CALCULUS
Continuity
OLD2
0210-1. a. At which numbers is the function $f$, shown above, discontinuous?

b. For each of the numbers, given in Part a, where $f$ is discontinuous, state whether or not $f$ is continuous from the LEFT at that number.

c. For each of the numbers, given in Part a, where $f$ is discontinuous, state whether or not $f$ is continuous from the RIGHT at that number.
0210-2 Display the graph of a function \( f \)

s.t. \( \lim_{x \to -1^-} f(x) = -3 \), \( \lim_{x \to -1^+} f(x) = 1 \),

and s.t. \( f(-1) = 1 \),

and s.t. \( \lim_{x \to 1} f(x) = -\infty \), \( f(1) = 2 \),

and s.t. \( \lim_{x \to 2} f(x) = 1 \), \( f(2) = 0 \),

and s.t. \( \lim_{x \to -\infty} f(x) = -1 \), \( \lim_{x \to \infty} f(x) = -4 \).
0210-3. Let \( f(t) = (4t^{2/3} + 3)^{85}. \)
Using the properties of limits, show that \( f \) is continuous at 7.

0210-4.
Let \( f(x) = \begin{cases} 
2x + 5, & \text{if } x < -1 \\
3, & \text{if } x = -1 \\
x^2 + 4, & \text{if } x > -1.
\end{cases} \)

a. Does \( \lim_{x \to -1} f(x) \) exist? If so, compute it.

b. Is \( f \) continuous from the left at \(-1\)?
Let $g(x) = \begin{cases} 
\cos(2x), & \text{if } x < 0 \\
1, & \text{if } x = 0 \\
x^2 + 1, & \text{if } x > 0.
\end{cases}$

a. Does $\lim_{x \to 0} g(x)$ exist? If so, compute it.

b. Is $g$ continuous at 0?
Let \( g(x) = \begin{cases} 
\cos(2x), & \text{if } x < 0 \\
1, & \text{if } x = 0 \\
x^2 + 1, & \text{if } x > 0.
\end{cases} \)

a. Does \( \lim_{{x \to -1}} g(x) \) exist? If so, compute it.

b. Is \( g \) continuous at \(-1\)?
0210-7. Let \( f(x) = \sqrt[3]{x} \).

a. Is \( f \) continuous at 0?
b. Is \( f \) continuous on \([0, \infty)\)?
c. Is \( f \) continuous?

0210-8. Let \( g(x) = \frac{1}{\sqrt[3]{x}} \).

a. Is \( g \) continuous at 0?
b. Is \( g \) continuous on \((0, \infty)\)?
c. Is \( g \) continuous?

0210-9. Compute \( \lim_{{x \to 27}} \frac{x + \sqrt[3]{x}}{(x - 20)^2 - 2x + 6} \).
Let \( f(x) = \begin{cases} 
  x^2 + 3, & \text{if } x < 2 \\
  2x + 2, & \text{if } 2 \leq x < 3 \\
  8[\cos(x - 3)], & \text{if } 3 \leq x. 
\end{cases} \)

a. At which numbers is the function \( f \) discontinuous?

b. For each of the numbers, given in Part a, where \( f \) is discontinuous, state whether or not \( f \) is continuous from the LEFT at that number.

c. For each of the numbers, given in Part a, where \( f \) is discontinuous, state whether or not \( f \) is continuous from the RIGHT at that number.
Let \( g(x) = \begin{cases} 
4e^x, & \text{if } x \leq 0 \\
(x + 2)^2, & \text{if } 0 < x < 1 \\
7x + 2, & \text{if } 1 < x.
\end{cases} \)

a. At which numbers is the function \( g \) discontinuous?

b. For each of the numbers, given in Part a, where \( g \) is discontinuous, state whether or not the discontinuity is removable.
0210-12. Find a number $a$ s.t.

$$f(x) = \begin{cases} 
  ae^x, & \text{if } x \leq 0 \\
  ax^3 + 3a + 8, & \text{if } 0 < x 
\end{cases}$$

is continuous at $x = 0$.

0210-13. Let $h(s) = \frac{s^2 + 5s - 6}{s - 1}$.

Find a function $p : \mathbb{R} \to \mathbb{R}$ such that $p$ is continuous at 1 and such that, \( \forall s \in \mathbb{R} \setminus \{1\} \), \( p(s) = h(s) \).
0210-14.
Using the Intermediate Value Theorem, show that \( x^3 + 2x - 8 = 0 \) has a solution \( x = c \) that satisfies \(-2 < c < 2\).

0210-15.
Using the Intermediate Value Theorem, show that \( 4e^x + \cos x = x + 6 \) has a sol’n \( x = c \) that satisfies \(-2 < c < 9\).