Financial Mathematics
Pricing/hedging in many subperiods
Part 2
\[ f(S) = (5000S - 5000) + \]

Goal: \textit{approximately}

\[ \text{Compute the expected value of } f(e^{Hu+Td}). \]
Then multiply by \( e^{-rN} = (e^r)^{-N} \).

\textbf{Coin-flipping game:} Flip a fair coin \( N \) times.
If \( H \) heads and \( T \) tails,
pay \( f(e^{Hu+Td}) \),
30 days from now.

\[ e^r = 1.000000001 \]
\[ N = 2,592,000 \]

\[ e^{rN}V = \text{expected payout} =: E = ??? \]
\[ V = e^{-rN}E \]

= discounted expected payout
\[ f(S) = (5000S - 5000) + \]

Goal: \( \text{approximately} \) \( E \)
Compute the expected value of \( f(e^{Hu + Td}) \).

Easier problem:
Compute the expected value of \( f(D_2) \).

\[ D_2 = H_2 - T_2 : \]

\[
\begin{array}{c|ccc}
   & 0.25 & 0.5 & 0.25 \\
\hline
2 & f(2) & f(0) & f(-2) \\
0 & & & \\
-2 & & & \\
\end{array}
\]

\[ [0.25][f(2)] + [0.5][f(0)] + [0.25][f(-2)] = 1,250 \]
Define: \[ g(S) = 5e^S + S^2 \]

Easier problem:
Compute the expected value of \( g(D_2) \).

\[ D_2 = H_2 - T_2 \]

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<tr>
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| 0.25 | -2 | \( g(-2) \) |

\[
[0.25][g(2)] + [0.5][g(0)] + [0.25][g(-2)] = \text{Exercise}
\]
Recall: \[ f(S) = (5000S - 5000)_+ \]

Goal: \( E \) approximately Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem: Compute the expected value of \( f(Z) \).

\[ Z: \quad \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx \]

\[ \int_{-\infty}^{\infty} \left[ f(x) \right] \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [f(x)] e^{-x^2/2} \, dx = \text{exercise} \]

\[ f(x) = (5000x - 5000)_+ \]
Recall: \( f(S) = (5000S - 5000)_+ \)

Goal: \( \text{approximately} \quad E \quad \) Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem: Compute the expected value of \( f(Z) \).

\[ Z: \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} \, dx \]  

\[ E[f(Z)] \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x'^2/2} dx' - \frac{x^2}{2} \, dx \]

Do this for all \( x \in \mathbb{R} \).
Recall: \( f(S) = (5000S - 5000)_+ \)

Goal: \( \overbrace{\text{approximately}}^E \) Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem:
Compute the expected value of \( f(Z) \).

\[ Z: \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} \, dx \quad x \quad f(x) \quad \text{Do this for all } x \in \mathbb{R} \]

works for any exp-bdd function \( f : \rightarrow g \)

\[ E[f(Z)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [f(x)] e^{-x^2/2} \, dx \]
Recall: \[ f(S) = (5000S - 5000)_{+} \]

Goal: \[ \text{approximately} \quad E \]

Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem:
Compute the expected value of \( g(Z) \).

\[ Z: \quad \int_{x}^{g(x)} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx \]

Do this for \( \text{all } x \in \mathbb{R} \)

works for any exp-bdd function \( g \)

temporary change of color...

\[ E[g(Z)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx \]
Recall: \( f(S) = (5000S - 5000)_+ \)

Goal: \( \underset{\text{approximately}}{\underbrace{\mathbb{E}}} \) Compute the expected value of \( f(e^{Hu + Td}) \).

New easier problem:
Compute the expected value of \( g(Z) \).

\[
Z: \quad \frac{1}{\sqrt{2\pi}} \int e^{-x^2/2} \, dx \bigg|_{x \to g(x)} \quad \text{Do this for all } x \in \mathbb{R}
\]

works for any exp-bdd function \( g \)

temporary change of color...

\[
E[g(Z)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \( f(S) = (5000S - 5000) \)

Goal: \( \text{approximately } E \) Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem:
Compute the expected value of \( g(Z) \).

\[
Z: \quad \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \quad \left| \begin{array}{c} x \quad g(x) \\ 0 \quad \infty \end{array} \right.
\]

Do this for all \( x \in \mathbb{R} \)

Change every \( Z \) to \( x \) and then integrate against \( h(x) \) \( dx \), from \( -\infty \) to \( \infty \).

\[
h(x) := \frac{e^{-x^2/2}}{\sqrt{2\pi}}
\]

works for any exp-bdd function \( g \)

\[
E[g(Z)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \[ f(S) = (5000S - 5000)_{+} \]

Goal: \( E \) approximately Compute the expected value of \( f(e^{Hu +Td}) \).

New easier problem:
Compute the expected value of \( g(Z) \).

\[ Z: \quad \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} \, dx \]

Do this for all \( x \in \mathbb{R} \)

works for any exp-bdd function \( g \)

\[ E[g(Z)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx \]
Recall: $f(S) = (5000S - 5000)_+$

Goal: \( \underbrace{\text{approximately}}_{E} \) Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem:
Compute approximately
the expected value of \( g(X) \).

\[
\begin{align*}
Z: & \\
\frac{1}{\sqrt{2\pi}} & \int e^{-x^2/2} \, dx \quad | \quad x \quad g(x) \\
X & \\
E[g(X)] \approx & \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\end{align*}
\]

Do this for all \( x \in \mathbb{R} \)
works for any exp-bdd function \( g \)
Recall: \( f(S) = (5000S - 5000)_+ \)

Goal: \( \underbrace{\text{approximately}}_E \) Compute the expected value of \( f(e^{Hu + Td}) \).

New easier problem: Subgoal: Choose \( g \) s.t.: \( \| \)
Compute \( \text{approximately} \) the expected value of \( g(X) \).

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: $f(s) = (5000s - 5000) +$ write $H,T$ as expr.s of $X$ 

Goal: 
Compute the expected value of $f(H_x + T_d)$.

Subgoal: Choose $g$ s.t. $g$ exp-bdd?

Compute the expected problem:
Compute the expected value of $g(X)$.

Approximately

$E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx$
Recall: \( f(S) = (5000S - 5000) \)

Goal: \( \text{approximately} \quad E \quad \text{Compute the expected value of } f(e^{Hx} + Td) \).

New easier problem: \( \text{Compute the expected value of } g(X) \).

\[
X = \frac{(H - T)}{\sqrt{N}} \times \sqrt{N} \\
H + T = N \\
H - T = X \sqrt{N} \\
2H = N + X \sqrt{N} \\
2T = N - X \sqrt{N}
\]

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \( f(S) = (5000S - 5000) + \)

Goal: \( E \) approximately Compute the expected value of \( f(e^{Hu +Td}) \).

New easier problem: Compute \( E \) approximately the expected value of \( g(X) \).

\[
H = \frac{[N + X\sqrt{N}]}{2} \quad T = \frac{[N - X\sqrt{N}]}{2} \tag{16}
\]

\[
N := 30 \times 24 \times 60 \times 60 = 2,592,000
\]

\[
2H = N + X\sqrt{N} \quad 2T = N - X\sqrt{N}
\]

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \( f(S) = (5000S - 5000) \)

Goal: \( \text{approximately } E \)

Compute the expected value of \( f(e^{Hu+Td}) \).

New easier problem:

Compute the expected value of \( g(X) \).

\[
H = \frac{[N + X\sqrt{N}]}{2} \quad \text{and} \quad T = \frac{[N - X\sqrt{N}]}{2}
\]

\[
Hu = \frac{[Nu + X\sqrt{N}u]}{2}, \quad Td = \frac{[Nd - X\sqrt{N}d]}{2}
\]

\[
Hu + Td = \frac{[N(u + d) + X\sqrt{N}(u - d)]}{2}
\]

\[
= \frac{[N(u + d)/2] + [X\sqrt{N}(u - d)/2]}{2}
\]

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \[ f(S) = (5000S - 5000) \]

Goal: compute the expected value of \( f(e^{Hu +Td}) \).

New easier problem: compute the expected value of \( g(X) \).

\[
H = [N + X \sqrt{N}] / 2 \\
T = [N - X \sqrt{N}] / 2
\]

\[
e^{Hu +Td} = [e^{N(u+d)/2}][e^{X \sqrt{N}(u-d)/2}] = [e^{N(u+d)/2}][e^{(\sqrt{N}(u-d)/2)X}]
\]

\[
Hu +Td = [N(u + d)/2] + [X \sqrt{N}(u - d)/2]
\]

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \( f(S) = (5000S - 5000) \)

Goal: approximately \( E \) Compute the expected value of \( f(e^{H \mu + T \delta}) \).

New easier problem: approximately Compute the expected value of \( g(X) \).

\[
H = \frac{[N + X\sqrt{N}]}{2} \quad T = \frac{[N - X\sqrt{N}]}{2}
\]

\[
e^{H \mu + T \delta} = \left[ e^{N(u + d)/2} \right] \left[ e^{\left( \sqrt{N}(u - d)/2 \right)X} \right] = Ce^{kX}
\]

\[
Hu + Td = \frac{[N(u + d)]}{2} + \frac{[X\sqrt{N}(u - d)]}{2}
\]

\[
E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \( f(S) = (5000S - 5000) \)

Goal: approximately Compute the expected value of \( f(e^{H u + T d}) \).

Restatement of goal: approximately Compute the expected value of \( g(X) \).

\[
H = \frac{[N + X \sqrt{N}]/2}{T = \frac{[N - X \sqrt{N}]/2}{
\]

\[
e^{H u + T d} = \left[e^{N(u+d)/2}\right]\left[e^{(\sqrt{N}(u-d)/2)X}\right] = Ce^{kX}
\]

\[
g(x) := f(Ce^{kx}) \quad g \exp-bdd? \quad \text{YES}
\]

\[
f(e^{H u + T d}) = f(Ce^{kX}) = g(X)
\]

\[
E = E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]

write \( H, T \) as expr.s of \( X \)
Recall: \( f(S) = (5000S - 5000)_+ \)

Goal: \( E \) approximately Compute the expected value of \( f(e^{Hu+Td}) \).

Restatement of goal: Compute approximately the expected value of \( g(X) \).

\[
N = 2,592,000 \quad u = 0.00003561536577 \\
u = 0.00003561463419 \\
d = -0.0573390439012 \\
1.00094857729 \approx C \\
\]

\[
e^{Hu+Td} = \left[ e^{N(u+d)/2} \right] \left[ e^{(\sqrt{N}(u-d)/2)} X \right] = Ce^{kX} \\

\]

\[
g(x) := f(Ce^{kx}) \\
f(e^{Hu+Td}) = f(Ce^{kX}) = g(X) \\
\]

\[
E = E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx \\
\]
Recall: \[ f(S) = (5000S - 5000)_+ \]
\[ = 5000(S - 1)_+ \]

\[ g(x) := f(Ce^{kx}) = 5000(Ce^{kx} - 1)_+ \]

\[ E \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 5000(Ce^{kx} - 1)_+ e^{-x^2/2} \, dx \]

\[ 1.00094857729 = C \]

\[ k = 0.0573390439012 \]

\[ g(x) := f(Ce^{kx}) \]

\[ E = E[g(X)] \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx \]
\[ E \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 5000(Ce^{kx} - 1) + e^{-x^2/2} \, dx \]

\[ = \frac{5000}{\sqrt{2\pi}} \left( \int_{-\infty}^{\infty} e^{kx} \, dx - 1 \right) + \int_{-\infty}^{\infty} e^{-x^2/2} \, dx + e^{-x^2/2} \, dx \]

\[ 1.00094857729 = C \]

\[ k = 0.0573390439012 \]
\[ E \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 5000(Ce^{kx} - 1) + e^{-x^2/2} \, dx \]

\[ = \frac{5000}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Ce^{kx} - 1) + e^{-x^2/2} \, dx \]

\[ = \frac{5000}{\sqrt{2\pi}} \int_{a}^{\infty} (Ce^{kx} - 1) e^{-x^2/2} \, dx \]

---

\[
\begin{align*}
Ce^{ka} - 1 &= 0 \\
Ce^{ka} &= 1 \\
e^{ka} &= 1/C \\
k &= \ln(1/C) = -\ln C \\
a &= -(\ln C)/k \\
C &= 0.0573390439012 \\
k &= 1.00094857729
\end{align*}
\]
\[ E \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 5000(e^{kx} - 1) + e^{-x^2/2} \, dx \]

\[ = \frac{5000}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Ce^{kx} - 1) + e^{-x^2/2} \, dx \]

\[ = \frac{5000}{\sqrt{2\pi}} \int_{a}^{\infty} (Ce^{kx} - 1) e^{-x^2/2} \, dx \]

\[ k = 0.0573390439012, \quad a = -0.0165354585751 \]

1.00094857729 = C
\[ E \approx \frac{5000}{\sqrt{2\pi}} \left[ C \int_a^\infty e^{kx} e^{-x^2/2} \, dx - \sqrt{2\pi} \Phi(-a) \right] \]
\[ E \approx \frac{5000}{\sqrt{2\pi}} C \left[ \int_{a}^{\infty} e^{kx} e^{-x^2/2} \, dx - \int_{a}^{\infty} e^{-x^2/2} \, dx \right] \]

\[ \int_{a-k}^{\infty} e^{k(x+k)} e^{-(x+k)^2/2} \, dx \]

\[ e^{k^2/2} \int_{a-k}^{\infty} e^{-x^2/2} \, dx \]

\[ \sqrt{2\pi} \Phi(k-a) \]

1.00094857729 = C  \quad k = 0.0573390439012  \quad a = -0.0165354585751
\[
E \approx \frac{5000}{\sqrt{2\pi}} \left[ C \int_a^\infty e^{kx} e^{-x^2/2} \, dx - \int_a^\infty e^{-x^2/2} \, dx \right] \\
\int_a^\infty e^{k(x+k)} e^{-(x+k)^2/2} \, dx \\
\int_a^{a-k} e^{k^2} e^{-x^2/2} e^{-k^2/2} \\
\sqrt{2\pi} \Phi(k-a)
\]

\[
C = 1.00094857729 \\
k = 0.0573390439012 \\
a = -0.0165354585751
\]
\[ E \approx \frac{5000}{\sqrt{2\pi}} \left[ C \int_a^{\infty} e^{kx} e^{-x^2/2} \, dx - \int_a^{\infty} e^{-x^2/2} \, dx \right] \]

\[ e^{k^2/2} \sqrt{2\pi} \Phi(k - a) \]

\[ C = 1.00094857729 \]

\[ k = 0.0573390439012 \]

\[ a = -0.0165354585751 \]
\[ E \approx \frac{5000}{\sqrt{2\pi}} \left[ C \int_{a}^{\infty} e^{kx} e^{-x^2/2} \, dx \right] - \int_{a}^{\infty} e^{-x^2/2} \, dx \]

\[ = 5000 \left[ C e^{k^2/2} \left( \Phi(k-a) \right) \right] - \left( \Phi(-a) \right) \]

\[ = 121.07046876 \]

\[ C = 1.00094857729 \]

\[ k = 0.0573390439012 \]

\[ a = -0.0165354585751 \]
Coin-flipping game: Flip a fair coin $N$ times. If $H$ heads and $T$ tails, pay $f(u^H d^T)$, 30 days from now.

$$e^r = 1.000000001$$
$$N = 2,592,000$$
$$E \approx$$

$e^r N V = \text{expected payout} =: E = ???$ apprx.

$V = e^{-rN} E$
$$= \text{discounted expected payout}$$

$V = e^{-rN} E \approx 120.757060394$

$1.000000001 \equiv e^r$

$e^{-rN} = 0.997411356336$

exact answer? soon...

$E \approx 121.07046876$

$N := 30 \times 24 \times 60 \times 60 = 2,592,000$
Scenarios with $j$ upticks and $N-j$ downticks:

There are $\binom{N}{j}$ of them.

Each has (risk-neutral) probability: $(0.5)^j(0.5)^{N-j}$

$$\text{prob. } j, N-j = \binom{N}{j} (0.5)^j(0.5)^{N-j}$$

$$V = e^{-rN} E \approx 120.757060394$$

$$e^{-rN} = 0.997411356336$$

$$E \approx 121.07046876$$

$N := 30 \times 24 \times 60 \times 60 = 2,592,000$
Scenarios with \( j \) upticks and \( N - j \) downticks:

\[
f(S) = 5000(S - 1)^+ \quad \text{ends at } ju + (N - j)d
\]

stock price ends at \( e^{ju + (N-j)d} \)

option pays \( 5000(e^{ju + (N-j)d} - 1)^+ \)

\[
\text{prob. } j, N-j = \binom{N}{j} (0.5)^j (0.5)^{N-j}
\]

ending value of hedge

\[
V = e^{-rN} E \approx 120.757060394
\]

\[
e^{-rN} = 0.997411356336
\]

exact answer?

\[
E \approx 121.07046876
\]

\[
N := 30 \times 24 \times 60 \times 60 = 2,592,000
\]
Scenarios with $j$ upticks and $N-j$ downticks:

$E := \text{(risk-neutral) expected ending value of hedge}$

to compute it, **multiply** this by this,
then **add** over $j = 0, \ldots, N$.

Option pays

$$5000(e^{ju} + (N-j)d - 1)$$

Ending value of hedge

V = e^{-rN}E \approx 120.757060394

$$1.000000001 \approx e^r$$

**Exact Answer?**

$E \approx 121.07046876$

$e^{-rN} = 0.997411356336$

$N := 30 \times 24 \times 60 \times 60 = 2,592,000$
Scenarios with $j$ upticks and $N-j$ downticks:

$$E := \text{(risk-neutral) expected ending value of hedge}$$

To compute it, **multiply** this by this, then **add** over $j = 0, \ldots, N$.

Option pays:

$$\text{prob. } j, N-j = \binom{N}{j} (0.5)^j (0.5)^{N-j}$$

$$5000(e^{ju} + (N-j)d - 1)$$

Ending value of hedge:

$$V = e^{-rN} E \approx 120.757060394$$

**Exact answer:**

$$E = \sum_{j=0}^{N} \binom{N}{j} [(0.5)^j (0.5)^{N-j}] 5000(e^{ju} + (N-j)d - 1)$$

Exact answer?

$$E \approx 121.07046876$$

$$N := 30 \times 24 \times 60 \times 60 = 2,592,000$$
\[ V = e^{-r^N}E \approx 120.757060394 \]

Exact answer:

\[
E = \sum_{j=0}^{N} \binom{N}{j} \left[ (0.5)^j (0.5)^{N-j} \right] 5000(e^{ju} + (N-j)d - 1) + e^{-r^N} \approx 121.07046876
\]

\[ N := 30 \times 24 \times 60 \times 60 = 2,592,000 \]
Another Central Limit Theorem application:

Recall: \( S_0 = 1 \) is the initial price of the stock.

Let \( S_1 \) denote the price after one year.

Exercise: Compute \( \mathbb{E}[S_1] \), approximately.

\[
V = e^{-rN}E \approx 120.757060394
\]

Exact answer:

\[
E = \sum_{j=0}^{N} \binom{N}{j} \left[ (0.5)^j (0.5)^{N-j} \right] 5000(e^{ju} + (N-j)d - 1) + e^{-rN} = 0.997411356336
\]

\[
E \approx 121.07046876
\]

\[
V = e^{-rN}E = 120.7994402
\]
Market analyst: annual vol = 0.200002881086
annual drift = 0.03399864624

Recall: $S_0 = 1$ is the initial price of the stock.
Let $S_1$ denote the price after one year.

Exercise: Compute $\mathbb{E}[S_1]$, approximately.

\[
V = e^{-rN}E \approx 120.757060394
\]

Exact answer:

\[
E = \sum_{j=0}^{N} \binom{N}{j} [(0.5)^j (0.5)^{N-j}] 5000(e^{ju}+(N-j)d - 1) + e^{-rN} = 0.997411356336
\]

\[
E \approx 121.07046876
\]

\[
V = e^{-rN}E = 120.7994402
\]
Market analyst: annual vol = 0.200002881086  
annual drift = 0.03399864624

Recall: \( S_0 = 1 \) is the initial price of the stock. 
Let \( S_1 \) denote the price after one year. 
Exercise: Compute \( E[S_1] \), approximately.

Solution: By the Central Limit Theorem, 
\( \ln S_1 \), being a large sum of iid PCRVs, 
is approximately normal.

Then exponentiation nearly almost commutes with expectation. 
That is, \( E[e^{\ln S_1}] \approx e^{AE[\ln S_1]} \).

That is, \( E[S_1] \approx e^{E[\ln S_1] + \frac{1}{2} \text{Var}[\ln S_1]} \).
Market analyst: annual vol = 0.200002881086
annual drift = 0.03399864624

Recall: $S_0 = 1$ is the initial price of the stock.
Let $S_1$ denote the price after one year.

Exercise: Compute $E[S_1]$, approximately.

Solution:

$$E[S_1] \approx e^{E[\ln S_1] + \frac{1}{2}\text{Var}[\ln S_1]}$$

$$= e^{(0.03399864624) + \frac{1}{2}(0.200002881086)^2}$$
Market analyst: annual vol = 0.200002881086
annual drift = 0.03399864624

Recall: $S_0 = 1$ is the initial price of the stock.
Let $S_1$ denote the price after one year.
Exercise: Compute $E[S_1]$, approximately.
Solution:

$$E[S_1] \approx e^{E[\ln S_1] + \frac{1}{2} \text{Var}[\ln S_1]}$$

$$= e^{(0.03399864624) + \frac{1}{2}(0.200002881086)^2}$$

$$= 1.05548378145$$

Expected annual return is about 5.5%.
Annual drift is 0.03399864624.
Annual augmented drift is, by definition,

$$\left(0.03399864624\right) + \frac{1}{2}(0.200002881086)^2$$

annual risk-free factor $= e$ annual augmented drift