CALCULUS
Average rates of change
NEW
Water is being drained from a tub; the amount in the tub is constantly monitored, and is tabulated against time as follows:

<table>
<thead>
<tr>
<th>hrs</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>liters</td>
<td>101</td>
<td>79</td>
<td>33</td>
<td>25</td>
</tr>
</tbody>
</table>

Let $W$ be the amount in the tank at time $t$.
Let $B = (6, 33)$, a point on the graph of $W$.

a. Find the slope of the secant lines between $B$ and the other points on the graph of $W$ appearing in the table above.

b. Estimate the slope of the tangent line to the graph of $W$ at the point $B$, by averaging the following two numbers:
   - the slope of the secant line between $B$ and $(4, 79)$
   - and the slope of the secant line between $B$ and $(8, 25)$. 
Let $A$ be the point $(1, 4)$ on the graph of $y = 5 - x^3$. Let $B$ be a variable point $(x, 5 - x^3)$ on the same graph.

a. Compute the slope of the secant line between $A$ and $B$, when $x$ is equal to

   (i) $2$  
   (ii) $1.1$  
   (iii) $1.01$  
   (iv) $0$  
   (v) $0.9$  
   (vi) $0.99$  
   (vii) $1 + h$, with $h \neq 0$

b. Guess the slope of the tangent line to $y = 5 - x^3$ at $A$.

c. Using b, write an equation of the tangent line to $y = 5 - x^3$ at $A$. 
A tennis player, in a fit of rage over a lost point, throws his racquet into the air. Assume that its distance, in feet, above the ground, \( t \) seconds later, is \( 8 + 30t - 16t^2 \).

a. Find its average velocity over the time period starting at time 1, and continuing for the following number of seconds:

(i) 1  (ii) 0.5  (iii) 0.01
(iv) 0.001  (v) 0.0001  (vi) 0.00005
(vii) \( \Delta t \), with \( \Delta t \neq 0 \)

b. Guess its instantaneous velocity 1 second after it’s thrown.