CALCULUS
Antidifferentiation problems
OLD2
0560-1. Find all antiderivatives in $x$ of

$$-2x^3 + x^2 + 5.$$

**ANSWER:**

all antiderivatives: $$-rac{x^4}{2} + rac{x^3}{3} + 5x + C$$

0560-2. Find all antiderivatives in $t$ of

$$\left(-2 \sqrt[4]{t} + 7 \sqrt[6]{t}\right) t^3.$$

**ANS:**

$$\left(-2 \sqrt[4]{t} + 7 \sqrt[6]{t}\right) t^3 = -2t^{13/4} + 7t^{19/6}$$

all antiderivatives: $$-\frac{2t^{17/4}}{17/4} + \frac{7t^{25/6}}{25/6} + C$$
0560-3. Find all antiderivatives in $t$ of
\[
\frac{3\sqrt{t} + 8\sqrt[5]{t}}{\sqrt[5]{t}}.
\]

**ANSWER:**
\[
\frac{3\sqrt{t} + 8\sqrt[5]{t}}{\sqrt[5]{t}} = t^{2/15} + 8t^{-2/35}
\]

all antiderivatives: \[
\frac{t^{17/15}}{17/15} + \frac{8t^{33/35}}{33/35} + C
\]

0560-4. Find all antiderivatives in $\nu$ of
\[
\frac{4e^{\nu} - \cos \nu}{3}.
\]

**ANSWER:** all antiderivatives: \[
\frac{4e^{\nu} - \sin \nu}{3} + C
\]
0560-5. Find the unique \( f(x) \) such that
\[
f'(x) = -5x^4 - 9x^2 + 2 \quad \text{and} \quad f(0) = 4.
\]
**ANSWER:**
\[
f(x) = -x^5 - 3x^3 + 2x + 4
\]

0560-6. Find the unique \( f(x) \) such that
\[
f'(x) = \frac{3x^2 + 4}{x \sqrt[6]{x}} \quad \text{and} \quad f(1) = 0.
\]
**ANSWER:**
\[
f'(x) = \frac{3x^2 + 4}{x \sqrt[6]{x}} = 3x^{5/6} + 4x^{-7/6}
\]
\[
f(x) = \frac{3x^{11/6}}{11/6} + \frac{4x^{-1/6}}{-1/6} - \frac{3}{11/6} - \frac{4}{-1/6}
\]
0560-7. Find the unique \( h(t) \) such that \( h'(t) = 2 \sin t - 7 \cos t \) and \( h(0) = -4 \).

**ANSWER:**

\[ h(t) = -2 \cos t - 7 \sin t - 2 \]

0560-8. Find the unique \( p(t) \) such that \( p''(t) = -e^t + 12t^3, \ p'(0) = 3 \) and \( p(0) = 1 \).

**ANSWER:**

\[ p'(t) = -e^t + 3t^4 + 4 \]

\[ p(t) = -e^t + \frac{3t^5}{5} + 4t + 2 \]
0560-9. The graph of $f$ is shown below.

Which of the following could be the graph of an antiderivative of $f$?
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**ANSWER:**
0560-10. The graph of \( f \) is shown below.

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**ANSWER:**
The graph of $f$ is shown below.

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**ANSWER:**
The graph of $f$ is shown below.

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**Answer:**
0560-13. A particle travels on a number line.

Suppose its acceleration at time \( t \) is \( 3t^2 + 2t - 6 \), its position at time 0 is 2 and its velocity at time 0 is \(-3\).

Find an expression for its position at time \( t \).

**ANSWER:**

velocity at time \( t \): \[ t^3 + t^2 - 6t - 3 \]

position at time \( t \): \[ \frac{t^4}{4} + \frac{t^3}{3} - 3t^2 - 3t + 2 \]
We drop a heavy ball out of a window in a tall building. Its speed at the moment of impact with the ground is 160 feet per second. From what height was it dropped?

**Answer:**

- Acceleration is: \( 32 \text{ ft/sec}^2 \)
- Velocity at time \( t \) seconds after release is: \( 32t \text{ ft/sec} \)
- Distance fallen in the first \( t \) seconds after release is: \( 16t^2 \text{ ft} \)

Let \( t_0 \) be the number of seconds between release and impact.

\[ 32t_0 = 160, \text{ so } t_0 = 5 \text{ seconds.} \]

Distance from release point to the ground is

\[ 16t_0^2 = 16 \cdot 5^2 = 400 \text{ ft.} \]