MATH 1271 Spring 2012, Midterm #2
Handout date: Thursday 29 March 2012

PRINT YOUR NAME:

Solutions
Version C

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Find the logarithmic derivative of $x^2 + 7x - 8$ w.r.t. x.

(a) $\dfrac{2x+7}{x^2+7x-8}$

(b) $\dfrac{x^2+7x-8}{2x+7}$

(c) $(\ln(x^2)) + 7(\ln x) - (\ln 8)$

(d) $\ln(2x+7)$

(e) NONE OF THE ABOVE

$-(x^2-4x+3) = -(x-1)(x-3)$

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B. (5 pts) (no partial credit) Suppose $f''(x) = -x^2 + 4x - 3$. At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

(a) f is concave up on $(-\infty, 1]$, down on $[1, 3]$ and up on $[3, \infty)$.

(b) f is concave down on $(-\infty, 1]$, up on $[1, 3]$ and down on $[3, \infty)$.

(c) f is concave up on $(-\infty, -3]$, down on $[-3, -1]$ and up on $[-1, \infty)$.

(d) f is concave down on $(-\infty, -3]$, up on $[-3, -1]$ and down on $[-1, \infty)$.

(e) NONE OF THE ABOVE

f'' neg 0 pos 0 neg

1 3

C. (5 pts) (no partial credit) Compute $[d/dx][\sin(\cos(e^x + 3))]$.

$\|$

$[\cos(\cos(e^x+3))][-\sin(e^x+3)][e^x]$

(a) $\cos(\cos(e^x + 3))$

(b) 0

(c) $[\cos(\cos(e^x + 3))][\cos(e^x + 3)][e^x + 3]$

(d) $[\cos(\cos(e^x + 3))][-\sin(e^x + 3)][e^x]$

(e) NONE OF THE ABOVE

D. (5 pts) (no partial credit). Find an equation of the tangent line to $4x^2y - 2y^3 = 2$ at the point $(1, 1)$.

(a) $y - 1 = 0$

(b) $y - 1 = x - 1$

(c) $y - 1 = 2(x - 1)$

(d) $y - 1 = 3(x - 1)$

(e) NONE OF THE ABOVE

$8xy + 4x^2y' - 6y^2y' = 0$

$y' = \dfrac{-8xy}{4x^2 - 6y^2}$

$y-1 = 4(x-1)$

slope $= \dfrac{-8}{4-6} = \dfrac{-8}{-2} = 4$

E. (5 pts) (no partial credit) Find the logarithmic derivative of $(2 + x^4)^{\cos x}$ w.r.t. x.

(a) $(\cos x)(\ln(2 + x^4))$

(b) $(-\sin x)(\ln(2 + x^4)) + (\cos x)(4x^3/(2 + x^4))$

(c) $(-\sin x)(4x^3/(2 + x^4))$

(d) $(\cos x)(\ln(2 + x^4)) + (-\sin x)(4x^3/(2 + x^4))$

(e) NONE OF THE ABOVE

$\dfrac{d}{dx}\left[(\cos x)(\ln(2+x^4))\right]$

F. (5 pts) (no partial credit) Find the derivative of $(2 + x^4)^{\cos x}$ w.r.t. x.

(a) $[(2 + x^4)^{\cos x}][(\cos x)(\ln(2 + x^4))]$

(b) $[(2 + x^4)^{\cos x}][(-\sin x)(\ln(2 + x^4)) + (\cos x)(4x^3/(2 + x^4))]$

(c) $[(2 + x^4)^{\cos x}][(-\sin x)(4x^3/(2 + x^4))]$

(d) $[(2 + x^4)^{\cos x}][(\cos x)(\ln(2 + x^4)) + (-\sin x)(4x^3/(2 + x^4))]$

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) Assume that $\lim_{x \to a} [f(x)] = 0$ and that $\lim_{x \to a} [g(x)] = 0$. Assume also that $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ does not exist. Then $\lim_{x \to a} \frac{f(x)}{g(x)}$ does not exist.

F

b. (5 pts) If $f'(7) = 0$ and $f''(7) > 0$, then f has a local minimum at 7.

$\cup$ T

c. (5 pts) Every local extremum occurs at critical number.

T

d. (5 pts) If $f' = g'$ on an interval I, then $f - g$ is constant on I.

T

e. (5 pts) If f is concave up on an interval I, then $f'' > 0$ on I.

F

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION C

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1ab.

III. 2.

III. 3,4.

III. 5.

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. a. (5 pts) Compute $\dfrac{d}{dx}\left[\dfrac{2x^3-8}{9+(\arctan(2x))}\right]$.

$$\| $$

$$\frac{[9+(\arctan(2x))][6x^2]-[2x^3-8]\left[\frac{1}{1+(2x)^2}\right][2]}{[9+(\arctan(2x))]^2}$$

b. (5 pts) Compute $\dfrac{d}{dx}\left[(4-\sin x)^x\right]$.

$$\| $$

$$[(4-\sin x)^x]\left[\frac{d}{dx}\left[x(\ln(4-\sin x))\right]\right]$$

$$\| $$

$$[(4-\sin x)^x]\left[(\ln(4-\sin x))+x\left(\frac{-\cos x}{4-\sin x}\right)\right]$$

2. (10 pts) Using implicit differentiation, find $y' = dy/dx$, assuming that $(x - y^2)^5 = x$.

$$5(x-y^2)^4(1-2yy') = 1$$

$$\left[5(x-y^2)^4\right] - \left[10y(x-y^2)^4\right]y' = 1$$

$$y' = \frac{1-5(x-y^2)^4}{-10y(x-y^2)^4}$$

3. (5 pts) Let $f(x) = 5x + 3x^5$. Then f is a one-to-one function. Let $g := f^{-1}$. Then $f(1) = 8$, so $g(8) = 1$. Compute $g'(8)$.

$$g'(8) = \frac{1}{f'(1)} = \frac{1}{\left[5+15x^4\right]_{x:\to 1}} = \frac{1}{20}$$

4. (10 pts) Find the maximal intervals of concavity for $f(x) = -3x^5 + 20x^4 - 8x + 4$. For each interval, state clearly whether f is concave up or concave down on that interval.

$$f'(x) = -15x^4 + 80x^3 - 8$$

$$f''(x) = -60x^3 + 240x^2$$

$$= -60x^2(x-4)$$

f'' pos 0^2 pos 0 neg

0 4

f is concave up on $(-\infty, 4]$

f is concave down on $[4, \infty)$

5. (10 pts) Compute $\lim_{x\to 1}\left[\dfrac{\ln x}{\cos(\pi x/2)}\right]$.

$$\frac{0}{0}$$

$$\parallel \text{l'H}$$

$$\lim_{x\to 1}\frac{1/x}{[-\sin(\pi x/2)][\pi/2]}$$

$$\parallel$$

$$\frac{1/1}{[-\sin(\pi/2)][\pi/2]}$$

$$\parallel$$

$$\frac{1}{[-1][\pi/2]}$$

$$\parallel$$

$$-\frac{2}{\pi}$$