PRINT YOUR NAME:
SOLUTIONS
Version B

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:
I. Multiple choice

A. (5 pts) (no partial credit) Compute \( \frac{d}{dx}[\sin^2(xy)] \). Circle one of the following answers:

(a) \( 2[\sin(xy)][\cos(xy)][y + xy'] \)
(b) \( [\cos^2(xy)][y + xy'] \)
(c) \( 2[\sin(xy)][y + xy'] \)
(d) \( 2[\sin(xy)][\cos(y + xy')] \)
(e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) Find the logarithmic derivative of \((2 + \sin(2x))^\cos x\) w.r.t. \(x\). Circle one of the following answers:

(a) \( (\cos x)(\ln(2 + \sin(2x))) + (-\sin x) \left( \frac{2\cos(2x)}{2 + \sin(2x)} \right) \)
(b) \( (-\sin x)(\ln(2 + \sin(2x))) + (\cos x) \left( \frac{2\cos(2x)}{2 + \sin(2x)} \right) \)
(c) \( (\cos x)(\ln(2 + \sin(2x))) \)
(d) \( (-\sin x) \left( \frac{2\cos(2x)}{2 + \sin(2x)} \right) \)
(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Find the derivative of \((2 + \sin(2x))^\cos x\) w.r.t. \(x\). Circle one of the following answers:

(a) \( [(2 + \sin(2x))^\cos x] \left[ (\cos x)(\ln(2 + \sin(2x))) + (-\sin x) \left( \frac{2\cos(2x)}{2 + \sin(2x)} \right) \right] \)
(b) \( [(2 + \sin(2x))^\cos x] \left[ (-\sin x)(\ln(2 + \sin(2x))) + (\cos x) \left( \frac{2\cos(2x)}{2 + \sin(2x)} \right) \right] \)
(c) \( [(2 + \sin(2x))^\cos x][(\cos x)(\ln(2 + \sin(2x)))] \)
(d) \( [(2 + \sin(2x))^\cos x] \left[ (-\sin x) \left( \frac{2\cos(2x)}{2 + \sin(2x)} \right) \right] \)
(e) NONE OF THE ABOVE
D. (5 pts) (no partial credit) Suppose \( f'(x) = (x - 1)^2(x - 2)(x - 3)^2 \). Which of the following is a maximal interval of increase for \( f \)? Circle one of the following answers:

(a) \([2, \infty)\)
(b) \((2, \infty)\)
(c) \([1, \infty)\)
(d) \((\infty, 1]\)
(e) NONE OF THE ABOVE

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E. (5 pts) (no partial credit) Compute the derivative of \( \ln(x^{\arctan x}) \), with respect to \( x \), on the interval \( x > 0 \). Circle one of the following answers:

(a) \( \frac{x^{1/(1+x^2)}}{x^{\arctan x}} \)
(b) \( \frac{1}{x^{\arctan x}} \)
(c) \( \frac{1}{x^{\sec^2 x}} \)
(d) \( x^{\sec^2 x} \)
(e) NONE OF THE ABOVE

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F. (5 pts) (no partial credit) Suppose \( f''(x) = -x^2 - 4x - 3 \). At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

(a) \( f \) is concave up on \((\infty, 1]\), down on \([1, 3]\) and up on \([3, \infty)\).
(b) \( f \) is concave down on \((\infty, 1]\), up on \([1, 3]\) and down on \([3, \infty)\).
(c) \( f \) is concave up on \((\infty, -3]\), down on \([-3, -1]\) and up on \([-1, \infty)\).
(d) \( f \) is concave down on \((\infty, -3]\), up on \([-3, -1]\) and down on \([-1, \infty)\).
(e) NONE OF THE ABOVE

\[ -(x^2 + 4x + 3) = -(x + 3)(x + 1) \]
II. True or false (no partial credit):

a. (5 pts) Assume that $\lim_{x \to 0} [f(x)] = 0 = \lim_{x \to 0} [g(x)]$. Assume also that $\lim_{x \to 0} \left[ \frac{f'(x)}{g'(x)} \right]$ does not exist. Then $\lim_{x \to 0} \left[ \frac{f(x)}{g(x)} \right]$ does not exist.

\text{False}

b. (5 pts) Assume that $\lim_{x \to 3} [f(x)] = 0 = \lim_{x \to 3} [g(x)]$. Assume also that $\lim_{x \to 3} \frac{f'(x)}{g'(x)} = 7$. Then $\lim_{x \to 3} \frac{f(x)}{g(x)} = 7$.

\text{True}

c. (5 pts) If $f' > 0$ on an interval $I$, then $f$ is increasing on $I$.

\text{True}

d. (5 pts) If $f$ is increasing on an interval $I$, then $f' > 0$ on $I$.

\text{False}

e. (5 pts) If $f$ and $g$ are differentiable at a number $a$, then $fg + f + g$ is differentiable at $a$.

\text{True}

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION B

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1,2.

III. 3.

III. 4.

III. 5.
III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (5 pts) Compute \( \frac{d}{dx} \left[ \frac{e^{x^4} - 8}{5 + \csc(x^2)} \right] \). (Here \( e^{x^4} \) means \( e^{(x^4)} \).)

\[
\left[ 5 + \csc(x^2) \right] \left[ 4x^3 e^{x^4} \right] - \left[ e^{x^4} - 8 \right] \left[ -\csc(x^2) \right] [\cot(x^2)] [2x] \\
\left[ 5 + \csc(x^2) \right]^2
\]

2. (5 pts) Compute \( \frac{d}{dx} \left[ (5 - \sin x)^7 \arccos x \right] \).

\[
\left[ (5 - \sin x)^7 \arccos x \right] \left[ \frac{d}{dx} \left[ (7 \arccos x)(\ln(5 - \sin x)) \right] \right] \\
\left[ (5 - \sin x)^7 \arccos x \right] \left[ \left( \frac{-7}{\sqrt{1-x^2}} \right) (\ln(5 - \sin x)) + (7 \arccos x) \left( \frac{-\cos x}{5 - \sin x} \right) \right]
\]
3. (10 pts) Find an equation for the tangent line to $x^3 + xy + y^3 = 11$ at $(1, 2)$.

\[ 3x^2 + y + xy' + 3y^2y' = 0 \]

\[
y' = \frac{-3x^2 - y}{x + 3y^2} \]

\[
\text{Slope} = \frac{-3 \cdot 1 - 2}{1 + 3 \cdot 4} = \frac{-5}{13}
\]

Eqn: \[ y - 2 = -\frac{5}{13} (x - 1) \]
4. (15 pts) Compute \( \lim_{x \to 0} ((\cos x) - (\sin x))^{3/x} \).

\[
\lim_{x \to 0} \left( \frac{3}{x} \right) \left( \ln((\cos x) - (\sin x)) \right)
\]

\[
\lim_{x \to 0} \frac{3 \left( \ln((\cos x) - (\sin x)) \right)}{x}
\]

\[
\lim_{x \to 0} \frac{3 \left( \frac{-(\sin x) - (\cos x)}{(\cos x) - (\sin x)} \right)}{1}
\]

\[
\lim_{x \to 0} \frac{3 \left( \frac{0 - 1}{1 - 0} \right)}{1}
\]

\[
e^{-3}
\]
5. (10 pts) Find the global maximum and minimum value of \( f(x) = x^3 - 3x^2 + 3x + 9 \) on the interval \(-1 \leq x \leq 1\).

\[
f'(x) = 3x^2 - 6x + 3 \\
     = 3(x^2 - 2x + 1) \\
     = 3(x - 1)^2
\]

\[f'\text{ pos } 0^2 \text{ pos}\]

\[
t_1
\]

\(f\) is increasing on \( IR = (-\infty, \infty)\)

On \(-1 \leq x \leq 1\):

\(f\) attains global min at \(x = -1\),
with global min value \(f(-1) = -1 - 3 - 3 + 9\)
\[
     = -4 + 6 = 2, \text{ and}
\]

\(f\) attains global max at \(x = 1\),
with global max value \(f(1) = 1 - 3 + 3 + 9\)
\[
     = 1 + 9 = 10.
\]