CALCULUS
The limit game and
the exact definition of a limit
OLD2
0150-1. For the function \( g \) graphed below, what is the largest number \( \delta \) such that

\[
0 < |t - 4| < \delta \quad \Rightarrow \quad |(g(t)) - 5| < 0.6
\]

**ANSWER:**

\[
4 - 3.3 = 0.7 \quad \text{and} \quad 6.5 - 4 = 2.5
\]

Since \( \delta \) must be smaller than both, the largest possibility is

\[
\delta = 0.7.
\]
0150-2. Let \( f(x) = 3x - 4 \).

Show a graph of \( y = f(x) \) that includes the points \((1, -1), (2, 2)\) and \((3, 5)\).

Find the largest number \( \delta \) such that
\[
|x - 2| < \delta \implies |(f(x)) - 2| < 0.9.
\]

\[\text{ANSWER:} \]

\[
\varepsilon = 0.9 \\
\varepsilon = 0.9
\]

\[
\text{run} = \frac{\text{rise}}{\text{slope}} = \frac{\pm 0.9}{3} = \pm 0.3
\]

The largest possibility: \( \delta = 0.3 \)
Let \( g(x) = [3x - 4] \left[ \frac{x - 2}{x - 2} \right] \).

Show a graph of \( y = g(x) \) that includes the points \((1, -1)\) and \((3, 5)\).

Find the largest number \( \delta \) such that
\[
0 < |x - 2| < \delta \quad \Rightarrow \quad |(g(x)) - 2| < 0.9.
\]

**ANSWER:**

\[
\text{run} = \frac{\text{rise}}{\text{slope}} = \pm \frac{0.9}{3} = \pm 0.3
\]

The largest possibility: \( \delta = 0.3 \).
In shop class, you are asked to build a square sheet of metal of area 400 square inches. The area can be slightly off, but must be between 399 and 401 square inches. Say you have access to a machine that will punch out a perfect square, and the side length (in inches) is controlled by a dial. How close to 20 must you set the dial to get the area to be in the specified range?

**Answer:** 399 < $s^2$ < 401

19.97498 ≈ $\sqrt{399}$ < $s$ < $\sqrt{401}$ ≈ 20.02498

20.02498 – 20 = 0.02498

20 – 19.97498 = 0.02502

Must be within 0.02498.
0150-5. Prove that \( \lim_{x \to 6} 4x = 24 \).

Your writeup should read:

Given \( \varepsilon > 0 \).

Let \( \delta = \ldots \).

Assume \( 0 < |x - 6| < \delta \).

Then \( |4x - 24| < 4\delta \).

Then \( |4x - 24| < \varepsilon \).

All you need do is fill in the ellipsis (\( \ldots \)) with a carefully chosen expression of \( \varepsilon \).

**Hint:** The last sentence in the writeup clearly follows from the penultimate sentence if \( 4\delta = \varepsilon \).

**ANSWER:** \( \delta = \frac{\varepsilon}{4} \)