion of a measure.
extension theorem 1. extension theorem 2.
\textbf{non-example} of extending ctbly additive from
a collection of sets to the gen’d $\sigma$-alg.

two Borel spaces that are \textbf{not} isomorphic.
two msr spaces that are \textbf{not} isomorphic.
two prob. spaces that are \textbf{not} isomorphic.
Discussion:
Example of a set that's **not** measurable.
Example of a msbl set that's **not** Borel.
Example of a Borel set that's **not** msbl.
Example of a fn that's **not** measurable.
Example of a measurable fn that's **not** Borel.
Example of a Borel fn that's **not** measurable.

Def'n of the Lebesgue integral, \( \int_M f \, d\mu \).

\[ f \text{ simple. } \quad f \geq 0. \quad f \text{ general.} \]

Same as Riemann integral? Differences?

Riemann: \( M = [a, b], f \) continuous.

\[ \int_M \lim_{n \to \infty} f_n \, d\mu = \lim_{n \to \infty} \int_M f_n \, d\mu ? \]
Discussion:
Fourier transform of a (probability) msr.
Convergence of measures
iff convergence of Fourier transforms.
Equality of measures
iff equality of Fourier transforms.
Push forward of measure.
Product of function and measure.
CDF of a measure on $\mathbb{R}$. Always continuous?
Left continuous? Right continuous?
Def’n of integrable or $L^1$.
Def’n of square integrable or $L^2$.
Def’n of $L^p$. Def’n of $L^\infty$.
$L^1 \Rightarrow L^2$?
$L^2 \Rightarrow L^1$?
Fubini’s theorem.
Discussion:
Radon-Nikodym derivative
PDF of a measure on $\mathbb{R}$
CDF of a measure on $\mathbb{R}$
Which functions are CDFs?
connection between CDF and PDF
convergence of measures on $\mathbb{R}$
Discussion:

Def’n of $dg$, for $g : [a, b] \to \mathbb{R}$ nondecreasing.

Def’n of bounded variation, for $v : [a, b] \to \mathbb{R}$.

Def’n of $dv$, for $v : [a, b] \to \mathbb{R}$ of bdd var.

Def’n of $E[X]$.

Def’n of $Var[X]$.

Def’n of $Cov[X, Y]$.

Def’n of independent.

\[
\delta_X := \ldots.
\]

\[
\delta_f(X) := \ldots.
\]

\[
\delta_{X+Y} := \ldots.
\]

Def’n of $E[X|S]$. 
Discussion:
Lebesgue measure on $\mathbb{R}$. measurable rectangle. product measure. (Borel-theoretic) atom.
Finite $\sigma$-algebras $\leftrightarrow$ partitions.
$$\text{size of } \sigma\text{-alg} = 2^{\text{size of part'n}}.$$ Finite generating set implies finite $\sigma$-algebra. $X, Y$ uncorrelated does NOT imply $f(X), g(Y)$ uncorrelated.

$X, Y$ independent implies $f(X), g(Y)$ independent, which implies $f(X), g(Y)$ uncorrelated.
Discussion:
convergence a.s.
convergence in probability
convergence in distribution

Which implies which?

\[ X_n \quad \Rightarrow \quad A = B \]
Discussion:
grade of a RV
distribution of the grade
copula of a family of RVs
marginals of the copula