PRINT YOUR NAME:
SOLUTIONS
Version A

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:
I. Multiple choice

A. (5 pts) (no partial credit) Find the logarithmic derivative of \((2 + \sin(2x))^\cos x\) w.r.t. \(x\). Circle one of the following answers:

\[
\frac{d}{dx} \left[ \cos x \left( \ln (2 + \sin(2x)) \right) \right]
\]

(a) \((- \sin x) \left( \frac{2 \cos(2x)}{2 + \sin(2x)} \right)\)

(b) \((\cos x)(\ln(2 + \sin(2x))) + (- \sin x) \left( \frac{2 \cos(2x)}{2 + \sin(2x)} \right)\)

(c) \((\cos x)(\ln(2 + \sin(2x)))\)

(d) \((- \sin x)(\ln(2 + \sin(2x))) + (\cos x) \left( \frac{2 \cos(2x)}{2 + \sin(2x)} \right)\)

(e) NONE OF THE ABOVE


B. (5 pts) (no partial credit) Find the derivative of \((2 + \sin(2x))^\cos x\) w.r.t. \(x\). Circle one of the following answers:

\[
[(2 + \sin(2x))^\cos x] \left[ (- \sin x) \left( \frac{2 \cos(2x)}{2 + \sin(2x)} \right) \right]
\]

(a) \([(2 + \sin(2x))^\cos x] \left[ (- \sin x) \left( \frac{2 \cos(2x)}{2 + \sin(2x)} \right) \right]\)

(b) \([(2 + \sin(2x))^\cos x] \left[ (\cos x)(\ln(2 + \sin(2x))) + (- \sin x) \left( \frac{2 \cos(2x)}{2 + \sin(2x)} \right) \right]\)

(c) \([(2 + \sin(2x))^\cos x] \left[ (\cos x)(\ln(2 + \sin(2x))) \right]\)

(d) \([(2 + \sin(2x))^\cos x] \left[ (- \sin x)(\ln(2 + \sin(2x))) + (\cos x) \left( \frac{2 \cos(2x)}{2 + \sin(2x)} \right) \right]\)

(e) NONE OF THE ABOVE

\[-(x^2 - 4x + 3) = -(x-1)(x-3)\]

C. (5 pts) (no partial credit) Suppose \(f''(x) = -x^2 + 4x - 3\). At most one of the following statements is true. If one is, circle it. Otherwise, circle “NONE OF THE ABOVE”.

(a) \(f\) is concave down on \((-\infty, 1]\), up on \([1, 3]\) and down on \([3, \infty)\).

(b) \(f\) is concave up on \((-\infty, 1]\), down on \([1, 3]\) and up on \([3, \infty)\).

(c) \(f\) is concave down on \((-\infty, -3]\), up on \([-3, -1]\) and down on \([-1, \infty)\).

(d) \(f\) is concave up on \((-\infty, -3]\), down on \([-3, -1]\) and up on \([-1, \infty)\).

(e) NONE OF THE ABOVE

\[
\begin{array}{cccc}
f'' & \text{neg} & 0 & \text{pos} & 0 & \text{neg} \\
& 1 & 3 &
\end{array}
\]
D. (5 pts) (no partial credit) Suppose \( f'(x) = (x - 1)^2(x - 2)(x - 3)^2 \). Which of the following is a maximal interval of increase for \( f' \)? Circle one of the following answers:

(a) \((-\infty, 1]\)
(b) \([1, \infty)\)
(c) \((2, \infty)\)
(d) \([2, \infty)\)
(e) NONE OF THE ABOVE

\[
\begin{array}{cccccc}
\text{neg} & \text{neg} & \text{neg} & \text{pos} & \text{pos} & \text{pos} \\
1 & 2 & 3 \\
\end{array}
\]

E. (5 pts) (no partial credit) Compute \( \frac{d}{dx}[\sin^2(xy)] \). Circle one of the following answers:

(a) \(2[\sin(xy)][\cos(xy)]\)
(b) \([\cos^2(xy)][y + xy']\)
(c) \(2[\sin(xy)][y + xy']\)
(d) \(2[\sin(xy)][\cos(y + xy')]\)
(e) NONE OF THE ABOVE

\[
\frac{d}{dx}[\sin(xy)] = 2[\sin(xy)][\cos(xy)][y + xy']
\]

F. (5 pts) (no partial credit) Compute the derivative of \( \frac{\ln(x\arctan x)}{4} \), with respect to \( x \), on the interval \( x > 0 \). Circle one of the following answers:

(a) \(\frac{1}{x\arctan x}\)
(b) \(\ln x \frac{\arctan x}{1 + x^2} + \frac{\arctan x}{x}\)
(c) \(\frac{1}{x^2 \sec^2 x}\)
(d) \(x^2 \sec^2 x\)
(e) NONE OF THE ABOVE
II. True or false (no partial credit):

a. (5 pts) If $f$ and $g$ are differentiable at a number $a$, then $fg + f + g$ is differentiable at $a$.

True

b. (5 pts) If $f$ is increasing on an interval $I$, then $f' > 0$ on $I$.

False

c. (5 pts) If $f' > 0$ on an interval $I$, then $f$ is increasing on $I$.

True

d. (5 pts) Assume that $\lim_{x \to 3} [f(x)] = 0 = \lim_{x \to 3} [g(x)]$. Assume also that $\lim_{x \to 3} \frac{f'(x)}{g'(x)} = 7$. Then $\lim_{x \to 3} \frac{f(x)}{g(x)} = 7$.

True

e. (5 pts) Assume that $\lim_{x \to 0} [f(x)] = 0 = \lim_{x \to 0} [g(x)]$. Assume also that $\lim_{x \to 0} \left[ \frac{f'(x)}{g'(x)} \right]$ does not exist. Then $\lim_{x \to 0} \left[ \frac{f(x)}{g(x)} \right]$ does not exist.

False

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION A
I. A,B,C
II. D,E,F
II. a,b,c,d,e
III. 1,2.
III. 3.
III. 4.
III. 5.
III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (5 pts) Compute \( \frac{d}{dx} \left[ \frac{e^{x^4} - 8}{5 + \sec(x^2)} \right]. \) (Here \( e^{x^4} \) means \( e^\left(x^4\right) \).)

\[
\left(5 + \sec(x^2)\right)^2 \left[4x^3 e^{x^4} - \left(e^{x^4} - 8\right)\sec(x^2)\tan(x^2)\right]
\]

\[
\left(5 + \sec(x^2)\right)^2
\]

2. (5 pts) Compute \( \frac{d}{dx} \left[(5 - \sin x)^7 \arctan x\right]. \)

\[
\left[(5 - \sin x)^7 \arctan x\right] \left[7 \arctan x \ln(5 - \sin x) - \frac{1}{1 + x^2} \ln(5 - \sin x) + \frac{\arctan x}{5 - \sin x} \right]
\]
3. (10 pts) Find an equation for the tangent line to \( x^3 + xy + y^3 = 11 \) at \((2, 1)\).

\[
3x^2 + y + xy' + 3y^2y' = 0
\]

\[
y' = \frac{-3x^2 - y}{x + 3y^2}
\]

\[
\text{slope} = \frac{-3 \cdot 2^2 - 1}{2 + 3 \cdot 1} = \frac{-13}{5}
\]

eq'n: \[ y - 1 = -\frac{13}{5} (x - 2) \]
4. (15 pts) Compute \( \lim_{x \to 0} ((\cos x) + (\sin x))^{5/x} \).

\[
\lim_{x \to 0} (5/x) \ln ((\cos x) + (\sin x))
\]

\[
\lim_{x \to 0} \frac{5 \ln((\cos x) + (\sin x))}{x}
\]

\[
\lim_{x \to 0} 5 \left( \frac{-\sin x + \cos x}{\cos x + \sin x} \right)
\]

\[
\lim_{x \to 0} \frac{5(-\sin 0 + \cos 0)}{1}
\]

\[
\lim_{x \to 0} \frac{5}{1 + 0}
\]

\[
\lim_{x \to 0} e^5
\]
5. (10 pts) Find the global maximum and minimum value of \( f(x) = x^3 - 3x^2 + 3x + 4 \) on the interval \( 0 \leq x \leq 2 \).

\[
f'(x) = 3x^2 - 6x + 3 \\
= 3(x^2 - 2x + 1) \\
= 3(x - 1)^2
\]

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\( f \) is increasing on \( \mathbb{R} = (-\infty, \infty) \)

On \( 0 \leq x \leq 2 \),

\( f \) attains global min at \( x = 0 \)

with global min value \( f(0) = 4 \), and

\( f \) attains global max at \( x = 2 \)

with global max value \( f(2) = 8 - 3 \cdot 4 + 6 + 4 \)

\[= 8 - 12 + 10 = 6\]