PRINT YOUR NAME:

SOLUTIONS
Version D

PRINT YOUR X.500 ID:

PRINT YOUR TA’S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.
I. Multiple choice

A. (5 pts) (no partial credit) Compute $\frac{d}{dx}[2e^3 + 5 \sin x]$. Circle one of the following answers:

(a) $5 \cos x$
(b) $-5 \cos x$
(c) $6e^2 + 5 \cos x$
(d) $6e^3 + 5 \cos x$
(e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) Compute $\left[ \frac{d}{dx} \left( \frac{e^x}{x^4 - 8x} \right) \right]$. Circle one of the following answers:

(a) $\frac{(e^x)(4x^3 - 8) - (x^4 - 8x)(e^x)}{(x^4 - 8x)^2}$
(b) $\frac{(x^4 - 8x)(e^x) - (e^x)(4x^3 - 8)}{(x^4 - 8x)^2}$
(c) $\frac{xe^{x-1}}{4x^3 - 8}$
(d) $\frac{e^x}{4x^3 - 8}$
(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Which is the intuitive definition of $\lim_{x \to \infty} (f(x)) = -\infty$? Circle one of the following answers:

(a) If $x$ is very positive, then $f(x)$ is very negative.
(b) If $x$ is very negative, then $f(x)$ is very positive.
(c) If $f(x)$ is very negative, then $x$ is very positive.
(d) If $f(x)$ is very positive, then $x$ is very negative.
(e) NONE OF THE ABOVE
D. (5 pts) (no partial credit) Compute $\Delta(x^3 - x^2)$. Circle one of the following answers:

(a) $3x^2 - 2x$

(b) $3x^2 + 3x(\Delta x) + (\Delta x)^2 - 2x - (\Delta x)$

(c) $3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - 2x(\Delta x)$

(d) $(3x^2 - 2x)(\Delta x)$

(e) NONE OF THE ABOVE

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E. (5 pts) (no partial credit) Let $f(t) = \tan^2 t$. Compute $f'(\pi/4)$.
(Hint: $f(t) = (\tan t)(\tan t)$.) Circle one of the following answers:

(a) $-\sqrt{2}/2$

(b) $-1$

(c) $1$

(d) $4$

(e) NONE OF THE ABOVE

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F. (5 pts) (no partial credit) Let $g(x) = [8 - 3x] \left[ \frac{x - 5}{x - 5} \right]$. What is the largest $\delta > 0$ such that $0 < |x - 5| < \delta \implies |(g(x)) + 7| < 0.6$? Circle one of the following answers:

(a) $0.3$

(b) $-0.3$

(c) $1.8$

(d) $0.2$

(e) NONE OF THE ABOVE
II. True or false (no partial credit):

a. (5 pts) \( \frac{d}{dx} \left[ \frac{\sin x}{x^2} \right] = \frac{\cos x}{2x} \).

\[ \text{False} \]

b. (5 pts) If \( f \) and \( g \) are both differentiable at 3, then \( 2f^9g^8 \) is also differentiable at 3.

\[ \text{True} \]

c. (5 pts) If \( P \) is any polynomial of degree 5 and \( Q \) is any polynomial of degree 3, then

\[ \lim_{x \to \infty} \left[ \frac{P(x)}{Q(x)} \right] = \infty. \]

\[ \frac{-x^5}{x^3} = -x^2 \xrightarrow{x \to \infty} -\infty \]

\[ \text{False} \]

d. (5 pts) \( \lim_{x \to 0} \frac{1 - \cos x}{x} = 0. \)

\[ \frac{1-\cos x}{x} \sim \frac{x^2/2}{x} \xrightarrow{x \neq 0} \frac{x}{2} \xrightarrow{x \to 0} 0 \]

\[ \text{True} \]

e. (5 pts) Let \( f \) and \( g \) be any two functions such that \( f'(5) = 50 \) and \( g'(3) = 30 \). Then \( (f - g)'(2) = 20. \)

\[ \text{False} \]
III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Compute \[ \frac{d}{dx} \left( \frac{(2x^3 + x)(4 + 7e^x)}{\cot x} \right). \]

\[ \cot x \left[(6x^2 + 1)(4 + 7e^x) + (2x^3 + x)(7e^x)\right] - [(2x^3 + x)(4 + 7e^x)]\left[-\csc^2 x\right] \]

\[ \cot^2 x \]
2. (10 pts) Compute \( \lim_{x \to 0} \frac{(\sin^2(4x))(\tan x)}{(\sin(2x))(\cos(3x))(3x^5 - 2x^4 - 4x^2)}. \)

\[
\begin{align*}
\lim_{x \to 0} & \quad \frac{(4x)^2 \left( \frac{x}{1} \right)}{(2x)(1)(-4x^2)} \\
& = \frac{16x^3}{-8x^3} \\
& = -2
\end{align*}
\]

\( \therefore \) \( x \neq 0 \)
3. Let \( f(x) = -x^6 + 6x^4 + (\tan(x)) \).

a. (5 pts) Find all \( a \in \mathbb{R} \) such that the graph of \( f \) has a horizontal tangent line at \((a, f(a))\).

\[
\begin{align*}
f'(x) &= -6x^5 + 24x^3 \\
      &= -6x^3(x^2 - 4) \\
      &= -6x^3(x+2)(x-2)
\end{align*}
\]

\( (a = 0) \) \quad \text{or} \quad \( (a = -2) \) \quad \text{or} \quad \( (a = 2) \)

b. (5 pts) Find all the maximal intervals on which \( f' \) is negative.

\[
\begin{array}{cccccc}
& & \text{pos} & 0 & \text{neg} & \text{pos} & 0 & \text{neg} \\
& -2 & & 0 & & 2 & & \\
\end{array}
\]

\( f' \) is negative on \((-2, 0)\)

and on \((2, \infty)\).
4. Let \( y = 3x^3 - 5x \). Then \( \Delta y = ax^2(\Delta x) + bx(\Delta x)^2 + c(\Delta x)^3 + k(\Delta x) \), for some real numbers \( a, b, c, k \).

a. (5 pts) Compute \( a, b, c \) and \( k \).

\[
\Delta (x^3) = 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3
\]

\[
\Delta y = 9x^2(\Delta x) + 9x(\Delta x)^2 + 3(\Delta x)^3 - 5(\Delta x)
\]

\[
\begin{array}{cccc}
a & b & c & k \\
11 & 12 & 11 & 11 \\
9 & 9 & 3 & -5 \\
\end{array}
\]

b. (5 pts) Assuming \( \Delta x \neq 0 \), compute \( \frac{\Delta y}{\Delta x} \).

\[
\frac{\Delta y}{\Delta x} \bigg|_{\Delta x \neq 0}
\]

\[
9x^2 + 9x(\Delta x) + 3(\Delta x)^2 - 5
\]

c. (5 pts) Compute \( \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \).

\[
\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 9x^2 - 5
\]