PRINT YOUR NAME:

SOLUTIONS

Version C

PRINT YOUR X.500 ID:

PRINT YOUR TA’S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.
I. Multiple choice

A. (5 pts) (no partial credit) Let \( y = x^2 + x \). Compute \( dy \), evaluated at \( x = 10, \Delta x = 0.1 \). Circle one of the following answers:
   
   (a) 1.22
   (b) 1.2
   (c) 2.11
   (d) 2.1
   (e) NONE OF THE ABOVE

\[
\left(2x + 1\right) \Delta x
\]

\[
(2 \cdot 10 + 1)(0.1) = (21)(0.1)
\]

\[= 2.1\]

B. (5 pts) (no partial credit) Find the derivative of \( (2 + x^4)^{\sin x} \) w.r.t. \( x \). Circle one of the following answers:

   (a) \([(2 + x^4)^{\sin x}][(\sin x)(\ln(2 + x^4)) + (\cos x)(4x^3/(2 + x^4))]\]
   \[\text{(b)}\][(2 + x^4)^{\sin x}][(\cos x)(\ln(2 + x^4)) + (\sin x)(4x^3/(2 + x^4))]\]
   (c) \([(2 + x^4)^{\sin x}][(\sin x)(\ln(2 + x^4))]\]
   (d) \([(2 + x^4)^{\sin x}][(\cos x)(4x^3/(2 + x^4))]\]
   (e) NONE OF THE ABOVE

\[
\frac{d}{dx} \left[ (2 + x^4)^{\sin x} \right] = \left[ \frac{\sin x}{\ln (2 + x^4)} \right] \left[ (2 + x^4)^{\sin x} \right]
\]

C. (5 pts) (no partial credit) Suppose \( f''(x) = (x - 7)^3(x - 8)^4 \). At most one of the following statements is true. If one is, circle it. Otherwise, circle “NONE OF THE ABOVE”.

   (a) \( f \) is concave down on \((-\infty, 7]\) and up on \([7, \infty])\).
   (b) \( f \) is concave up on \((-\infty, 7]\) and down on \([7, \infty])\).
   (c) \( f \) is concave up on \((-\infty, 7]\), down on \([7, 8]\) and up on \([8, \infty)]\).
   (d) \( f \) is concave down on \((-\infty, 7]\), up on \([7, 8]\) and down on \([8, \infty)]\).
   (e) NONE OF THE ABOVE

\[f'' \quad \text{neg} \quad 0^3 \quad \text{pos} \quad 0^4 \quad \text{pos}\]

\[
7 \quad - \quad 8
\]
D. (5 pts) (no partial credit) Let \( f(x) = e^{3x-4} \). Recall that \( L_2S_5^5 f \) denotes the left endpoint Riemann sum, from 1 to 5, of \( f \), with two subintervals. Which of these is equal to \( L_2S_5^5 f \)?

Circle one of the following answers:

(a) \( e^5 + e^{11} \)
(b) \( e^2 + e^8 \)
(c) \( 2(e^5 + e^{11}) \)
(d) \( 2(e^2 + e^8) \)
(e) NONE OF THE ABOVE

\[ f(1) = e^{3-4} = e^{-1} \]
\[ f(3) = e^{9-4} = e^5 \]

E. (5 pts) (no partial credit) Let \( f(x) = e^{2x} + 3x \). What is the iterative formula of Newton’s method used to solve \( f(x) = 0 \)? Circle one of the following answers:

(a) \( x_{n+1} = x_n - \frac{e^{2x_n} + 3x_n}{2e^{2x_n} + 3} \)
(b) \( x_{n+1} = x_n - \frac{2e^{2x_n} + 3}{e^{2x_n} + 3x_n} \)
(c) \( x_{n+1} = x_n - \frac{e^{2x_n} + 3x_n}{e^{2x_n} + 3x_n} \)
(d) \( x_{n+1} = x_n - \frac{e^{2x_n} + 3}{e^{2x_n} + 3x_n} \)
(e) NONE OF THE ABOVE

\[ x_{n+1} = x_n - \frac{e^{2x_n} + 3x_n}{2e^{2x_n} + 3} \]

F. (5 pts) (no partial credit) Let \( f(x) = \tan^2(5x^4 + 1) \). Compute \( \int_{-1}^{1} f(x) \, dx \). Circle one of the following answers:

(a) \(-1\)
(b) \(-\sqrt{2}/2\)
(c) 0
(d) 20
(e) NONE OF THE ABOVE
II. True or false (no partial credit):

a. (5 pts) Let \( f, g : \mathbb{R} \to \mathbb{R} \) be any two differentiable functions such that, for all \( x \in \mathbb{R} \), \( f'(x) = g'(x) \). Then \( f = g \).

\[ f(x) = \frac{1}{2}, \quad g(x) = 2 \quad \text{False} \]

b. (5 pts) Let \( f : \mathbb{R} \to \mathbb{R} \) be any function such that \( f''(2) = 0 \) and \( f''(2) > 0 \). Assume that \( f'' \) is defined on \( \mathbb{R} \). Then \( f \) has a local minimum at 2.

(2nd derivative test) \[ f \]

\( x = 2 \quad \text{True} \)

c. (5 pts) Assume that \( \lim_{x \to a} f(x) = 1 = \lim_{x \to a} g(x) \). Assume also that \( \lim_{x \to a} \frac{f'(x)}{g'(x)} = 3 \). Then

\[ \lim_{x \to a} \frac{f(x)}{g(x)} = 3 \quad \text{False} \]

\[ \frac{1}{1} = 1 \]

d. (5 pts) If \( f \) is continuous on \([a, b]\), then \( \int_a^b f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^n f(a_i) \Delta x \).

Definition of \( \int_a^b f(x) \, dx \quad \text{True} \)

e. (5 pts) \( \frac{d}{dx} \left[ \int_1^x \sin(e^t) \, dt \right] = \cos(e^x) \).

FTC \quad \text{False} \]

THE BOTTOM OF THIS PAGE IS FOR TOTALING SCORES
PLEASE DO NOT WRITE BELOW THE LINE

VERSION C

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1.

III. 2.

III. 3.

III. 4.
III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Find an antiderivative w.r.t. $x$ of $\cos^2(x-3)$. (Hint: $\cos(2\theta) = -1 + 2(\cos^2 \theta)$.)

$$\int \left[ \cos(2\theta) \right] = 2(\cos^2 \theta)$$

\[
\frac{1}{2} + \frac{1}{2} \left[ \cos(2\theta) \right] = \cos^2 \theta
\]

$\theta \rightarrow x - 3$

\[
\frac{1}{2} + \frac{1}{2} \left[ \cos(2x-6) \right] = \cos^2 (x-3)
\]

Antiderivative:

\[
\frac{1}{2} x + \frac{1}{2} \left[ \frac{\sin(2x-6)}{2} \right]
\]
2. (10 pts) Let \( f(x) = \int_0^{x^4} \sqrt{2t + 8t^2 + 6} \, dt \). Compute \( f'(1) \).

\[
G'(t)
\]

\[
f(x) \overset{\text{FTC}}{=} (G(x^4)) - (G(x))
\]

\[
f'(x) \overset{\text{CR}}{=} (G'(x^4))(4x^3) - (G'(x))
\]

\[
f'(1) = (G'(1))(4) - (G'(1))
\]

\[
= (G'(1))(4 - 1) = (G'(1))(3)
\]

\[
= (\sqrt{2 + 8 + 6})(3) = (\sqrt{16})(3)
\]

\[
= 12
\]
3. (15 pts) We are asked to design a large cup in the shape of an inverted (i.e., upside down) cone. The cup is to have an open top, and must contain \( \pi/3 \) cubic feet of volume inside. Let \( r \) be the radius of the top of the cup. On the interval \( r > 0 \), find the choice of \( r \) (in feet) that minimizes the surface area, \( A \), of the cup. (HINT: Our local precalculus expert shows us the formula that relates \( A \) to \( r \). It is \( A = (\pi r^2 + r^{-4}) \)).

\[
A = \pi^2 + r^{-4} 
\]

\[
\frac{dA}{dr} = \pi \sqrt{w^7} + \left( \pi r \right) \left( \frac{2n^{-4}n^{-5}}{2\sqrt{w^5}} \right) 
\]

\[
= \pi \left( \sqrt{\frac{w}{n}} \right) + \left( \pi r \right) \left( \frac{n^{-2} - 2n^{-5}}{\sqrt{n}} \right) 
\]

\[
= \pi \left( \frac{n^2 + n^{-4}}{\sqrt{n^7}} \right) + \pi \left( \frac{n^2 - 2n^{-4}}{\sqrt{n^5}} \right) 
\]

\[
= \pi \left( \frac{2n^2 - n^{-4}}{\sqrt{n^7}} \right) n^4 = \pi \left( \frac{2n^6 - 1}{n^4 \sqrt{n^3}} \right) 
\]

\[
\frac{dA}{dr} \quad \text{neg} \quad 0 \quad \text{pos} 
\]

\[
\begin{array}{c|c|c}
\text{r} & \text{neg} & \text{pos} \\
\hline
0 & 0 & \frac{1}{\sqrt{2}} \\
\end{array} 
\]

On \( n > 0 \), \( A \) attains a global minimum only at \( n = \frac{\sqrt{2}}{2} \) ft.
4. (10 pts) A square-based pyramid is growing. Its height is always equal to the length, $s$, of the sides of its base. Assume that its volume is always growing at a rate of 10 cubic feet per minute. Find the rate of growth in $s$ (in feet per minute) at the moment when the volume is 9 cubic feet. (HINT: According to our local precalculus expert, its volume, $V$, is given by $V = \frac{s^3}{3}$.)

$$9 = V_* = \frac{A_*^3}{3}$$

$$27 = A_*^3$$

$$3 = A_*$$

$$10 = \dot{V} = \frac{1}{3} A_*^2 \dot{s} / 3 = \frac{A_*^2}{3} \dot{s}$$

$$10 = A_*^2 \dot{s}_* = (9)(?)$$

$$? = \frac{10}{9} \text{ ft/min}$$