MATH 1271 Spring 2014, Midterm \#2
Handout date: Thursday 17 April 2014
Instructor: Scot Adams

PRINT YOUR NAME:

PRINT YOUR X. 500 ID:

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.
I. Multiple choice
A. (5 pts) (no partial credit) Let $f(x)=e^{2 x}+3 x$. What is the iterative formula of Newton's method used to solve $f(x)=0$ ? Circle one of the following answers:
(a) $x_{n+1}=x_{n}-\frac{e^{2 x_{n}}+3 x_{n}}{e^{2 x_{n}}+3}$
(b) $x_{n+1}=x_{n}-\frac{2 e^{2 x_{n}}+3}{e^{2 x_{n}}+3 x_{n}}$
(c) $x_{n+1}=x_{n}-\frac{e^{2 x_{n}}+3 x_{n}}{2 e^{2 x_{n}}+3}$
(d) $x_{n+1}=x_{n}-\frac{e^{2 x_{n}}+3}{e^{2 x_{n}}+3 x_{n}}$
(e) NONE OF THE ABOVE
B. (5 pts) (no partial credit) Let $y=x^{2}+x$. Compute $d y$, evaluated at $x=10, d x=0.1$. Circle one of the following answers:
(a) 1.2
(b) 2.1
(c) 1.22
(d) 2.11
(e) NONE OF THE ABOVE
C. (5 pts) (no partial credit) Find the derivative of $\left(2+x^{4}\right)^{\sin x}$ w.r.t. $x$. Circle one of the following answers:
(a) $\left[\left(2+x^{4}\right)^{\sin x}\right]\left[(\sin x)\left(\ln \left(2+x^{4}\right)\right)\right]$
(b) $\left[\left(2+x^{4}\right)^{\sin x}\right]\left[(\cos x)\left(4 x^{3} /\left(2+x^{4}\right)\right)\right]$
(c) $\left[\left(2+x^{4}\right)^{\sin x}\right]\left[(\sin x)\left(\ln \left(2+x^{4}\right)\right)+(\cos x)\left(4 x^{3} /\left(2+x^{4}\right)\right)\right]$
(d) $\left[\left(2+x^{4}\right)^{\sin x}\right]\left[(\cos x)\left(\ln \left(2+x^{4}\right)\right)+(\sin x)\left(4 x^{3} /\left(2+x^{4}\right)\right)\right]$
(e) NONE OF THE ABOVE
D. (5 pts) (no partial credit) Let $f(x)=\cot ^{2}\left(5 x^{4}+1\right)$. Compute $\int_{5}^{5} f(x) d x$. Circle one of the following answers:
(a) 20
(b) 6
(c) 2
(d) 0
(e) NONE OF THE ABOVE
E. (5 pts) (no partial credit) Suppose $f^{\prime \prime}(x)=(x-7)^{2}(x-8)^{4}$. At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".
(a) $f$ is concave down on $(-\infty, 7]$ and up on $[7, \infty)]$.
(b) $f$ is concave up on $(-\infty, 7]$ and down on $[7, \infty)]$.
(c) $f$ is concave up on $(-\infty, 7]$, down on $[7,8]$ and up on $[8, \infty)$.
(d) $f$ is concave down on $(-\infty, 7]$, up on $[7,8]$ and down on $[8, \infty)$.
(e) NONE OF THE ABOVE
F. (5 pts) (no partial credit) Let $f(x)=e^{3 x-4}$. Recall that $M_{2} S_{1}^{5} f$ denotes the midpoint Riemann sum, from 1 to 5 , of $f$, with two subintervals. Which of these is equal to $M_{2} S_{1}^{5} f$ ? Circle one of the following answers:
(a) $2\left(e^{2}+e^{8}\right)$
(b) $e^{2}+e^{8}$
(c) $2\left(e^{5}+e^{11}\right)$
(d) $e^{5}+e^{11}$
(e) NONE OF THE ABOVE
II. True or false (no partial credit):
a. (5 pts) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be any function such that $f^{\prime}(4)=0$ and $f^{\prime \prime}(4)<0$. Assume that $f^{\prime \prime}$ is defined on $\mathbb{R}$. Then $f$ has a global maximum at 4 .
b. (5 pts) Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be any two differentiable functions such that, for all $x \in \mathbb{R}$, $f^{\prime}(x)=g^{\prime}(x)$. Then $f-g$ is a constant.
c. $(5 \mathrm{pts})$ Assume that $\lim _{x \rightarrow a}[f(x)]=0=\lim _{x \rightarrow a}[g(x)]$. Assume also that $\lim _{x \rightarrow a} \frac{f^{\prime}(x)}{g^{\prime}(x)}=-\infty$. Then $\lim _{x \rightarrow a} \frac{f(x)}{g(x)}=-\infty$.
d. $(5 \mathrm{pts}) \frac{d}{d x}\left[\int_{1}^{x} \sin \left(e^{t}\right) d t\right]=\cos \left(e^{x}\right)$.
e. (5 pts) If $f$ is continuous on $[a, b]$, then $\int_{a}^{b}(f(x)) d x=\lim _{n \rightarrow \infty}\left[R_{n} S_{a}^{b} f\right]$.

## VERSION D

I. $A, B, C$
I. D,E,F
II. a,b,c,d,e
III. 1.
III. 2.
III. 3.
III. 4.
III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Find an antiderivative w.r.t. $x$ of $\cos ^{2}(2 x+3)$. (Hint: $\cos (2 \theta)=-1+2\left(\cos ^{2} \theta\right)$.)
2. (10 pts) Let $f(x)=\int_{2+5 x}^{1+e^{x}} \sqrt{t^{3}+1} d t$. Compute $f^{\prime}(0)$.
3. ( 15 pts ) We are asked to design a large cup in the shape of a cylinder. The cup is to have an open top, and must contain $2 \pi$ cubic feet of volume inside. Let $r$ be the radius of the top of the cup. On the interval $r>0$, find the choice of $r$ (in feet) that minimizes the surface area, $A$, of the cup. (HINT: Our local precalculus expert shows us the formula that relates $A$ to $r$. It is $A=\pi r^{2}+(4 \pi / r)$.)
4. (10 pts) A conical pile of sand is growing. Its height is always equal to the radius, $r$, of its base. Assume that its volume is always growing at a rate of $5 \pi$ cubic feet per minute. Find the rate of growth in $r$ (in feet per minute) at the moment when the volume is $9 \pi$ cubic feet. (HINT: According to our local precalculus expert, its volume, $V$, is given by $V=\pi r^{3} / 3$.)
