MATH 1271 Fall 2012, Midterm #2
Handout date: Thursday 8 November 2012

PRINT YOUR NAME:

SOLUTIONS
Version B

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Compute $[d/dx][\sin^2(xy)]$. Circle one of the following answers:

(a) $2[\sin(xy)][\cos(xy)][y + xy']$

(b) $[\cos^2(xy)][y + xy']$

(c) $2[\sin(xy)][y + xy']$

(d) $2[\sin(xy)][\cos(y + xy')]$

(e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) Find the logarithmic derivative of $(2 + \sin(2x))^{\cos x}$ w.r.t. x. Circle one of the following answers:

(a) $(\cos x)(\ln(2 + \sin(2x))) + (-\sin x)\left(\dfrac{2\cos(2x)}{2 + \sin(2x)}\right)$

(b) $(-\sin x)(\ln(2 + \sin(2x))) + (\cos x)\left(\dfrac{2\cos(2x)}{2 + \sin(2x)}\right)$

(c) $(\cos x)(\ln(2 + \sin(2x)))$

(d) $(-\sin x)\left(\dfrac{2\cos(2x)}{2 + \sin(2x)}\right)$

$\frac{d}{dx}\left[(\cos x)(\ln(2+\sin(2x)))\right]$

(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Find the derivative of $(2 + \sin(2x))^{\cos x}$ w.r.t. x. Circle one of the following answers:

(a) $[(2 + \sin(2x))^{\cos x}]\left[(\cos x)(\ln(2 + \sin(2x))) + (-\sin x)\left(\dfrac{2\cos(2x)}{2 + \sin(2x)}\right)\right]$

(b) $[(2 + \sin(2x))^{\cos x}]\left[(-\sin x)(\ln(2 + \sin(2x))) + (\cos x)\left(\dfrac{2\cos(2x)}{2 + \sin(2x)}\right)\right]$

(c) $[(2 + \sin(2x))^{\cos x}][(\cos x)(\ln(2 + \sin(2x)))]$

(d) $[(2 + \sin(2x))^{\cos x}]\left[(-\sin x)\left(\dfrac{2\cos(2x)}{2 + \sin(2x)}\right)\right]$

(e) NONE OF THE ABOVE

D. (5 pts) (no partial credit) Suppose $f'(x) = (x-1)^2(x-2)(x-3)^2$. Which of the following is a maximal interval of increase for f? Circle one of the following answers:

(a) $[2, \infty)$

(b) $(2, \infty)$

(c) $[1, \infty)$

(d) $(-\infty, 1]$

(e) NONE OF THE ABOVE

f' neg 0^2 neg 0 pos 0^2 pos

1 2 3

E. (5 pts) (no partial credit) Compute the derivative of $\ln\left(x^{\arctan x}\right)$, with respect to x, on the interval $x > 0$. Circle one of the following answers:

(a) $\dfrac{x^{1/(1+x^2)}}{x^{\arctan x}}$

(b) $\dfrac{1}{x^{\arctan x}}$

(c) $\dfrac{1}{x^{\sec^2 x}}$

(d) $x^{\sec^2 x}$

(e) NONE OF THE ABOVE

$$\frac{d}{dx}\left[(\arctan x)(\ln x)\right]$$

$$\| $$

$$\frac{\ln x}{1+x^2} + \frac{\arctan x}{x}$$

$-(x^2+4x+3) = -(x+3)(x+1)$

F. (5 pts) (no partial credit) Suppose $f''(x) = -x^2 - 4x - 3$. At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

(a) f is concave up on $(-\infty, 1]$, down on $[1, 3]$ and up on $[3, \infty)$.

(b) f is concave down on $(-\infty, 1]$, up on $[1, 3]$ and down on $[3, \infty)$.

(c) f is concave up on $(-\infty, -3]$, down on $[-3, -1]$ and up on $[-1, \infty)$.

(d) f is concave down on $(-\infty, -3]$, up on $[-3, -1]$ and down on $[-1, \infty)$.

(e) NONE OF THE ABOVE

f'' neg 0 pos 0 neg

-3 -1

II. True or false (no partial credit):

a. (5 pts) Assume that $\lim_{x\to 0}[f(x)] = 0 = \lim_{x\to 0}[g(x)]$. Assume also that $\lim_{x\to 0}\left[\frac{f'(x)}{g'(x)}\right]$ does not exist. Then $\lim_{x\to 0}\left[\frac{f(x)}{g(x)}\right]$ does not exist.

False

b. (5 pts) Assume that $\lim_{x\to 3}[f(x)] = 0 = \lim_{x\to 3}[g(x)]$. Assume also that $\lim_{x\to 3}\frac{f'(x)}{g'(x)} = 7$. Then $\lim_{x\to 3}\frac{f(x)}{g(x)} = 7$.

True

c. (5 pts) If $f' > 0$ on an interval I, then f is increasing on I.

True

d. (5 pts) If f is increasing on an interval I, then $f' > 0$ on I.

False

e. (5 pts) If f and g are differentiable at a number a, then $fg + f + g$ is differentiable at a.

True

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION B

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1,2.

III. 3.

III. 4.

III. 5.

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (5 pts) Compute $\frac{d}{dx}\left[\frac{e^{x^4}-8}{5+\csc(x^2)}\right]$. (Here e^{x^4} means $e^{(x^4)}$.)

$$\| $$

$$\frac{[5+\csc(x^2)][4x^3e^{x^4}]-[e^{x^4}-8][-\csc(x^2)][\cot(x^2)][2x]}{[5+\csc(x^2)]^2}$$

2. (5 pts) Compute $\frac{d}{dx}\left[(5-\sin x)^{7\arccos x}\right]$.

$$\|$$

$$\left[(5-\sin x)^{7\arccos x}\right]\left[\frac{d}{dx}\left[(7\arccos x)(\ln(5-\sin x))\right]\right]$$

$$\|$$

$$\left[(5-\sin x)^{7\arccos x}\right]\left[\left(\frac{-7}{\sqrt{1-x^2}}\right)(\ln(5-\sin x))+(7\arccos x)\left(\frac{-\cos x}{5-\sin x}\right)\right]$$

3. (10 pts) Find an equation for the tangent line to $x^3 + xy + y^3 = 11$ at $(1, 2)$.

$$3x^2 + y + xy' + 3y^2y' = 0$$

1 2

$$y' = \frac{-3x^2 - y}{x + 3y^2}$$

1 2

$$\text{slope} = \frac{-3\cdot 1 - 2}{1 + 3\cdot 4} = \frac{-5}{13}$$

eq'n: $y - 2 = -\frac{5}{13}(x - 1)$

4. (15 pts) Compute $\lim_{x\to 0} ((\cos x) - (\sin x))^{3/x}$.

$$\| $$

$$e^{\lim_{x\to 0} (3/x)(\ln((\cos x)-(\sin x)))}$$

$$\| $$

$$e^{\lim_{x\to 0} \frac{3(\ln((\cos x)-(\sin x)))}{x}}$$

$$\| \; l'H \; \tfrac{0}{0}$$

$$e^{\lim_{x\to 0} \frac{3\left(\frac{-(\sin x)-(\cos x)}{(\cos x)-(\sin x)}\right)}{1}}$$

$$\| $$

$$e^{\frac{3\left(\frac{-0-1}{1-0}\right)}{1}}$$

$$\| $$

$$e^{-3}$$

5. (10 pts) Find the global maximum and minimum value of $f(x) = x^3 - 3x^2 + 3x + 9$ on the interval $-1 \le x \le 1$.

$$f'(x) = 3x^2 - 6x + 3$$
$$= 3(x^2 - 2x + 1)$$
$$= 3(x-1)^2$$

f' pos 0^2 pos

1

f is increasing on $\mathbb{R} = (-\infty, \infty)$

On $-1 \le x \le 1$,

f attains global min at $x = -1$,
with global min value $f(-1) = -1 - 3 - 3 + 9$
$= -4 + 6 = 2$, and

f attains global max at $x = 1$
with global max value $f(1) = 1 - 3 + 3 + 9$
$= 1 + 9 = 10.$