PRINT YOUR NAME:

SOLUTIONS

Version D

PRINT YOUR TA’S NAME:

WHAT SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:
I. Multiple choice

A. (5 pts) (no partial credit) Compute \( \lim_{x \to 0} \frac{2x^3 - 5x^2}{7x(\sin x)} \). Circle one of the following answers:

(a) 0
(b) \( \infty \)
(c) \( \frac{5}{7} \)
(d) \( \frac{2}{7} \)

(e) NONE OF THE ABOVE

\[
\lim_{x \to 0} \frac{-5x^2}{7x \cdot x} = -\frac{5}{7}
\]

B. (5 pts) (no partial credit) Compute the largest \( \delta > 0 \) such that: \( 0 < |x - 1| < \delta \) implies \( |(5x + 4) - 9| < 0.05 \). Circle one of the following answers:

(a) 0.2
(b) 0.1
(c) 0.025
(d) 0.01
(e) NONE OF THE ABOVE

\[
\frac{0.05}{15} = 0.01
\]

C. (5 pts) (no partial credit) Let \( y = x^2 - x \). Find \( \Delta y \). Circle one of the following answers:

(a) \((x + \Delta x)^2 - (x + \Delta x)\)
(b) \([x + \Delta x)^2 - (x + \Delta x)] + [x^2 - x]\)
(c) \([x + \Delta x)^2 - (x + \Delta x)] - [x^2 - x]\)
(d) \([x^2 - x] - [(x + \Delta x)^2 - (x + \Delta x)]\)
(e) NONE OF THE ABOVE
D. (5 pts) (no partial credit) Assume that \( \lim_{x \to 200} f(x) = 4 \) and \( \lim_{x \to 200} g(x) = 5 \). At most one of the following statements must follow. If one does, circle it. Otherwise, circle Answer e.

(a) \( \lim_{z \to 200} [(f(x)) + (g(x))] = 9 \)

(b) \( \lim_{x \to 400} [(f(x)) + (g(x))] = 9 \)

(c) \( \lim_{x \to 1} \left[ \frac{f(x)}{g(x)} \right] = \frac{4}{5} \)

(d) \( \lim_{x \to 300} [(f(x)) + (g(x))] \) does not exist

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) A line passes through (1, 40) and (5, 80). Find its slope. Circle one of the following answers:

(a) 10

(b) 20

(c) 30

(d) 40

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) A particle travels along a number line. Its position at time 1 is 40 and its position at time 5 is 80. Find its average velocity between time 1 and time 5. Circle one of the following answers:

(a) 10

(b) 20

(c) 30

(d) 40

(e) NONE OF THE ABOVE
II. True or false (no partial credit):
a. (5 pts) The function \( f(x) = |x| \) is differentiable at every real number.
   \[ \text{F} \]
b. (5 pts) If a function is differentiable at 0, then it is continuous at 0.
   \[ \text{T} \]
c. (5 pts) A tangent line to the graph of a function cannot intersect the graph of the function more than once.
   \[ \text{F} \]
d. (5 pts) For every real number \( x \), \( \ln(e^x) = x \).
   \[ \text{T} \]
e. (5 pts) There is a function with two horizontal asymptotes.
   \[ \text{T} \]

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VERSION D
I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1a,b

III. 2

III. 3

III. 4a,b
III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. a. (5 pts) Compute \( \lim_{h \to 0} \frac{\sqrt{5+2h} - \sqrt{5-h}}{h} \cdot \frac{\sqrt{5+2h} + \sqrt{5-h}}{\sqrt{5+2h} + \sqrt{5-h}} \)

\[ \frac{(\sqrt{5+2h})^2 - (\sqrt{5-h})^2}{h} \]

\[ \frac{\sqrt{5+2h} - \sqrt{5-h}}{h} \cdot \frac{1}{\sqrt{5+2h} + \sqrt{5-h}} \]

\[ \frac{3h}{h} \cdot \frac{1}{\sqrt{5+2h} + \sqrt{5-h}} \]

\[ \frac{3}{\sqrt{5} + \sqrt{5}} \]

\[ \frac{3}{2 \sqrt{5}} \]

b. (5 pts) Compute \( \lim_{h \to 0} \frac{\frac{5+2h}{h} - \frac{1}{5-h}}{h} \)

\[ \frac{(5-h) - (5+2h)}{h(5+2h)(5-h)} \]

\[ \frac{-3h}{h(5+2h)(5-h)} \]

\[ \lim_{h \to 0} \]

\[ \frac{-3}{5 \cdot 5} \]

\[ = -\frac{3}{25} \]
2. (10 pts) Find all the horizontal asymptotes to \( y = \frac{\sqrt{9x^2 + 5}}{x + 1} \).

\[
\lim_{x \to \pm \infty} f(x) = \lim_{x \to \pm \infty} \frac{\sqrt{9x^2}}{x} = \lim_{x \to \pm \infty} \frac{(3)(\pm x)}{x} = \pm 3
\]

\( y = -3 \) and \( y = 3 \) are the horizontal asymptotes.
3. (10 pts) Compute \( \lim_{x \to 0} \left( \frac{7x^3 + 4x^2}{8x \sin x} \right) \).

\[
\lim_{x \to 0} \frac{4x}{8x \sin x} = \frac{4}{8} = \frac{1}{2}
\]
4. On the planet of Gallifrey, in an alternate universe, a dropped object travels $t^3 + t^2$ feet during its first $t$ seconds of free fall.

a. (10 pts) For $h \neq 0$, the average velocity between time $t = 2$ seconds and time $t = 2 + h$ seconds is given by a quadratic polynomial in $h$ of the form $ah^2 + bh + c$. Find the coefficients $a$, $b$ and $c$.

\[
\frac{\left[ \frac{t^3 + t^2}{t_1 \to 2 + h} \right]_{t_1 \to 2}}{\left[ \frac{t}{t_1 \to 2 + h} \right]_{t_1 \to 2}} = \frac{\left[ (2+h)^3 + (2+h)^2 \right] - \left[ 2^3 + 2^2 \right]}{h}
\]

\[
= \frac{2^6 + 3 \cdot 2^5 \cdot h + 3 \cdot 2^4 \cdot h^2 + h^3 + 2^2 \cdot 2 \cdot h + h^2 - 2^2 - 2}{h}
\]

\[
= \frac{12h + 6h^2 + h^3 + 4h + h^2}{h} = \frac{h^3 + 7h^2 + 16h}{h} \xrightarrow{h \to 0} 7h + 16 \text{ ft/sec}
\]

$a = 1, b = 7, c = 16$

b. (5 pts) Find the instantaneous velocity at time $t = 2$ seconds.

\[
\lim_{h \to 0} \frac{\left[ \frac{t^3 + t^2}{t_1 \to 2 + h} \right]_{t_1 \to 2}}{\left[ \frac{t}{t_1 \to 2 + h} \right]_{t_1 \to 2}} = \lim_{h \to 0} \left( h^2 + 7h + 16 \right) = 16 \text{ ft/sec}
\]