PRINT YOUR NAME:

SOLUTIONS
Version C

PRINT YOUR TA’S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:
I. Multiple choice

A. (5 pts) (no partial credit) Suppose \( f'(x) = -(x - 1)^2(x - 2)(x - 3)^2 \). Which of the following is a maximal interval of increase for \( f' \)? Circle one of the following answers:

(a) \((-\infty, 2]\)
(b) \([1, \infty)\)
(c) \((2, \infty)\)
(d) \([3, \infty)\)
(e) NONE OF THE ABOVE

\[\frac{(x-1)(x-3)}{x^2-4x+3}\]

B. (5 pts) (no partial credit) Suppose \( f''(x) = x^2 - 4x + 3 \). At most one of the following statements is true. If one is, circle it. Otherwise, circle “NONE OF THE ABOVE”.

(a) \( f \) is concave down on \((-\infty, 1]\), up on \([1, 3]\) and down on \([3, \infty)\).
(b) \( f \) is concave down on \((-\infty, \infty)\).
(c) \( f \) is concave down on \((-\infty, -3]\), up on \([-3, -1]\) and down on \([-1, \infty)\).
(d) \( f \) is concave up on \((-\infty, -3]\), down on \([-3, -1]\) and up on \([-1, \infty)\).
(e) NONE OF THE ABOVE

\[f \text{ cc up on } (-\infty, 1], \text{ dn on } [1, 3], \text{ up on } [3, \infty)\]

C. (5 pts) (no partial credit) Compute \([d/dx][\sin^2(xy)]\). Circle one of the following answers:

(a) \(2[\sin(xy)][y + xy']\)
(b) \([\cos^2(xy)][y + xy']\)
(c) \(2[\sin(xy)][\cos(xy)][y + xy']\)
(d) \(2[\sin(xy)][\cos(y + xy')]\)
(e) NONE OF THE ABOVE
D. (5 pts) (no partial credit) Find the logarithmic derivative of \((2 + \sin(2x))^{\cos x}\) w.r.t. \(x\). Circle one of the following answers:

(a) \((\cos x)(\ln(2 + \sin(2x))) + (-\sin x)\left(\frac{2\cos(2x)}{2 + \sin(2x)}\right)\)

(b) \((-\sin x)\left(\frac{2\cos(2x)}{2 + \sin(2x)}\right)\)

(c) \((-\sin x)(\ln(2 + \sin(2x))) + (\cos x)\left(\frac{2\cos(2x)}{2 + \sin(2x)}\right)\)

(d) \((\cos x)(\ln(2 + \sin(2x)))\)

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Find the derivative of \((2 + \sin(2x))^{\cos x}\) w.r.t. \(x\). Circle one of the following answers:

(a) \([\ln((2 + \sin(2x)))^{\cos x}] \left[(\cos x)(\ln(2 + \sin(2x))) + (-\sin x)\left(\frac{2\cos(2x)}{2 + \sin(2x)}\right)\right]\)

(b) \([\ln((2 + \sin(2x)))^{\cos x}] \left((-\sin x)\left(\frac{2\cos(2x)}{2 + \sin(2x)}\right)\right]\)

(c) \([\ln((2 + \sin(2x)))^{\cos x}] \left((-\sin x)(\ln(2 + \sin(2x))) + (\cos x)\left(\frac{2\cos(2x)}{2 + \sin(2x)}\right)\right]\)

(d) \([\ln((2 + \sin(2x)))^{\cos x}] \left[(\cos x)(\ln(2 + \sin(2x)))\right]\)

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Compute the derivative of \(\ln(x^{\text{arctan } x})\), with respect to \(x\), on the interval \(x > 0\). Circle one of the following answers:

(a) \(\frac{1}{x^2}\)

(b) \(x^{\sec^2 x}\)

(c) \(\frac{1}{\text{arctan } x}\)

(d) \(\ln x \left(\frac{\text{arctan } x}{x} + \frac{1}{1 + x^2}\right)\)

(e) NONE OF THE ABOVE
II. True or false (no partial credit):

a. (5 pts) Assume that \( \lim_{x \to 0} [f(x)] = 0 = \lim_{x \to 0} [g(x)] \). Assume also that \( \lim_{x \to 0} \left[ \frac{f'(x)}{g'(x)} \right] \) does not exist. Then \( \lim_{x \to 0} \left[ \frac{f(x)}{g(x)} \right] \) does not exist.

\[ \text{False} \]

b. (5 pts) Assume that \( \lim_{x \to 3} [f(x)] = 0 = \lim_{x \to 3} [g(x)] \). Assume also that \( \lim_{x \to 3} \frac{f'(x)}{g'(x)} = 7 \). Then \( \lim_{x \to 3} \frac{f(x)}{g(x)} = 7 \).

\[ \text{True} \]

c. (5 pts) If \( f \) and \( g \) are differentiable at a number \( a \), then \( fg + f + g \) is differentiable at \( a \).

\[ \text{True} \]

d. (5 pts) If \( f \) is increasing on an interval \( I \), then \( f' > 0 \) on \( I \).

\[ \text{False} \]

e. (5 pts) If \( f' > 0 \) on an interval \( I \), then \( f \) is increasing on \( I \).

\[ \text{True} \]

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION C

I. A,B,C

II. D,E,F

II. a,b,c,d,e

III. 1,2.

III. 3.

III. 4.

III. 5.
III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (5 pts) Compute $\frac{d}{dx} \left[ \frac{e^{x^2} - 8}{5 + \csc(x^2)} \right]$. (Here $e^{x^2}$ means $e^{(x^2)}$.)

\[
\frac{\left[5 + \csc(x^2)\right] \left[4x^3 e^{x^2}\right] - \left[e^{x^2} - 8\right] \left[-\csc(x^2)\right] \left[\cot(x^2)\right]}{(2x)^2} \left[\frac{5 + \csc(x^2)}{2}\right]^2
\]

2. (5 pts) Compute $\frac{d}{dx} \left[ (5 - \sin x)^{7 \arctan x} \right]$.

\[
\left[ (5 - \sin x)^{7 \arctan x} \right] \left[ \frac{d}{dx} \left[ (7 \arctan x) (\ln (5 - \sin x)) \right] \right]
\]

\[
\left[ (5 - \sin x)^{7 \arctan x} \right] \left[ \frac{7}{1 + x^2} (\ln (5 - \sin x)) + (7 \arctan x) \left( \frac{-\cos x}{5 - \sin x} \right) \right]
\]
3. (10 pts) Find an equation for the tangent line to \( x^3 + xy + y^3 = 11 \) at \((2, 1)\).

\[
3x^2 + y + xy' + 3y^2 y' = 0
\]

\[
y' = \frac{-3x^2 - y}{x + 3y^2}
\]

\[
\text{Slope} = \frac{-3 \cdot 2 - 1}{2 + 3 \cdot 1} = \frac{-13}{5}
\]

\[
eq n' : \quad y - 1 = -\frac{13}{5}(x - 2)
\]
4. (15 pts) Compute $\lim_{x \to 0} ((\cos x) + (\sin x))^{-2/x}$.

\[ e^{\lim_{x \to 0} (-2/x) \left( \ln ((\cos x) + (\sin x)) \right)} \]

\[ e^{\lim_{x \to 0} \frac{-2(\ln ((\cos x) + (\sin x)))}{x}} \]

\[ e^{\lim_{x \to 0} \frac{-2((\sin x) + (\cos x))}{\cos x + \sin x}} \]

\[ e^{\lim_{x \to 0} \frac{-2\left(\frac{-x + 1}{1 + \delta}\right)}{1}} \]

\[ e^{-2} \]
5. (10 pts) Find the global maximum and minimum value of \( f(x) = -x^3 + 3x^2 - 3x - 3 \) on the interval \( 0 \leq x \leq 1 \).

\[
\begin{align*}
f'(x) &= -3x^2 + 6x - 3 \\
&= -3(x^2 - 2x + 1) \\
&= -3(x - 1)^2
\end{align*}
\]

\[
\begin{array}{ccc}
\text{neg} & 0^2 & \text{neg} \\
\hline
1 & \text{neg} & \text{neg}
\end{array}
\]

\( f \) is decreasing on \( IR = (-\infty, \infty) \)

On \( 0 \leq x \leq 1 \),

\( f \) attains global min at \( x = 1 \)

with global min value \( f(1) = -1 + 3 - 3 - 3 = -1 - 3 = -4 \) and

\( f \) attains global max at \( x = 0 \)

with global max value \( f(0) = -3 \).