PRINT YOUR NAME:

PRINT YOUR TA’S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:
I. Multiple choice

A. (5 pts) (no partial credit) Suppose $f''(x) = -x^2 + 4x - 3$. At most one of the following statements is true. If one is, circle it. Otherwise, circle “NONE OF THE ABOVE”.

   (a) $f$ is concave up on $(-\infty, 1]$, down on $[1, 3]$ and up on $[3, \infty)$.
   (b) $f$ is concave down on $(-\infty, 1]$, up on $[1, 3]$ and down on $[3, \infty)$.
   (c) $f$ is concave up on $(-\infty, -3]$, down on $[-3, -1]$ and up on $[-1, \infty)$.
   (d) $f$ is concave down on $(-\infty, -3]$, up on $[-3, -1]$ and down on $[-1, \infty)$.
   (e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) Find the logarithmic derivative of $x^2 + 7x - 8$ w.r.t. $x$.

   (a) $\frac{x^2 + 7x - 8}{2x + 7}$
   (b) $(\ln(x^2)) + 7(\ln x) - (\ln 8)$
   (c) $\ln(2x + 7)$
   (d) $\frac{2x + 7}{x^2 + 7x - 8}$
   (e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Find an equation of the tangent line to $4x^2y - 2y^3 = 2$ at the point $(1, 1)$.

   (a) $y - 1 = x - 1$
   (b) $y - 1 = 2(x - 1)$
   (c) $y - 1 = 3(x - 1)$
   (d) $y - 1 = 4(x - 1)$
   (e) NONE OF THE ABOVE
D. (5 pts) (no partial credit) Find the logarithmic derivative of \((2 + x^4)\cos x\) w.r.t. \(x\).

(a) \((\cos x)(\ln(2 + x^4)) + (-\sin x)(4x^3/(2 + x^4))\)

(b) \((-\sin x)(4x^3/(2 + x^4))\)

(c) \((-\sin x)(\ln(2 + x^4)) + (\cos x)(4x^3/(2 + x^4))\)

(d) \((\cos x)(\ln(2 + x^4))\)

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Find the derivative of \((2 + x^4)\cos x\) w.r.t. \(x\).

(a) \([(2 + x^4)\cos x][\cos x(\ln(2 + x^4)) + (-\sin x)(4x^3/(2 + x^4))]\)

(b) \([(2 + x^4)\cos x][(-\sin x)(4x^3/(2 + x^4))]\)

(c) \([(2 + x^4)\cos x][(-\sin x)(\ln(2 + x^4)) + (\cos x)(4x^3/(2 + x^4))]\)

(d) \([(2 + x^4)\cos x][(\cos x)(\ln(2 + x^4))]\)

(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Compute \([d/dx][\sin(\cos(e^x + 3))]\).

(a) \(\cos(\cos(e^x + 3))\)

(b) \([\cos(\cos(e^x + 3))][\cos(e^x + 3)][e^x + 3]\)

(c) \([\cos(\cos(e^x + 3))][-\sin(e^x + 3)][e^x + 3]\)

(d) 0

(e) NONE OF THE ABOVE
II. True or false (no partial credit):

a. (5 pts) If \( f'(7) = 0 \) and \( f''(7) > 0 \), then \( f \) has a local maximum at 7.

b. (5 pts) Assume that \( \lim_{x \to a} f(x) = 0 \) and that \( \lim_{x \to a} g(x) = 0 \). Assume also that \( \lim_{x \to a} \frac{f'(x)}{g'(x)} \) does not exist. Then \( \lim_{x \to a} \frac{f(x)}{g(x)} \) does not exist.

c. (5 pts) Every local extremum occurs at a critical number.

d. (5 pts) If \( f \) is concave up on an interval \( I \), then \( f'' > 0 \) on \( I \).

e. (5 pts) If two functions have the same derivative, then they are equal.
III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. a. (5 pts) Compute \( \frac{d}{dx} \left[ \frac{2x^3 - 8}{6 + (\arctan(2x))} \right] .

b. (5 pts) Compute \( \frac{d}{dx} [(4 - \sin x)^2] .\)
2. (10 pts) Using implicit differentiation, find \( y' = dy/dx \), assuming that \((x - y^2)^5 = x\).
3. (5 pts) Let \( f(x) = 4x + 4x^5 \). Then \( f \) is a one-to-one function. Let \( g := f^{-1} \). Then \( f(1) = 8 \), so \( g(8) = 1 \). Compute \( g'(8) \).

4. (10 pts) Find the maximal intervals of concavity for \( f(x) = -3x^5 + 20x^4 + 4x - 8 \). For each interval, state clearly whether \( f \) is concave up or concave down on that interval.
5. (10 pts) Compute \( \lim_{x \to 1} \left[ \frac{\ln x}{\cos(\pi x/2)} \right] \).