CALCULUS
The derivative of a function is a function
OLD2
The graph of $f$ is shown above. Which of the following is the graph of $f'$?

Choose red, green or purple.
The graph of $f$ is shown above. Which of the following is the graph of $f'$?

**Answer:** Choose red, green or purple.
The graph of $f$ is shown above. Which of the following is the graph of $f'$?

Choose red, green or purple.
The graph of \( f \) is shown above. Which of the following is the graph of \( f' \)?

**ANSWER:**

Choose red, green or **purple**.
The graph of $f$ is shown above. 
Freehand a sketch of the graph of $f'$. 
On your graph, indicate 1 and $-1$ on the horizontal axis.
0280-3.

ANSWER:
The graph of \( f \) is shown above.

a. At which of the numbers \(-3, -2, -1, 0, 1, 2, 3\) is \( f \) not defined?

b. At which of the numbers \(-3, -2, -1, 0, 1, 2, 3\) is \( f \) not continuous?

c. At which of the numbers \(-3, -2, -1, 0, 1, 2, 3\) is \( f \) not differentiable?
The graph of $f$ is shown above.

a. At which of the numbers $-3, -2, -1, 0, 1, 2, 3$ is $f$ not defined?  \( \text{ANS: } -1, 2 \)

b. At which of the numbers $-3, -2, -1, 0, 1, 2, 3$ is $f$ not continuous?  \( \text{ANS: } -1, 2, 3 \)

c. At which of the numbers $-3, -2, -1, 0, 1, 2, 3$ is $f$ not differentiable?  \( \text{ANS: } -2, -1, 1, 2, 3 \)
The graphs of $f$, $f'$ and $f''$ are shown above. Which is which?

State the color of $f$, the color of $f'$ and the color of $f''$. 

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The graphs of $f$, $f'$ and $f''$ are shown above. Which is which?

State the color of $f$, the color of $f'$ and the color of $f''$. 
0280-6. Let \( f(s) = 5s^3 - 4s \).

a. What is the domain of \( f \)?

b. Using the definition of the derivative, and using the cubic binomial formula
\[
(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3,
\]
compute \( f'(s) \).

c. What is the domain of the derivative \( f' \)?
0280-6. Let $f(s) = 5s^3 - 4s$.

a. What is the domain of $f$?

**ANSWER:**

a. $\mathbb{R} = (-\infty, \infty)$
Let $f(s) = 5s^3 - 4s$.

b. Using the definition of the derivative, and using the cubic binomial formula

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3,$$

compute $f'(s)$.

**ANSWER:**

\[
\begin{align*}
\text{b. } f'(s) &= \lim_{h \to 0} \frac{[5(s + h)^3 - 4(s + h)] - [5s^3 - 4s]}{h} \\
&= \lim_{h \to 0} \frac{[5(s^3 + 3s^2h + 3sh^2 + h^3) - 4(s + h)] - [5s^3 - 4s]}{h} \\
&= \lim_{h \to 0} \frac{5(3s^2h + 3sh^2 + h^3) - 4h}{h} \\
&= \lim_{h \to 0} 5(3s^2 + 3sh + h^2) - 4 \\
&= 5(3s^2) - 4 = 15s^2 - 4
\end{align*}
\]
0280-6. Let \( f(s) = 5s^3 - 4s \).

c. What is the domain of the derivative \( f' \)?
0280-7. Let \( f(x) = \frac{1 + 3x}{4 - 2x} \).

a. What is the domain of \( f \)?

b. Using the definition of the derivative, compute \( f'(x) \).

c. What is the domain of the derivative \( f' \)?
Let \( f(x) = \frac{1 + 3x}{4 - 2x} \).

a. What is the domain of \( f \)?

**ANSWER:**

a. \( \mathbb{R} \setminus \{2\} \)
0280-7. Let \( f(x) = \frac{1 + 3x}{4 - 2x} \).

b. Using the definition of the derivative, compute \( f'(x) \).

\[
\begin{align*}
\text{ANS: b.} \quad & \frac{(f(x + h)) - (f(x))}{h} = \frac{1}{h} \left[ \frac{1 + 3x + 3h}{4 - 2x - 2h} - \frac{1 + 3x}{4 - 2x} \right] \\
& = \frac{1}{h} \left[ \frac{(1 + 3x + 3h)(4 - 2x) - (4 - 2x - 2h)(1 + 3x)}{(4 - 2x - 2h)(4 - 2x)} \right] \\
& = \frac{3h(4 - 2x) + 2h(1 + 3x)}{h(4 - 2x - 2h)(4 - 2x)} \\
& = \frac{14h}{h(4 - 2x - 2h)(4 - 2x)} \\
& \quad \quad \quad \quad \quad \quad \xrightarrow{h \to 0} \frac{14}{(4 - 2x)^2}
\end{align*}
\]
Let \( f(x) = \frac{1 + 3x}{4 - 2x} \). 

c. What is the domain of the derivative \( f' \)?

**ANSWER:** 
\[ f'(x) = \frac{14}{(4 - 2x)^2} \]

c. \( \mathbb{R} \setminus \{2\} \)
Let \( f(x) = |x^2 - 3x - 4| \).

At which numbers is \( f \) not differentiable?

**Hint:** Determine the (maximal) intervals where \( x^2 - 3x - 4 \)

is positive and negative.

Sketch the graph of \( y = x^2 - 3x - 4 \).

Sketch the graph of \( y = f(x) \).

**GENERAL RULE:**

At numbers \( x \) where \( x^2 - 3x - 4 \) has a root of multiplicity one, \( f \) is not differentiable. Everywhere else, \( f \) is differentiable.
Let \( f(x) = |x^2 - 3x - 4| \).

At which numbers is \( f \) not differentiable?

**Answer:**

\( x^2 - 3x - 4 = (x + 1)(x - 4) \)

- Positive on \( 4 < x \)
- Negative on \( -1 < x < 4 \)
- Positive on \( x < -1 \)

\[ y = x^2 - 3x - 4 \]
0280-8. Let \( f(x) = |x^2 - 3x - 4| \).

At which numbers is \( f \) not differentiable?

**ANSWER:** \( x^2 - 3x - 4 = (x + 1)(x - 4) \)
- positive on \( 4 < x \)
- negative on \( -1 < x < 4 \)
- positive on \( x < -1 \)

\[ y = f(x) = |x^2 - 3x - 4| \]

\( f \) is not differentiable at \(-1\) and \(4\).

**GENERAL RULE:**
At numbers \( x \) where \( x^2 - 3x - 4 \) has a root of multiplicity one, \( f \) is **not** differentiable. Everywhere else, \( f \) is differentiable.
0280-9. Let \( f(x) = |x^4 - 3x^3 - 4x^2| \).
At which numbers is \( f \) not differentiable?

Hint:
\( y = x^4 - 3x^3 - 4x^2 \) is hard to graph, but you don't have to; just use the...

GENERAL RULE:
At numbers \( x \) where \( x^4 - 3x^3 - 4x^2 \) has a root of multiplicity one, \( f \) is not differentiable. Everywhere else, \( f \) is differentiable.
0280-9. Let \( f(x) = |x^4 - 3x^3 - 4x^2| \).

At which numbers is \( f \) not differentiable?

**ANSWER:** \( x^4 - 3x^3 - 4x^2 = (x + 1)x^2(x - 4) \)

\( x^4 - 3x^3 - 4x^2 \) has a root of multiplicity one at \( x = -1 \) and \( x = 4 \),

so

\( f \) is not differentiable at \(-1\) and \(4\).

Everywhere else, \( f \) is differentiable.

**GENERAL RULE:**

At numbers \( x \) where \( x^4 - 3x^3 - 4x^2 \) has a root of multiplicity one, \( f \) is not differentiable.

Everywhere else, \( f \) is differentiable.