MATH 1271 Fall 2012, Midterm #1
Handout date: Thursday 4 October 2012

PRINT YOUR NAME:

SOLUTIONS
Version A

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Compute $\lim_{x\to 0}\left[\frac{3x^4+2x^3}{7x(\sin^2 x)}\right]$. Circle one of the following answers:

$\underbrace{}_{? \, x\to 0}$

$$\frac{2x^3}{7x(x^2)} \overset{x\neq 0}{=} \frac{2}{7} \xrightarrow{x\to 0} \frac{2}{7}$$

(a) 0

(b) ∞

(c) 5/7

(d) 2/7

(e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) Compute $\lim_{x\to -\infty}\left[\frac{\sqrt{16x^6-x}}{16x^3+x}\right]$. Circle one of the following answers:

$\underbrace{}_{? \, x\to -\infty}$

$$\frac{\sqrt{16x^6}}{16x^3} \overset{x<0}{=} \frac{-4\cancel{x^3}}{16\cancel{x^3}} = -\frac{1}{4}$$

$$\Big\downarrow x\to -\infty$$

$$-\frac{1}{4}$$

(a) 1/4

(b) $-1/4$

(c) 1/2

(d) $-1/2$

(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Which is the intuitive definition of $\lim_{x\to 3}(g(x)) = 8$? Circle one of the following answers:

(a) If $g(x)$ is close to 3, then x is close to 8.

(b) If x is close to 3, but not equal to 3, then $g(x)$ is close to 8, but not equal to 8.

(c) If $g(x)$ is close to 8, but not equal to 8, then x is close to 3.

(d) If x is close to 3, but not equal to 3, then $g(x)$ is close to 8.

(e) NONE OF THE ABOVE

D. (5 pts) (no partial credit) Compute $\lim_{t\to 3}\left[\underbrace{\frac{t^2+t-12}{t-3}}_{\parallel}\right]$. Circle one of the following answers:

(a) 3

(b) 4

(c) 5

(d) 6

(e) NONE OF THE ABOVE

$$\frac{(t-3)(t+4)}{t-3} \overset{t\neq 3}{=} t+4 \xrightarrow{t\to 3} 7$$

E. (5 pts) (no partial credit) Compute $\lim_{x\to 0}\left[\underbrace{\frac{x^3+2x^2-4x}{\sin(8x)}}_{\wr\, x\to 0}\right]$ Circle one of the following answers:

(a) 2/3

(b) −1/2

(c) 1/2

(d) −2/3

(e) NONE OF THE ABOVE

$$\frac{-4x}{8x} \overset{x\neq 0}{=} \frac{-4}{8} = -\frac{1}{2} \xrightarrow{x\to 0} -\frac{1}{2}$$

F. (5 pts) (no partial credit) Compute $\lim_{h\to 0}\left[\underbrace{\frac{\sqrt{9+h}-\sqrt{9+4h}}{3h}}_{\parallel}\right]$. Circle one of the following answers:

(a) 1/6

(b) −1/6

(c) 1/9

(d) This limit does not exist.

(e) NONE OF THE ABOVE

$$\underbrace{\frac{\cancel{9}+h-\cancel{9}-4h}{3h}}_{\parallel} \cdot \underbrace{\frac{1}{\sqrt{9+h}+\sqrt{9+4h}}}_{\downarrow h\to 0}$$

$$\frac{-3h}{3h} \overset{h\neq 0}{=} -1 \qquad \frac{1}{\sqrt{9}+\sqrt{9}} = \frac{1}{6}$$

$$\boxed{\text{ANSWER: } (-1)(1/6) = -1/6}$$

II. True or false (no partial credit):

a. (5 pts) For every $x < 0$, $\sqrt{x^4} = -x^2$.

False

b. (5 pts) Let $f(x) = x^3$. Then f is a one-to-one function.

True

c. (5 pts) Let $f(x) = |x|$. Then f is continuous at every real number.

True

d. (5 pts) If a function f is continuous at a number a, then f is differentiable at a.

False

e. (5 pts) Let $f(x) = |x|$. Then the domains of f and of f' are equal.

False

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION A

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Find all horizontal asymptotes to

$$y = \underbrace{\frac{\sqrt{9x^4+2x+5}}{2x^2-3}}_{=: f(x)}$$

(NOTE: A horizontal asymptote is a line; your answers should be equations of lines, **NOT** numbers.)

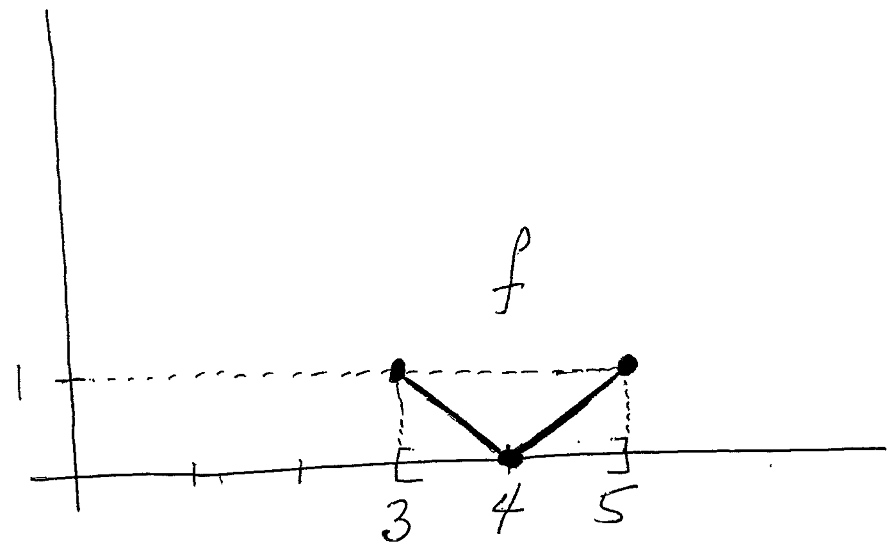
$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{\sqrt{9x^4}}{2x^2}$$

$$= \lim_{x \to \pm\infty} \frac{3\cancel{x^2}}{2\cancel{x^2}} = \frac{3}{2}$$

$\therefore$ $y = \frac{3}{2}$ is the only horizontal asymptote.

2. (15 pts) Draw a single graph showing a function $f : [3,5] \to \mathbb{R}$ with *all* of the following properties:

- (•) Its domain is the interval $[3,5]$.
- (•) It is continuous on $[3,5]$.
- (•) It is differentiable on $(3,4)$ and on $(4,5)$.
- (•) For all $x \in (3,4)$, we have: $f'(x) = -1$.
- (•) For all $x \in (4,5)$, we have: $f'(x) = 1$.
- (•) It is not differentiable at 4.
- (•) $f(4) = 0$.



3. (10 pts) Compute $\lim\limits_{x\to\infty} \underbrace{\left[\dfrac{x^2+\sin^2 x}{2x^2+1}\right]}_{\shortparallel \atop f(x)}$.

$$\left[\begin{array}{c} 1 \\ \geq \\ \sin^2 x \\ \geq \\ 0 \end{array}\right] \text{ so: } \left[\begin{array}{l} \dfrac{x^2+1}{2x^2+1} \overset{x\to\infty}{\sim} \dfrac{x^2}{2x^2} \overset{x\neq 0}{=\!=} \dfrac{1}{2} \xrightarrow{x\to\infty} \dfrac{1}{2} \\ \quad \geq \\ f(x) \\ \quad \geq \\ \dfrac{x^2+0}{2x^2+1} \overset{x\to\infty}{\sim} \dfrac{x^2}{2x^2} \overset{x\neq 0}{=\!=} \dfrac{1}{2} \xrightarrow{x\to\infty} \dfrac{1}{2} \end{array}\right.$$

By the Squeeze Theorem,

$$\lim_{x\to\infty} f(x) = \frac{1}{2}$$

4. (10 pts) Let $f(x) = (x+1)^3(x-2)^4(x-5)$. Find all of the maximum intervals of positivity and negativity for f.

$f(x)$	pos	0^3	neg	0^4	neg	0	pos
x		-1		2		5	

f is pos. on $(-\infty, -1)$

neg. on $(-1, 2)$

neg. on $(2, 5)$

pos. on $(5, \infty)$