CALCULUS
Definite integration and Riemann sum problems
OLD2
0590-1. Let \( f(x) = 5 - 5x^2 \).

a. Compute \( L_3S^1_{-1}f \).
   Sketch \( f \) over \([-1, 1]\) and add, into your sketch, the three rectangles represented by \( L_3S^1_{-1}f \).

b. Compute \( M_3S^1_{-1}f \).
   Sketch \( f \) over \([-1, 1]\) and add, into your sketch, the three rectangles represented by \( M_3S^1_{-1}f \).

c. Compute \( R_3S^1_{-1}f \).
   Sketch \( f \) over \([-1, 1]\) and add, into your sketch, the three rectangles represented by \( R_3S^1_{-1}f \). \( ^2 \)
0590-2. Let \( f(x) = e^x + 4 \).

a. Compute \( L_3 S_0^3 f \) to three decimal places.
b. Compute \( M_3 S_0^3 f \) to three decimal places.
c. Compute \( R_3 S_0^3 f \) to three decimal places.

0590-3. Let \( f(x) = \sin^2 x \).

a. Compute \( L_4 S_0^{\pi} f \) to three decimal places.
b. Compute \( M_4 S_0^{\pi} f \) to three decimal places.
c. Compute \( R_4 S_0^{\pi} f \) to three decimal places.
A car’s acceleration is positive from time 0 to time 18 seconds, and its velocity at various times is given in the table below.

<table>
<thead>
<tr>
<th>time (secs)</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity (ft/sec)</td>
<td>0</td>
<td>30</td>
<td>50</td>
<td>65</td>
<td>75</td>
<td>80</td>
<td>82</td>
</tr>
</tbody>
</table>

Find upper and lower estimates for the distance traveled by the car over these 18 seconds.
The graph of a function $f$ appears below.

Estimate $\int_{0}^{10} f(x) \, dx$ by computing

(a) $L_5 S_0^{10} f$,  \hspace{0.5cm} (b) $M_5 S_0^{10} f$

and (c) $R_5 S_0^{10} f$.  \hspace{0.5cm} 5
0590-6. Express the area under $y = e^{-x^2/2}$ from $x = -1$ to $x = 3$ as a limit of midpoint Riemann sums. (Don’t evaluate the limit.)

0590-7. Express the area under $y = \sqrt{x^3 + x + 9}$ from $x = 2$ to $x = 4$ as a limit of left endpoint Riemann sums. (Don’t evaluate the limit.)

0590-8. Express the area under $y = \cos(x^3 + x)$ from $x = -3$ to $x = 5$ as a limit of right endpt Riemann sums. (Don’t evaluate the limit.)
0590-9. Express \( \int_4^6 \frac{e^{-x^2/2}}{\sqrt{2\pi}} \, dx \) as a limit of midpoint Riemann sums. (Don’t evaluate the limit.)

0590-10. Let \( f(x) = x^3 - x \).

a. Write \( R_n S_0^2 f \) as a rational expression in \( n \) (i.e., as one polynomial in \( n \) divided by another).

b. Compute \( \lim_{n \to \infty} R_n S_0^2 f \).
The limit

$$\lim_{n \to \infty} \left[ \frac{9}{n} \sum_{j=0}^{n-1} \left( e^{- \cos(7 + j(9/n))} \right) \right]$$

represents the area under $y = f(x)$

from $x = a$ to $x = b$, for some choice of $f(x)$, $a$ and $b$.

a. Find $f(x)$, $a$ and $b$.

b. Express the limit as a definite integral.
0590-12. The limit

\[
\lim_{n \to \infty} \left[ \frac{5}{n} \sum_{j=1}^{n} \sin \left( e^{-2+j(5/n)} \right) \right]
\]

represents the area under \( y = f(x) \) from \( x = a \) to \( x = b \), for some choice of \( f(x) \), \( a \) and \( b \).

a. Find \( f(x) \), \( a \) and \( b \).

b. Express the limit as a definite integral.
Let \( f(x) = 2 + \sqrt{9 - x^2} \).

a. Sketch the graph of \( y = f(x) \).

b. Compute \( \int_{-3}^{3} f(x) \, dx \), by interpreting this integral as an area.