MATH 1271 Spring 2013, Midterm #2
Handout date: Thursday 4 April 2013

PRINT YOUR NAME:

SOLUTIONS
Version C

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Compute $[d/dx][\sin^2(xy)]$. Circle one of the following answers:

(a) $[\cos^2(xy)][y + xy']$

(b) $2[\sin(xy)][y + xy']$

(c) $2[\sin(xy)][\cos(y + xy')]$

(d) $2[\sin(xy)][\cos(xy)]$

(e) NONE OF THE ABOVE

$2[\sin(xy)][\cos(xy)][y + xy']$

B. (5 pts) (no partial credit) The Quotient Rule says that $(f/g)'$ is equal to what? Circle one of the following answers:

(a) $(gf' - fg')/g^2$

(b) $(fg' - gf')/g^2$

(c) g'/f'

(d) f'/g'

(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Let f be a function such that $f'(x) = 6e^{4x}$. Suppose, also, that $f(0) = 1$. Which of the following is an equation of the tangent line to the graph of f at $(0, 1)$. Circle one of the following answers:

(a) $y = 1 + 6x$

(b) $y = 6(x - 1)$

(c) $y - 1 = 6e^{4x}x$

(d) $y = 6e^{4x}(x - 1)$

(e) NONE OF THE ABOVE

slope $= f'(0) = 6e^{4\cdot 0} = 6$

$y - 1 = 6(x - 0)$

$y = 1 + 6x$

D. (5 pts) (no partial credit) Compute $\frac{d}{dx}\left[\ln|(2x+1)(3x-4)|\right]$. Circle one of the following answers:

(a) $\left|\frac{2}{2x+1}+\frac{3}{3x-4}\right|$

(b) $\left|\frac{6}{(2x+1)(3x+4)}\right|$

(c) $\frac{2}{2x+1}+\frac{3}{3x-4}$

(d) $\frac{6}{(2x+1)(3x+4)}$

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Suppose $f'(x) = -2(x-1)^5(x-2)^6(x-3)^8$. Which of the following is a maximal interval of increase for f? Circle one of the following answers:

(a) $(-\infty, 1]$

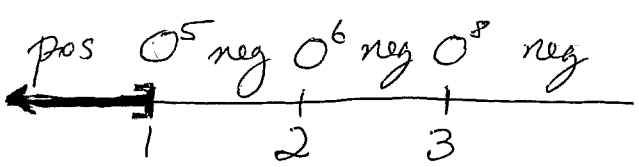
(b) $(-\infty, 2)$

(c) $[1, \infty)$

(d) $[2, \infty)$

(e) NONE OF THE ABOVE

f'



F. (5 pts) (no partial credit) Compute $\lim_{x\to\infty}(2x^2+4x-3e^x)e^{-x}$. Circle one of the following answers:

$= (2x^2+4x)e^{-x} - 3$

$\| \quad$

$\left[\lim_{x\to\infty}\frac{2x^2+4x}{e^x}\right] - 3$

(a) ∞

(b) 2

(c) 0

(d) -3

(e) NONE OF THE ABOVE

$\|$ l'H $\frac{\infty}{\infty}$ twice

$\left[\lim_{x\to\infty}\frac{4}{e^x}\right] - 3 = 0 - 3 = -3$

"$e^\infty = \infty$" and "$\frac{4}{\infty} = 0$"

II. True or false (no partial credit):

a. (5 pts) Let f and g be any two functions such that $\lim_{x\to 5} f(x) = 1$ and $\lim_{x\to 5} g(x) = 0$. Then $\lim_{x\to 5} \frac{f(x)}{g(x)} = \infty$.

False "$\frac{1}{0}$" is (slightly) indeterminate

b. (5 pts) Let u be any expression of x. Then $(d/dx)(e^u) = e^u$.

False $e^u(du/dx)$

c. (5 pts) If $f' > 0$ on an interval I, then f is increasing on I.

True

d. (5 pts) Let g be any function such that $\lim_{x\to\infty}[g(x)] = \infty$. Then $\lim_{x\to\infty}[(1/x)^{g(x)}] = 0$.

True "$1/\infty = 0^+$" and "$(0^+)^\infty = 0$"

e. (5 pts) Let f and g be any two functions such that $\lim_{x\to a}[f(x)] = \infty$ and $\lim_{x\to a}[g(x)] = \infty$. Then $\lim_{x\to a}[(f(x)) - (g(x))] = 0$.

False "$\infty - \infty$" is indeterminate

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION C

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1,2.

III. 3.

III. 4.

III. 5. a,b,c

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (5 pts) Compute $\frac{d}{dx}\left[\frac{e^{-x^4}}{4-\sin(x^2)}\right]$. (Here e^{-x^4} means $e^{(-x^4)}$.)

$$=$$

$$\frac{[4-\sin(x^2)][e^{-x^4}][-4x^3]-[e^{-x^4}][-\cos(x^2)][2x]}{[4-\sin(x^2)]^2}$$

2. (5 pts) Compute $\frac{d}{dx}\left[(2+3x^2)^{\tan x}\right]$.

$$=$$

$$[(2+3x^2)^{\tan x}]\left[\frac{d}{dx}[(\tan x)(\ln(2+3x^2))]\right]$$

$$=$$

$$[(2+3x^2)^{\tan x}]\left[(\sec^2 x)(\ln(2+3x^2))+(\tan x)\left(\frac{6x}{2+3x^2}\right)\right]$$

3. (10 pts) Find an equation for the tangent line to $-6e^x - 5xy + y^2 = 4x - y$ at $(0, 2)$.

$m :=$ slope of this tangent line

$$-6e^x - 5y - 5xy' + 2yy' = 4 - y'$$

$$0 \qquad 2 \qquad 0m \qquad 2m \qquad m$$

$$-6 - 10 - 0 + 4m = 4 - m$$

$$-16 + 4m = 4 - m$$

$$5m = 20$$

$$m = 4$$

$$\boxed{y - 2 = 4x}$$

4. (10 pts) Compute $\lim_{x\to 0} (2e^x - \cos x)^{3/x}$.

$$\| $$

$$e^{\lim_{x\to 0} (3/x)(\ln(2e^x - \cos x))}$$

$$\| $$

$$e^{\lim_{x\to 0} \frac{3(\ln(2e^x - \cos x))}{x}}$$

$$\| \text{l'H } \tfrac{0}{0}$$

$$e^{\lim_{x\to 0} \frac{3\left(\frac{2e^x + \sin x}{2e^x - \cos x}\right)}{1}}$$

$$\| $$

$$e^{\frac{3\left(\frac{2+0}{2-1}\right)}{1}} = e^6$$

5. Let $y = x^3$. Then $\triangle y = ax^2(\triangle x) + bx(\triangle x)^2 + c(\triangle x)^3$, for some real numbers a, b, c.

a. (5 pts) Compute a, b and c.

$$\triangle y = (x + \triangle x)^3 - x^3$$

$$= \cancel{x^3} + 3x^2(\triangle x) + 3x(\triangle x)^2 + (\triangle x)^3 - \cancel{x^3}$$

$a = 3$ $\quad b = 3$ $\quad c = 1$

b. (5 pts) Assuming $\triangle x \neq 0$, compute $\dfrac{\triangle y}{\triangle x}$.

$\| \triangle x \neq 0$

$$3x^2 + 3x(\triangle x) + (\triangle x)^2$$

c. (5 pts) Compute $\lim\limits_{\triangle x \to 0} \dfrac{\triangle y}{\triangle x}$.

$\|$

$3x^2$