SOLUTIONS TO PRACTICE TEST 3

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1. If S is a plane in Euclidean 3-space containing $(0,0,0)$, $(2,0,0)$ and $(0,0,1)$, then S is the

(A) xy-plane
(B) xz-plane
(C) yz-plane
(D) plane $y-z=0$
(E) plane $x+2y-2z=0$

Solution: The xy-plane is $z=0$ which does not contain $(0,0,1)$, so (A) is not correct. The yz-plane is $x=0$ which does not contain $(2,0,0)$, so (C) is not correct.

The plane $y-z=0$ is $(0,1,-1)\cdot(x,y,z)=0$, and we have

$$(0,1,-1) \quad \cdot \quad (0,0,1) \quad = \quad -1 \quad \neq \quad 0,$$

so (D) is not correct. The plane $x+2y-2z=0$ is $(1,2,-2)\cdot(x,y,z)=0$, and we have

$$(1,2,-2) \quad \cdot \quad (0,0,1) \quad = \quad -2 \quad \neq \quad 0,$$

so (E) is not correct.

The xz-plane is $y=0$ and this contains all three of the points $(0,0,0)$, $(2,0,0)$ and $(0,0,1)$. Answer: (B) □

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2. If a and b are real numbers, which of the following are necessarily true?

I. If $a < b$ and $ab \neq 0$, then $\dfrac{1}{a} > \dfrac{1}{b}$.
II. If $a < b$, then $ac < bc$, for all real numbers c.
III. If $a < b$, then $a + c < b + c$, for all real numbers c.
IV. If $a < b$, then $-a > -b$.

Choose one of these answers:

(A) I only
(B) I and III only
(C) III and IV only
(D) II, III and IV only
(E) I,II,III and IV

Solution: We see that I is false using $a = -3$ and $b = 2$.

We see that II is false using $a = 2$, $b = 3$ and $c = -1$.

On the other hand, III and IV are both basic facts about the real number system; they are both true. Answer: (C) □

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3. $\displaystyle\int_0^1 \int_0^x xy\,dy\,dx =$

(A) 0
(B) 1/8
(C) 1/3
(D) 1
(E) 3

Solution: We compute $\displaystyle\int_0^x xy\,dy = [x(y^2/2)]_{y:\to 0}^{y:\to x} = [x(x^2/2)] - 0 = x^3/2$.

Then $\displaystyle\int_0^1 \int_0^x xy\,dy\,dx = \int_0^1 (x^3/2)\,dx = [x^4/8]_{x:\to 0}^{x:\to 1} = [1/8] - 0 = 1/8$.

Answer: (B) □

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4. For $x \geq 0$, $\dfrac{d}{dx}\left(x^e \cdot e^x\right) =$
   (A) $x^e \cdot e^x + x^{e-1} \cdot e^{x+1}$
   (B) $x^e \cdot e^x + x^{e+1} \cdot e^{x-1}$
   (C) $x^e \cdot e^x$
   (D) $x^{e-1} \cdot e^{x+1}$
   (E) $x^{e+1} \cdot e^{x-1}$

Solution: Using the Product Rule,

$$\frac{d}{dx}\left(x^e \cdot e^x\right) = \left(ex^{e-1}\right)\left(e^x\right) + \left(x^e\right)\left(e^x\right) = x^{e-1} \cdot e^{x+1} + x^e \cdot e^x,$$

so $\dfrac{d}{dx}\left(x^e \cdot e^x\right) = x^e \cdot e^x + x^{e-1} \cdot e^{x+1}$. Answer: (A) □

5. All functions f defined on the xy-plane such that

$$\frac{\partial}{\partial x}[f(x,y)] = 2x + y \qquad \text{and} \qquad \frac{\partial}{\partial y}[f(x,y)] = x + 2y$$

are given by $f(x,y) =$

(A) $x^2 + xy + y^2 + C$
(B) $x^2 - xy + y^2 + C$
(C) $x^2 - xy - y^2 + C$
(D) $x^2 + 2xy + y^2 + C$
(E) $x^2 - 2xy + y^2 + C$

Solution: As $\dfrac{\partial}{\partial x}[f(x,y)] = 2x + y$ and $\dfrac{\partial}{\partial y}[f(x,y)] = x + 2y$, we see that there are functions $g, h : \mathbb{R} \to \mathbb{R}$ such that

$$\begin{aligned} f(x,y) &= x^2 + xy + [g(y)] \qquad \text{and} \\ f(x,y) &= xy + y^2 + [h(x)]. \end{aligned}$$

Subtracting, we see that

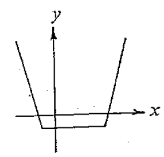
$$0 \quad = \quad [f(x,y)] - [f(x,y)] \quad = \quad x^2 + [g(y)] - y^2 - [h(x)],$$

so $[h(x)] - x^2 = [g(y)] - y^2$. Setting $x :\to 0$, we see that

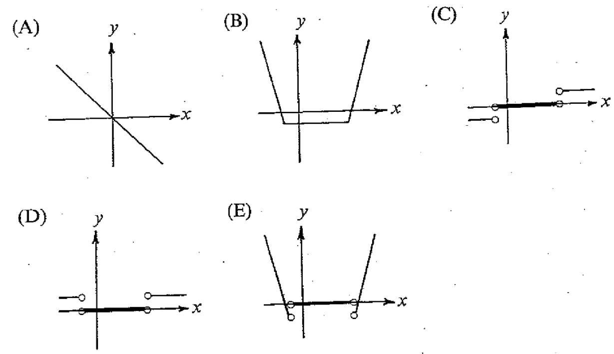
$$[h(0)] - 0^2 \quad = \quad [g(y)] - y^2.$$

Let $C := h(0)$. Then $C = [h(0)] - 0^2 = [g(y)] - y^2$, so $g(y) = y^2 + C$, so $f(x,y) = x^2 + xy + [g(y)] = x^2 + xy + y^2 + C$. Answer: (A) □

Alternate solution: Simply computing $\partial/\partial x$ and $\partial/\partial y$ of each of the answers (A)-(E) shows that only (A) works. Answer: (A) □



6. Which of the following could be the graph of the derivative of the function whose graph is shown in the figure above?



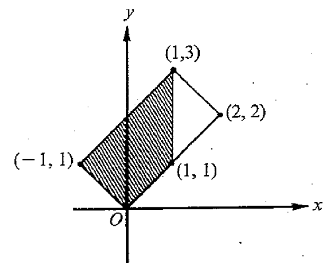
Solution: The graph consists, piecewise, of

- a negatively sloped half-line on the left,
- a horizontal line segment in the middle and
- a positively sloped half-line on the right.

The derivative will be, piecewise,

- a negative constant on the left,
- a zero constant in the middle and
- a positive constant on the right.

Answer: (C) □



7. Which of the following integrals represents the area of the shaded portion of the rectangle shown in the figure above?

(A) $\int_{-1}^{1} (x + 2 - |x|)\, dx$ (B) $\int_{-1}^{1} (|x| + x + 2)\, dx$ (C) $\int_{-1}^{1} (x + 2)\, dx$

(D) $\int_{-1}^{1} |x|\, dx$ (E) $\int_{-1}^{1} 2\, dx$

Solution: The line through $(-1, 1)$ and $(1, 3)$ is $y = x + 2$. The two line segments

- from $(-1, 1)$ to the origin and
- from the origin to $(1, 1)$

are both on the graph of $y = |x|$. Thus we need to compute the area of the region

between $y = x + 2$ and $y = |x|$

from $x = -1$ to $x = 1$. This is $\displaystyle\int_{-1}^{1} (x + 2 - |x|)\, dx$. Answer: (A) □

8. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1} =$

(A) $\frac{1}{e}$
(B) $\log 2$
(C) 1
(D) e
(E) $+\infty$

Solution: We have $\frac{n}{n+1} = \frac{n/n}{(n+1)/n} = \frac{1}{1+(1/n)} \to \frac{1}{1+0} = 1 > 0$, as $n \to \infty$. Thus the terms of the series tend toward a positive number, and it follows that the series tends toward $+\infty$. Answer: (E) $\square$