CALCULUS
Even more graphing problems
OLD2
0500-1. Let $f : [0, 15) \setminus \{8\} \to \mathbb{R}$ be as shown.

a. Find the maximal intervals on which
   (i) $f$ is increasing;
   (ii) $f$ is decreasing;
   (iii) $f$ is concave up;
   and (iv) $f$ is concave down.

b. Find all points of inflection for $f$. 
0500-2. Let $f : [0, 15) \setminus \{8\} \to \mathbb{R}$ be continuous from the right at 0. The graph of $f'$ is shown below.

Find the maximal intervals on which

(i) $f$ is concave up;
and (ii) $f$ is concave down.
0500-3. Let \( f : (0, 14] \to \mathbb{R} \) be as shown.

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.
0500-4. Let $f$ be continuous on $(0, 14]$.
The graph of $f'$ is shown below.

a. Find the maximal intervals on which
   (i) $f$ is concave up;
   and (ii) $f$ is concave down.

b. At what numbers does $f$ have
   (i) a local maximum?
   (ii) a local minimum?
Let \( f(x) = 3x^4 + 8x^3 - 48x^2 + 4 \).

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.

c. Find the maximal intervals on which
   (i) \( f \) is concave up;
   and (ii) \( f \) is concave down.
Let \( f(x) = (x^2 - 8)e^{-x} \).

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.

c. Find the maximal intervals on which
   (i) \( f \) is concave up;
   and (ii) \( f \) is concave down.

d. Find all points of inflection for \( f \).
Let \( f(x) = (x + 2)e^{-x^2/6} \).

a. Find all critical numbers for \( f \).

b. For each critical number for \( f \), use the Second Derivative Test to determine whether, at that number, the function \( f \) has a local maximum or a local minimum.
0500-8. Let $f(x) = x^5 e^{x^2}$.

a. Find all critical numbers for $f$.

b. For each critical number for $f$, what does the Second Derivative Test tell you about that critical number?

c. For each critical number for $f$, use the First Derivative Test to determine whether, at that number, the function $f$ has a local maximum or a local minimum.
Sketch the graph of a function

\[ H : [0, 8] \rightarrow \mathbb{R} \]

with the following properties:

- (●) \( H \) is continuous on \([0, 8]\);
- (●) \( H'' \) is continuous on \((0, 8)\);
- (●) \( H(0) = H(4) = H(8) = 0 \);
- (●) \( H'(2) = H'(6) = 0 \);
- (●) \( H'' > 0 \) on \((0, 4)\);
- (●) \( H'' < 0 \) on \((4, 8)\).

Find a cubic \( g(t) = at^3 + bt^2 + ct + d \)

s.t. \( g \) attains a local max value of 20 at \(-3\)

and a local min value of \(-88\) at 3.
Let \( f(x) = 2 + \cos^2(3x) \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).
   
   Also:
   
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-12. Let \( f(x) = \ln(x^2 + 4) \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-13. Let \( f(x) = \frac{x}{\sqrt{x^2 - 4}} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-14. Let \( f(x) = x^4 + 5x^3 \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).
   Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-15. Let \( f(x) = \frac{1}{x^2 - 9} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-16. Let \( f(x) = \sqrt{x^2 + 4x - 5} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-17. Let \( f(x) = 2x - \sin x \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?

(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).

(iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
Let \( f(x) = xe^{-x^2/8} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).
   Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-19. Let \( f(x) = \frac{x^2 + 3x + 12}{x - 1} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).