PRINT YOUR NAME:

PRINT YOUR X.500 ID:

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.
I. Multiple choice

A. (5 pts) (no partial credit) Let $y = x^2 + x$. Compute $dy$, evaluated at $x = 10$, $dx = 0.1$. Circle one of the following answers:

(a) 12
(b) 21
(c) 1.22
(d) 2.11
(e) NONE OF THE ABOVE

B. (5 pts) (no partial credit) Let $f(x) = e^{3x-4}$. Recall that $L_2S_5^1 f$ denotes the left endpoint Riemann sum, from 1 to 5, of $f$, with two subintervals. Which of these is equal to $L_2S_5^1 f$? Circle one of the following answers:

(a) $2(e^5 + e^{11})$
(b) $2(e^{-1} + e^5)$
(c) $e^5 + e^{11}$
(d) $2(e^2 + e^8)$
(e) NONE OF THE ABOVE

C. (5 pts) (no partial credit) Suppose $f''(x) = -(x - 7)^3(x - 8)^3$. At most one of the following statements is true. If one is, circle it. Otherwise, circle “NONE OF THE ABOVE”.

(a) $f$ is concave down on $(-\infty, 7]$ and up on $[7, \infty])$.
(b) $f$ is concave up on $(-\infty, 7]$ and down on $[7, \infty])$.
(c) $f$ is concave up on $(-\infty, 7]$, down on $[7, 8]$ and up on $[8, \infty)$.
(d) $f$ is concave down on $(-\infty, 7]$, up on $[7, 8]$ and down on $[8, \infty)$.
(e) NONE OF THE ABOVE
D. (5 pts) (no partial credit) Let \( f(x) = \cos^2(5x^4 + 1) \). Compute \( \int_3^3 f(x) \, dx \). Circle one of the following answers:

(a) \(-2\)
(b) 0
(c) 6
(d) 20
(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Let \( f(x) = e^{2x} + 3x \). What is the iterative formula of Newton’s method used to solve \( f(x) = 0 \)? Circle one of the following answers:

(a) \( x_{n+1} = x_n - \frac{e^{2x_n} + 3x_n}{2e^{2x_n} + 3x_n} \)
(b) \( x_{n+1} = x_n + \frac{2e^{2x_n} + 3}{e^{2x_n} + 3x_n} \)
(c) \( x_{n+1} = x_n - \frac{e^{2x_n} + 3x_n}{2e^{2x_n} + 3} \)
(d) \( x_{n+1} = x_n + \frac{2e^{2x_n} + 3x_n}{2e^{2x_n} + 3x_n} \)
(e) NONE OF THE ABOVE

F. (5 pts) (no partial credit) Find the derivative of \((2 + x^4)^{\cos x}\) w.r.t. \( x \). Circle one of the following answers:

(a) \( [(2 + x^4)^{\cos x}][(-\sin x)(\ln(2 + x^4)) + (\cos x)(4x^3/(2 + x^4))] \)
(b) \( [(2 + x^4)^{\cos x}][(-\sin x)(4x^3/(2 + x^4))] \)
(c) \( [(2 + x^4)^{\cos x}][\cos x(\ln(2 + x^4)) + (-\sin x)(4x^3/(2 + x^4))] \)
(d) \( [(2 + x^4)^{\cos x}][\cos x(\ln(2 + x^4))] \)
(e) NONE OF THE ABOVE
II. True or false (no partial credit):

a. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be any function such that $f'(8) = 0$ and $f''(8) > 0$. Assume that $f''$ is defined on $\mathbb{R}$. Then $f$ has a local maximum at 8.

b. (5 pts) Let $f, g : \mathbb{R} \to \mathbb{R}$ be any two differentiable functions such that, for all $x \in \mathbb{R}$, $f'(x) = g'(x)$. Then $f = g$.

c. (5 pts) Assume that $\lim_{x \to a} [f(x)] = 1 = \lim_{x \to a} [g(x)]$. Assume also that $\lim_{x \to a} \frac{f'(x)}{g'(x)} = 3$. Then $\lim_{x \to a} \frac{f(x)}{g(x)} = 3$.

d. (5 pts) $\frac{d}{dx} \left[ \int_1^x \sin(e^t) \, dt \right] = \sin(e^x)$.

e. (5 pts) If $f$ is continuous on $[a, b]$, then $\int_a^b (f(x)) \, dx = \lim_{n \to \infty} [M_n S_a^b f]$.
III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Find an antiderivative w.r.t. $x$ of $\sin^2(2x - 3)$. (Hint: $\cos(2\theta) = 1 - 2(\sin^2 \theta)$.)
2. (10 pts) Let $f(x) = \int_{2x-1}^{e^{x-1}} \sqrt{2t^6 - 2t^2 + 4} \, dt$. Compute $f'(1)$. 
3. (15 pts) We are asked to design a large cup in the shape of a cylinder. The cup is to have an open top, and must contain $2\pi$ cubic feet of volume inside. Let $r$ be the radius of the top of the cup. On the interval $r > 0$, find the choice of $r$ (in feet) that minimizes the surface area, $A$, of the cup. (HINT: Our local precalculus expert shows us the formula that relates $A$ to $r$. It is $A = \pi r^2 + (4\pi/r)$.)
4. (10 pts) A conical pile of sand is growing. Its height is always equal to the radius, \( r \), of its base. Assume that its volume is always growing at a rate of 10 cubic feet per minute. Find the rate of growth in \( r \) (in feet per minute) at the moment when the volume is 9\( \pi \) cubic feet. (HINT: According to our local precalculus expert, its volume, \( V \), is given by \( V = \pi r^3/3 \).)