CALCULUS
Maxima and minima
OLD2
0450-1. **a.** Sketch the graph of a continuous function whose domain is $[2, \infty)$, and which has exactly two global minima, and **exactly one** local minimum.

**b.** Sketch the graph of a continuous function whose domain is $(2, 4]$, and which has **exactly two** local minima, and **no** global minima.
0450-2. Let \( f : [a, z] \to \mathbb{R} \) be the function whose graph is shown below.

i. For each number \( a, b, c, d, e, r, s, t, u, v, w, x, z \), state whether or not \( f \) has, at that number,
- a global maximum
- a global minimum
- a local maximum
- a local minimum.

ii. Which of \( a, b, c, d, e, r, s, t, u, v, w, x, z \) is a critical number for \( f \)?
0450-3. Let $f : [-4, 4] \to \mathbb{R}$ be the function whose graph is displayed below.

a. At what numbers $x$ does $f(x)$ have a local minimum?

b. What are the corresponding local minimum values?

c. At what numbers $x$ does $f(x)$ have a local maximum?

d. What are the corresponding local maximum values?
0450-4. Let \( f : [-4, 4] \rightarrow \mathbb{R} \) be the function whose graph is displayed below.

a. At what numbers \( x \) does \( f(x) \) have a global minimum?

b. What is the corresponding global minimum value?

c. At what numbers \( x \) does \( f(x) \) have a global maximum?

d. What is the corresponding global maximum value?
Sketch a graph of a continuous function whose domain is \((-3, 2)\) and which has no global maxima, two local maxima, and two global minima.

Sketch a graph of a function whose domain is \([1, 4]\), which is continuous on \([1, 4]\) and which has two global maxima, one local maximum and two local minima.
0450-7. Define \( f : [-2, 2] \rightarrow \mathbb{R} \) by \( f(x) = x^4 \).

a. Sketch the graph of \( f \).

b. Does \( f \) have a global maximum? If so, at what number(s)?

c. Does \( f \) have a global minimum? If so, at what number(s)?

d. Does \( f \) have a local maximum? If so, at what number(s)?

e. Does \( f \) have a local minimum? If so, at what number(s)?

f. What are the critical numbers of \( f \)?
0450-8. Define \( f : (-2, 2) \rightarrow \mathbb{R} \) by \( f(x) = x^4 \).

a. Sketch the graph of \( f \).

b. Does \( f \) have a global maximum? If so, at what number(s)?

c. Does \( f \) have a global minimum? If so, at what number(s)?

d. Does \( f \) have a local maximum? If so, at what number(s)?

e. Does \( f \) have a local minimum? If so, at what number(s)?

f. What are the critical numbers of \( f \)?
Define \( f : [-2, 2] \rightarrow \mathbb{R} \) by \( f(x) = x \).

a. Sketch the graph of \( f \).

b. Does \( f \) have a global maximum? If so, at what number(s)?

c. Does \( f \) have a global minimum? If so, at what number(s)?

d. Does \( f \) have a local maximum? If so, at what number(s)?

e. Does \( f \) have a local minimum? If so, at what number(s)?

f. What are the critical numbers of \( f \)?
0450-10. Define \( f : (-2, 2) \rightarrow \mathbb{R} \) by \( f(x) = x \).

a. Sketch the graph of \( f \).

b. Does \( f \) have a global maximum? If so, at what number(s)?

c. Does \( f \) have a global minimum? If so, at what number(s)?

d. Does \( f \) have a local maximum? If so, at what number(s)?

e. Does \( f \) have a local minimum? If so, at what number(s)?

f. What are the critical numbers of \( f \)?
0450-11. Define $f : (-2, 2) \to \mathbb{R}$ by
\[ f(x) = 3 - 2x^2. \]

a. Sketch the graph of $f$.

b. Does $f$ have a global maximum? If so, at what number(s)?

c. Does $f$ have a global minimum? If so, at what number(s)?

d. Does $f$ have a local maximum? If so, at what number(s)?

e. Does $f$ have a local minimum? If so, at what number(s)?

f. What are the critical numbers of $f$?
0450-12. Define $f : [-2, 2] \rightarrow \mathbb{R}$ by
\[
    f(x) = 3 - 2x^2.
\]

a. Sketch the graph of $f$.
b. Does $f$ have a global maximum? If so, at what number(s)?
c. Does $f$ have a global minimum? If so, at what number(s)?
d. Does $f$ have a local maximum? If so, at what number(s)?
e. Does $f$ have a local minimum? If so, at what number(s)?
f. What are the critical numbers of $f$?
0450-13. Find the critical numbers of
\[ f(x) = 2x^3 + 3x^2 - 72x + 12. \]

0450-14. Find the critical numbers of
\[ f(x) = |x^2 - 10x + 21|. \]

0450-15. Find the critical numbers of
\[ f(x) = |x^2 - 2x + 1|. \]

0450-16. Find the critical numbers of
\[ f(x) = \cos x. \]

0450-17. Find the critical numbers of
\[ f(x) = |\cos x|. \]
0450-18. Find the global maximum and minimum values of
\[ f(x) = x^3 - 15x^2 + 48x - 6 \]
on \( 0 \leq x \leq 10 \).

0450-19. Find the global maximum and minimum values of
\[ g(t) = \frac{3t}{t^2 + 9} \]
on \( -5 \leq t \leq 5 \).

0450-20. Find the global maximum and minimum values of
\[ H(s) = se^{-s^2/8} \]
on \( -4 \leq s \leq 8 \).