PRINT YOUR NAME:

PRINT YOUR TA’S NAME:

WHAT SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:
I. Multiple choice

A. (5 pts) (no partial credit) Find the logarithmic derivative of \(x^2 + 3x - 8\) w.r.t. \(x\).

(a) \(\frac{2x + 3}{x^2 + 3x - 8}\)

(b) \(\frac{x^2 + 3x - 8}{2x + 3}\)

(c) \((\ln(x^2)) + 3(\ln x) - (\ln 8)\)

(d) \(\ln(2x + 3)\)

(e) NONE OF THE ABOVE

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B. (5 pts) (no partial credit) Compute \(\lim_{x \to 0} \left[ \frac{\sin^2 x}{4x^3 + 2x^2} \right]\).

(a) 2

(b) 1

(c) 1/2

(d) 1/4

(e) NONE OF THE ABOVE

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C. (5 pts) (no partial credit) Suppose \(f'(x) = -x^2 + 3x - 2\). At most one of the following statements is true. If one is, circle it. Otherwise, circle “NONE OF THE ABOVE”.

(a) \(f\) is increasing on \((−\infty, 1]\), decreasing on \([1, 2]\) and increasing on \([2, \infty)\).

(b) \(f\) is increasing on \((−\infty, −2]\), decreasing on \([-2, −1]\) and increasing on \([-1, \infty)\).

(c) \(f\) is decreasing on \((−\infty, 1]\), increasing on \([1, 2]\) and decreasing on \([2, \infty)\).

(d) \(f\) is decreasing on \((−\infty, −2]\), increasing on \([-2, −1]\) and decreasing on \([-1, \infty)\).

(e) NONE OF THE ABOVE
D. (5 pts) (no partial credit) Find the slope of the tangent line to \( y = (x^3 + 4)e^{2x} \) at the point \((0, 4)\).

(a) 2
(b) 4
(c) 6
(d) 8
(e) NONE OF THE ABOVE

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E. (5 pts) (no partial credit) Find the logarithmic derivative of \((2 + \sin x)^x\) w.r.t. \(x\).

(a) \(\ln(\cos x)\)
(b) \(\cos x\)
(c) \([(2 + \sin x)^x] \left[ \ln(2 + \sin x) + \left( \frac{x \cos x}{2 + \sin x} \right) \right]\)
(d) \(\ln(2 + \sin x) + \left( \frac{x \cos x}{2 + \sin x} \right)\)
(e) NONE OF THE ABOVE

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F. (5 pts) (no partial credit) Find the derivative of \((2 + \sin x)^x\) w.r.t. \(x\).

(a) \(\ln(\cos x)\)
(b) \(\cos x\)
(c) \([(2 + \sin x)^x] \left[ \ln(2 + \sin x) + \left( \frac{x \cos x}{2 + \sin x} \right) \right]\)
(d) \(\ln(2 + \sin x) + \left( \frac{x \cos x}{2 + \sin x} \right)\)
(e) NONE OF THE ABOVE
II. True or false (no partial credit):

a. (5 pts) If $f' > 0$ on an interval $I$, then $f$ is increasing on $I$.

b. (5 pts) If $f'(3) = 0$ and $f''(3) > 0$, then $f$ has a local maximum at 3.

c. (5 pts) If $f$ and $g$ are differentiable, then $\frac{d}{dx}[(f(x))(g(x))] = [f'(x)][g'(x)]$.

d. (5 pts) Every global extremum occurs at a critical number.

e. (5 pts) Every local extremum occurs at a critical number.
III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Compute \( \frac{d}{dx} \left[ 2x^3 - 8 \arctan x + xe^{\sin x} \right] \)
2. (10 pts) Using implicit differentiation (and logarithmic differentiation), find \( y' = \frac{dy}{dx} \), assuming that \((2 + y^2)x^y = 0\).
3. (5 pts) Suppose $f$ is 1-1 and $g = f^{-1}$ is the inverse of $f$. Suppose $f(3) = 4$ and $f'(3) = 91$. Compute $g(4)$ and $g'(4)$.

4. (10 pts) Find the maximal intervals of increase and decrease for $f(x) = x^3 - 6x^2 + 5$. 
5. (10 pts) Among all pairs of positive numbers \(x\) and \(y\) such that \(xy = 100\), find the
global maximum value of \(x + 4y\), provided it exists. Then find the global minimum value,
provided it exists. (NOTE: If the global maximum value does not exist, you need to state
that clearly to receive full credit. If it does exist, for full credit, you’ll need to compute
\(x + 4y\); computing \(x\) and/or \(y\) alone is insufficient. These same comments apply to the
global minimum value.)