MATH 1271 Fall 2012, Midterm #2
Handout date: Thursday 8 November 2012

PRINT YOUR NAME:
SOLUTIONS
Version A

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Find the logarithmic derivative of $(2+\sin(2x))^{\cos x}$ w.r.t. x. Circle one of the following answers:

$\frac{d}{dx}\left[(\cos x)(\ln(2+\sin(2x)))\right]$

(a) $(-\sin x)\left(\frac{2\cos(2x)}{2+\sin(2x)}\right)$

(b) $(\cos x)(\ln(2+\sin(2x))) + (-\sin x)\left(\frac{2\cos(2x)}{2+\sin(2x)}\right)$

(c) $(\cos x)(\ln(2+\sin(2x)))$

(d) $(-\sin x)(\ln(2+\sin(2x))) + (\cos x)\left(\frac{2\cos(2x)}{2+\sin(2x)}\right)$

(e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) Find the derivative of $(2+\sin(2x))^{\cos x}$ w.r.t. x. Circle one of the following answers:

(a) $[(2+\sin(2x))^{\cos x}]\left[(-\sin x)\left(\frac{2\cos(2x)}{2+\sin(2x)}\right)\right]$

(b) $[(2+\sin(2x))^{\cos x}]\left[(\cos x)(\ln(2+\sin(2x))) + (-\sin x)\left(\frac{2\cos(2x)}{2+\sin(2x)}\right)\right]$

(c) $[(2+\sin(2x))^{\cos x}][(\cos x)(\ln(2+\sin(2x)))]$

(d) $[(2+\sin(2x))^{\cos x}]\left[(-\sin x)(\ln(2+\sin(2x))) + (\cos x)\left(\frac{2\cos(2x)}{2+\sin(2x)}\right)\right]$

(e) NONE OF THE ABOVE

$-(x^2-4x+3) = -(x-1)(x-3)$

C. (5 pts) (no partial credit) Suppose $f''(x) = -x^2+4x-3$. At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

(a) f is concave down on $(-\infty, 1]$, up on $[1, 3]$ and down on $[3, \infty)$.

(b) f is concave up on $(-\infty, 1]$, down on $[1, 3]$ and up on $[3, \infty)$.

(c) f is concave down on $(-\infty, -3]$, up on $[-3, -1]$ and down on $[-1, \infty)$.

(d) f is concave up on $(-\infty, -3]$, down on $[-3, -1]$ and up on $[-1, \infty)$.

(e) NONE OF THE ABOVE

f'' neg 0 pos 0 neg

1 3

D. (5 pts) (no partial credit) Suppose $f'(x) = (x-1)^2(x-2)(x-3)^2$. Which of the following is a maximal interval of increase for f? Circle one of the following answers:

(a) $(-\infty, 1]$

(b) $[1, \infty)$

(c) $(2, \infty)$

(d) $[2, \infty)$

(e) NONE OF THE ABOVE

f' neg 0^2 neg 0 pos 0^2 pos

1 2 3

E. (5 pts) (no partial credit) Compute $[d/dx][\sin^2(xy)]$. Circle one of the following answers:

(a) $2[\sin(xy)][\cos(xy)]$

(b) $[\cos^2(xy)][y + xy']$

(c) $2[\sin(xy)][y + xy']$

(d) $2[\sin(xy)][\cos(y + xy')]$

(e) NONE OF THE ABOVE

$= 2[\sin(xy)][\cos(xy)][y + xy']$

F. (5 pts) (no partial credit) Compute the derivative of $\ln(x^{\arctan x})$, with respect to x, on the interval $x > 0$. Circle one of the following answers:

$= (\arctan x)(\ln x)$

(a) $\dfrac{1}{x^{\arctan x}}$

(b) $\dfrac{\ln x}{1+x^2} + \dfrac{\arctan x}{x}$

(c) $\dfrac{1}{x^{\sec^2 x}}$

(d) $x^{\sec^2 x}$

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) If f and g are differentiable at a number a, then $fg + f + g$ is differentiable at a.

True

b. (5 pts) If f is increasing on an interval I, then $f' > 0$ on I.

False

c. (5 pts) If $f' > 0$ on an interval I, then f is increasing on I.

True

d. (5 pts) Assume that $\lim_{x \to 3}[f(x)] = 0 = \lim_{x \to 3}[g(x)]$. Assume also that $\lim_{x \to 3} \frac{f'(x)}{g'(x)} = 7$. Then $\lim_{x \to 3} \frac{f(x)}{g(x)} = 7$.

True

e. (5 pts) Assume that $\lim_{x \to 0}[f(x)] = 0 = \lim_{x \to 0}[g(x)]$. Assume also that $\lim_{x \to 0} \left[\frac{f'(x)}{g'(x)}\right]$ does not exist. Then $\lim_{x \to 0} \left[\frac{f(x)}{g(x)}\right]$ does not exist.

False

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION A

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1,2.

III. 3.

III. 4.

III. 5.

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (5 pts) Compute $\dfrac{d}{dx}\left[\dfrac{e^{x^4}-8}{5+\sec(x^2)}\right]$. (Here e^{x^4} means $e^{(x^4)}$.)

$$\|$$

$$\frac{[5+\sec(x^2)][4x^3e^{x^4}]-[e^{x^4}-8][\sec(x^2)][\tan(x^2)][2x]}{[5+\sec(x^2)]^2}$$

2. (5 pts) Compute $\dfrac{d}{dx}\left[(5-\sin x)^{7\arctan x}\right]$.

$$\|$$

$$\left[(5-\sin x)^{7\arctan x}\right]\left[\frac{d}{dx}\left[(7\arctan x)(\ln(5-\sin x))\right]\right]$$

$$\|$$

$$\left[(5-\sin x)^{7\arctan x}\right]\left[\left(\frac{7}{1+x^2}\right)(\ln(5-\sin x))+(7\arctan x)\left(\frac{-\cos x}{5-\sin x}\right)\right]$$

3. (10 pts) Find an equation for the tangent line to $x^3 + xy + y^3 = 11$ at $(2, 1)$.

$$3x^2 + y + xy' + 3y^2y' = 0$$

2 1

$$y' = \frac{-3x^2 - y}{x + 3y^2}$$

2 1

$$\text{slope} = \frac{-3 \cdot 4 - 1}{2 + 3 \cdot 1} = \frac{-13}{5}$$

eq'n: $y - 1 = -\frac{13}{5}(x - 2)$

4. (15 pts) Compute $\lim_{x\to 0}((\cos x)+(\sin x))^{5/x}$.

$$\| $$

$$e^{\lim_{x\to 0}(5/x)(\ln((\cos x)+(\sin x)))}$$

$$\| $$

$$e^{\lim_{x\to 0}\frac{5(\ln((\cos x)+(\sin x)))}{x}}$$

$$\| \text{L'H } \tfrac{0}{0}$$

$$e^{\lim_{x\to 0}\frac{5\left(\frac{-(\sin x)+(\cos x)}{(\cos x)+(\sin x)}\right)}{1}}$$

$$\| $$

$$e^{\frac{5\left(\frac{-0+1}{1+0}\right)}{1}}$$

$$\| $$

$$e^{5}$$

5. (10 pts) Find the global maximum and minimum value of $f(x) = x^3 - 3x^2 + 3x + 4$ on the interval $0 \le x \le 2$.

$$f'(x) = 3x^2 - 6x + 3$$
$$= 3(x^2 - 2x + 1)$$
$$= 3(x-1)^2$$

f' pos 0^2 pos

1

f is increasing on $\mathbb{R} = (-\infty, \infty)$

On $0 \le x \le 2$,

f attains global min at $x = 0$,
with global min value $f(0) = 4$, and

f attains global max at $x = 2$
with global max value $f(2) = 8 - 3 \cdot 4 + 6 + 4$
$= 8 - 12 + 10 = 6$