

MATH 1271 Fall 2012, Midterm #1
Handout date: Thursday 4 October 2012

PRINT YOUR NAME:

SOLUTIONS
Version B

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Compute $\lim_{x \rightarrow 0} \left[\frac{x^3 + 2x^2 - 4x}{\sin(8x)} \right]$ Circle one of the following answers:

(a) $2/3$

(b) $-2/3$

(c) $-1/2$

(d) $1/2$

(e) NONE OF THE ABOVE

$$\lim_{x \rightarrow 0} \frac{x^3 + 2x^2 - 4x}{\sin(8x)}$$

$$\frac{-4x}{8x} \quad \underline{x \neq 0} \quad \frac{-4}{8} = -\frac{1}{2}$$

$$\downarrow x \rightarrow 0$$

$$-\frac{1}{2}$$

B. (5 pts) (no partial credit) Compute $\lim_{t \rightarrow 3} \left[\frac{t^2 + t - 12}{t - 3} \right]$. Circle one of the following answers:

(a) 3

(b) 4

(c) 5

(d) 6

(e) NONE OF THE ABOVE

$$\lim_{t \rightarrow 3} \frac{t^2 + t - 12}{t - 3}$$

$$\frac{(t-3)(t+4)}{t-3} \quad \underline{t \neq 3} \quad t+4$$

$$\downarrow t \rightarrow 3$$

$$7$$

C. (5 pts) (no partial credit) Compute $\lim_{x \rightarrow 0} \left[\frac{3x^4 + 2x^3}{7x(\sin^2 x)} \right]$. Circle one of the following answers:

(a) $2/7$

(b) 0

(c) $5/7$

(d) ∞

(e) NONE OF THE ABOVE

$$\lim_{x \rightarrow 0} \frac{3x^4 + 2x^3}{7x(\sin^2 x)}$$

$$\frac{2x^3}{7x(x^2)} \quad \underline{x \neq 0} \quad \frac{2}{7} \xrightarrow{x \rightarrow 0} \frac{2}{7}$$

D. (5 pts) (no partial credit) Compute $\lim_{x \rightarrow -\infty} \left[\frac{\sqrt{16x^6 - x}}{16x^3 + x} \right]$. Circle one of the following answers:

(a) 1/2

(b) -1/2

(c) 1/4

(d) -1/4

(e) NONE OF THE ABOVE

$$\begin{aligned} & \lim_{x \rightarrow -\infty} \frac{\sqrt{16x^6 - x}}{16x^3 + x} \\ & \stackrel{x < 0}{=} \frac{-4x^3}{16x^3} = -\frac{1}{4} \end{aligned}$$

$\downarrow x \rightarrow -\infty$
 $-\frac{1}{4}$

E. (5 pts) (no partial credit) Compute $\lim_{h \rightarrow 0} \left[\frac{\sqrt{9+h} - \sqrt{9+4h}}{3h} \right]$. Circle one of the following answers:

(a) 1/6

(b) -1/6

(c) 1/9

(d) This limit does not exist.

(e) NONE OF THE ABOVE

$$\begin{aligned} & \frac{\sqrt{9+h} - \sqrt{9+4h}}{3h} \cdot \frac{1}{\sqrt{9+h} + \sqrt{9+4h}} \\ & \stackrel{h \rightarrow 0}{=} \frac{-3h}{3h} \cdot \frac{1}{\sqrt{9} + \sqrt{9}} = \frac{-1}{6} \end{aligned}$$

$\downarrow h \neq 0$
ANSWER: $(-1)(1/6) = -1/6$

F. (5 pts) (no partial credit) Which is the intuitive definition of $\lim_{x \rightarrow 3} (g(x)) = 8$? Circle one of the following answers:

(a) If $g(x)$ is close to 3, then x is close to 8.

(b) If x is close to 3, but not equal to 3, then $g(x)$ is close to 8, but not equal to 8.

(c) If x is close to 3, but not equal to 3, then $g(x)$ is close to 8.

(d) If $g(x)$ is close to 8, but not equal to 8, then x is close to 3.

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) Let $f(x) = |x|$. Then f is continuous at every real number.

True

b. (5 pts) If a function f is continuous at a number a , then f is differentiable at a .

False

c. (5 pts) Let $f(x) = x^3$. Then f is a one-to-one function.

True

d. (5 pts) Let $f(x) = |x|$. Then the domains of f and of f' are equal.

False

e. (5 pts) For every $x < 0$, $\sqrt{x^6} = -x^3$.

True

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION B

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Find all horizontal asymptotes to

$$y = \frac{\sqrt{4x^4 + 2x + 5}}{5x^2 - 3}.$$

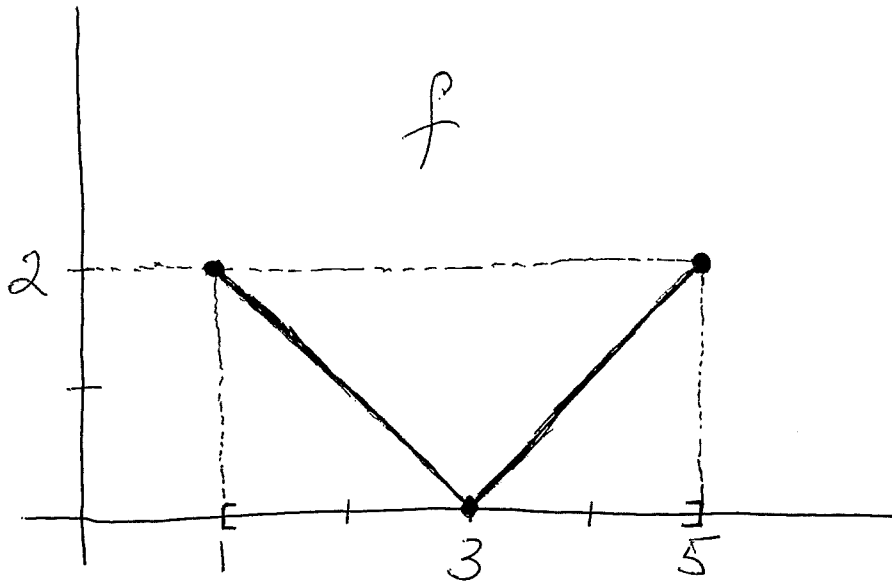
(NOTE: A horizontal asymptote is a line; your answers should be equations of lines, **NOT** numbers.)

$$\begin{aligned} \lim_{x \rightarrow \pm\infty} f(x) &= \lim_{x \rightarrow \pm\infty} \frac{\sqrt{4x^4}}{5x^2} \\ &= \lim_{x \rightarrow \pm\infty} \frac{2x^2}{5x^2} = \frac{2}{5} \end{aligned}$$

$y = \frac{2}{5}$ is the only horizontal asymptote.

2. (15 pts) Draw a single graph showing a function $f : [1, 5] \rightarrow \mathbb{R}$ with *all* of the following properties:

- (•) Its domain is the interval $[1, 5]$.
- (•) It is continuous on $[1, 5]$.
- (•) It is differentiable on $(1, 3)$ and on $(3, 5)$.
- (•) For all $x \in (1, 3)$, we have: $f'(x) = -1$.
- (•) For all $x \in (3, 5)$, we have: $f'(x) = 1$.
- (•) It is not differentiable at 3.
- (•) $f(3) = 0$.



3. (10 pts) Compute $\lim_{x \rightarrow \infty} \underbrace{\left[\frac{x^2 + \sin^2 x}{2x^2 + 1} \right]}_{f(x)}$.

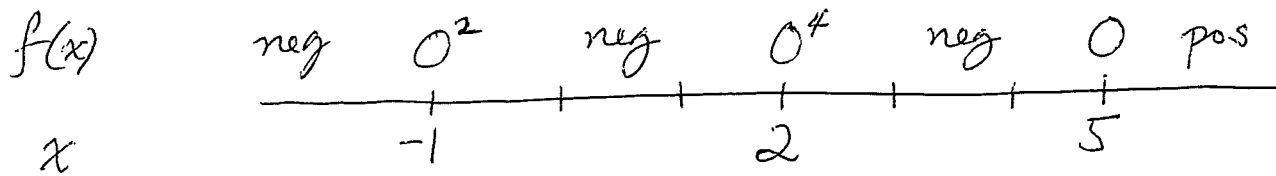
so:

$$\begin{array}{|c|} \hline 1 \\ \hline \forall \\ \hline \sin^2 x \\ \hline \forall \\ \hline 0 \\ \hline \end{array} \quad \left[\begin{array}{l} \frac{x^2 + 1}{2x^2 + 1} \underset{x \rightarrow \infty}{\sim} \frac{x^2}{2x^2} \underset{x \neq 0}{=} \frac{1}{2} \xrightarrow{x \rightarrow \infty} \frac{1}{2} \\ \forall \\ f(x) \\ \forall \\ \frac{x^2 + 0}{2x^2 + 1} \underset{x \rightarrow \infty}{\sim} \frac{x^2}{2x^2} \underset{x \neq 0}{=} \frac{1}{2} \xrightarrow{x \rightarrow \infty} \frac{1}{2} \end{array} \right]$$

By the Squeeze Theorem,

$$\lim_{x \rightarrow \infty} f(x) = \frac{1}{2}$$

4. (10 pts) Let $f(x) = (x + 1)^2(x - 2)^4(x - 5)$. Find all of the maximum intervals of positivity and negativity for f .



f is neg. on $(-\infty, -1)$
neg. on $(-1, 2)$
neg. on $(2, 5)$
pos. on $(5, \infty)$